Errata and Updates for the 1st Edition 2nd Printing of the ACTEX Manual for Exam FAM-S

(Last updated 03/06/2024)

Page 22 **Second paragraph from the bottom.**

Change it to:

If $\theta > 0$ is a real number, then the exponential distribution with mean θ has pdf $f(x) = \frac{e^{-\frac{x}{\theta}}}{\theta}$ for $x > 0$, and $f(x) = 0$, otherwise. The exponential distribution and generalizations of it are very important in actuarial modelling.

Page 28 **Third line of Example 2.6.**

The sentence should read:

Formulate the empirical distribution function $F_8(x)$ and draw the graph of $F_8(x)$.

Page 29 **Two lines above Equation 2.26.**

Change dx to dt. It should read $\int_0^\infty t^{\alpha-1}e^{-t/\theta}\,dt = \theta^\alpha \times \Gamma(\alpha)$.

Page 87 **Example 7.2. End of the solution.** Add:

Y has the same distribution as the original X.

Page 151 **Last line of Example 11.3.** The random variable is N, not X. The last line should read:

$$E\left[(N-2)_+\right] = E[N] - E[N \wedge 2] = 3 - \left[3e^{-3} + 2 \times \left(1 - 4e^{-3}\right)\right] = 1.2489.$$

Page 171 **Solution of Example 12.3.**

Typo with subscripts for both parts (a) and (b). f_{YP} should be changed to f_{Y_P} .

Page 172 **Equation (12.8).**

In the first integrant, it should be ${}_tp_z$ rather than just p_z. Also, the survival functions in the subsequent integrands should be capitalized (that is, S rather than s).

Page 192 **9th line from the bottom.** For the Single parameter Pareto distribution, change the second moment from $E\left[X^2\right] = \frac{2\theta^2}{\alpha-2}$ to $E\left[X^2\right] = \frac{\alpha\theta^2}{\alpha-2}$.

Page 211 **Question 8 in Section 14.** (iv) should read:

(iv) Michael pays 10% of the remaining repair costs each year.

Page 246 **At the end of the Section 16.3.** Add a new section:

16.4 The Convolution Method for Finding the Distribution of a Sum of Random Variables

Given a collection of random variables $X_1, X_2, \ldots, X_n$ there are a number of ways of finding the distribution of the sum $W = X_1 + X_2 + \cdots + X_n$. If the X's are independent of one another, than we might be able to use the **moment generating function relationship** $M_W(t) = \prod_{i=1}^n M_{X_i}(t)$ or the probability generating function relationship $P_W(r) = \prod_{i=1}^n P_{X_i}(r)$ to identify the distribution of W. This approach has not come up on the exam in recent years.

An alternative approach to find the distribution of a sum of random variables is the method of convolution. In this method, we would first find the distribution of $X_1 + X_2$. Then we would use that to find the distribution of $(X_1 + X_2) + X_3$, and so on. This method can be applied whether or not the X's are independent, but it is often the case that they are.

Convolution applied to discrete random variables

To apply the method of convolution to discrete integer-valued random variables, we apply a combinatorial approach. For instance, to find $P[X_1 + X_2 = k]$, we look at all X_1, X_2 pairs that add up to k, and add up those probabilities:

$$P[X_1 + X_2 = k] = \sum_{\text{all integers } j} P(X_1 = j \cap X_2 = k - j) \tag{0.1}$$

If X_1 and X_2 are independent, then the sum on the right in Equation (0.1) becomes

$$\sum_{\text{all integers } j} P[X_1 = j] \times P[X_2 = k - j]. \tag{0.2}$$

This approach can also be applied to find the distribution function of $X_1 + X_2$ if the pf of X_1 and the cdf of X_2 are known. If we let $Z = X_1 + X_2$, then

$$F_{X_1+X_2}(k) = F_Z(k) = P[X_1 + X_2 \leq k] = \sum_{\text{all integers } j} P[X_1 = j \cap X_2 \leq k - j]. \tag{0.3}$$

The notation $f_{X_1} * f_{X_2}(k)$ is used to denote the probability $f_{X_1+X_2}(k) = f_Z(k)$ and $F_{X_1} * F_{X_2}(k)$ is used to denote the probability $F_{X_1+X_2}(k) = F_Z(k)$. Assuming independence, (0.3) becomes

$$F_{X_1+X_2}(k) = F_Z(k) = \sum_{\text{all integers } j} f_{X_1}(j) \times F_{X_2}(k - j). \tag{0.4}$$

If X_1 and X_2 are ≥ 0, then Equation (0.2) becomes

$$f_{X_1+X_2}(k) = f_{X_1} * f_{X_2}(k) = P[X_1 + X_2 = k] = \sum_{j=0}^{k} f_1(j) \times f_2(k - j)$$

and Equation (0.4) becomes

$$P[X_1 + X_2 \leq k] = \sum_{j=0}^{k} f_1(j) \times F_2(k - j).$$

Example 16.5. Suppose that X and Y are independent discrete integer-valued random variables with X uniformly distributed on the integers 1 to 5, and Y having the following probability function:

$$f_Y(0) = 0.3,\ f_Y(1) = 0.5,\ f_Y(3) = 0.2$$

Let $Z = X + Y$. Find $P[Z = 5]$.

Solution.

Using the fact that $f_X(x) = 0.2$ for $x = 1, 2, 3, 4, 5$, and the convolution method for independent discrete random variables, we have

$$\begin{aligned} f_Z(5) &= \sum_{i=1}^{5} f_X(i) f_Y(5 - i) \\ &= (0.2)(0) + (0.2)(0.2) + (0.2)(0) + (0.2)(0.5) + (0.2)(0.3) = 0.20. \end{aligned}$$

□

Convolution applied to continuous random variables

To apply the method of convolution to continuous random variables, we apply an integral version of the summations in Equations (0.1) to (0.4). Suppose that X and Y are non-negative random variables with a joint pdf $f(x, y)$. If $Z = X + Y$, then the pdf of Z is

$$f_Z(z) = \int_0^z f(t, z-t)\, dt. \tag{0.5}$$

If X and Y are independent, then this becomes

$$\int_0^z f_X(t) \times f_Y(z-t)\, dt, \tag{0.6}$$

and the cdf of Z is

$$F_Z(z) = \int_0^z f_X(t) \times F_Y(z-t)\, dt. \tag{0.7}$$

Example 16.6. X_1 and X_2 are independent exponential random variables each with a mean of 1.

Find $P[X_1 + X_2 < 1]$.

Solution.

Using the convolution method, the density function of $Y = X_1 + X_2$ is

$$f_Y(y) = \int_0^y f_{X_1}(t) \times f_{X_2}(y-t)\, dt = \int_0^y e^{-t} \times e^{-(y-t)} dt = ye^{-y},$$

so that

$$P[X_1 + X_2 < 1] = P[Y < 1] = \int_0^1 ye^{-y} dy = [-ye^{-y} - e^{-y}]\Big|_{y=0}^{y=1} = 1 - 2e^{-1}$$

(the last integral required integration by parts).

Note that the sum of independent exponentials with the same mean has a gamma distribution. Since X_1 and X_2 are independent each with a mean of 1, Y has a gamma distribution with $\alpha = 2$ and $\theta = 1$. □

Convolution may arise in the context of **compound distributions**. If $X_1, X_2, \ldots, X_n$ are independent random variables that all have the same pdf or pf, then we may see the following notation used to described the distribution of $X_1 + X_2 + \cdots + X_n$, called the **n-fold convolution** of X:

$$P\left[\sum_{i=1}^{n} X_i \leq z\right] = F_X^{*n}(z) \tag{0.8}$$

In the case of continuous X, we have $F_X^{*n}(z) = \displaystyle\int_0^x F_X^{*(n-1)}(x-t) \times f_X(t)\, dt,$ (0.9)

and the pdf is $f_X^{*n}(x) = \displaystyle\int_0^x f^{*(n-1)}(x-y) \times f_X(y)\, dy.$ (0.10)

If X is discrete and integer-valued, then $F_X^{*n}(x) = \sum_{j=0}^{x} F_X^{*(n-1)}(x-j) \times f_X(j)$ is the cdf, and

$$f_X^{*n}(x) = \sum_{j=0}^{x} f_X^{*(n-1)}(x-j) \times f_X(j) \qquad \text{is the pf for } x = 0,\ 1,\ 2,\ \ldots. \tag{0.11}$$

Note that in these formulations,

$$f_X^{*(0)}(t) = 0,\ f_X^{*(1)}(t) = f_X(t),\ F_X^{*(0)}(t) = 1 \text{ and } F_X^{*(1)}(t) = F_X(t).$$

Page 259 **Last line of Problem 45.**

Change "1 weeks" to "13 weeks".

Page 262 **Line 4.**

Add the missing "=" so it becomes: $Var\,(S_2) = E(N) \times Var\,(X_2) + Var(N) \times [E\,(X_2)]^2$
$= E(N) \times \frac{\theta^2}{12} + Var(N) \times \frac{\theta^2}{4} = 351$ (Eq. 4)

Page 308 **Third line of Solution of Problem 13.**

Change $E[S^{(A)}]$ to $Var[S^{(A)}]$.

Page 318 **Problem 2.**

Change the question to:

The distribution of X is a mixture of two continuous random variables.
The mixing weight is a for random variable X_1 and the mixing weight is $1-a$ for random variable X_2, where $0 < a < 1$.

VaR_α is the 100αth percentile of the mixed distribution X.

Show that the following statement regarding the mixed random variable X false. Use X_1 and X_2 as exponential random variables with means 1 and 2 and with mixing weights .5 and .5 to construct an example to show the statement is false. The statement is:

"The VaR_α of X is the weighted average of the VaR_α of X_1 and the VaR_α of X_2 using the mixing weights a and $1-a$."

Page 325 **Solution of Problem 2.**

Change the solution to:

We will use the 50th percentile, but another would work just as well.
The 50th percentile of X_1 is c_1, where $e^{-c_1} = .5$, so that $c_1 = .693147$.
The 50th percentile of X_2 is c_2, where $e^{-c_2/2} = .5$, so that $c_1 = 1.386294$.
With mixing weights of .5 each, we get $.5(.693147) + .5(1.386294) = 1.03972$.
The cdf of the mixture distribution is $F_X(x) = .5(1-e^{-x}) + .5(1-e^{-x/2})$.
$F_X(1.03972) = .5(1-e^{-1.03972}) + .5(1-e^{-1.03972/2}) = .526$.
Therefore, 1.03972 is not the 50th percentile of the mixed distribution.

Page 328 **Example 21.1, 6th line from the bottom.**

The numerical value 0.0 should be 0.8. That part of the equation should read:

$$\cdots = \frac{1.5}{\lambda} + \frac{0.8}{\lambda} + \frac{0.7}{\lambda} + \frac{1.4}{\lambda} - 4 = 0$$

Page 337 **Solution of Problem 5.**

Change **Answer C** to **Answer D**.

Page 349 **Example 22.6, 9th line from the bottom.**

${}_{.4}q_{+.6}$ should be ${}_{.4}q_{x+.6}$. Delete the sentence "(this is the UDD probablity ${}_{.4}q_{+.6}$)"

Page 352 **Tabe in Problem 9.**

Change Year Reported of 1997, settled in 1999 from 11 to 1, namely,

Number of Claims Settled			
Year	Year Settled		
Reported	1997	1998	1999
1997	Unknown	3	1
1998		5	2
1999			4

Page 367 **Problem 7. Before the choices.**

Add the missing question:

Determine the difference between $\widehat{S}_1(1250)$ and $\widehat{S}_2(1250)$.

Page 372 **Solution of Problem 7. Third line.**

Change $y_2 = 1200$ to $y_4 = 1200$.

Page 383 **Tenth line from the top.**

Change the last n_1 to n_3, namely,

The sum over all Category 3 points is $\frac{n_3}{\alpha} + \Sigma\left[\ln\left(d_i+\theta\right) - \ln\left(x_i+\theta\right)\right] = \frac{n_3}{\alpha} - C_3$

Page 388 **Ninth Line from the bottom.**

In (c), change the second line to:

10 insurance payment amounts: 2, 4, 5, 5, 8, 10, 12, 15 and 2 limit payments of 20 each

Page 398 **Solution of Problem 15.**

Change the last four lines to:

$$Y = \ln\left(\frac{21}{20}\right) + \ln\left(\frac{22}{20}\right) + \ln\left(\frac{25}{20}\right) + \ln\left(\frac{26}{20}\right) + 2\times\ln\left(\frac{29}{20}\right) + \ln\left(\frac{33}{20}\right) + \ln\left(\frac{35}{20}\right)$$
$$+ 2\times\ln\left(\frac{30}{20}\right) + \ln\left(\frac{28}{25}\right) + \ln\left(\frac{30}{25}\right) + \ln\left(\frac{35}{25}\right) + \ln\left(\frac{42}{25}\right) + 2\times\ln\left(\frac{30}{25}\right) = 4.7596$$

and $Z = 8 + 4 = 12$ (number of non-censored values).

The mle of $\frac{1}{\alpha}$ is .397, so the mle of α is 2.52.

Page 408 **5th line from the bottom.** Typo "claimss" should be "claims"

Page 415 **Problem 21.**

Add the missing choices:

(A) Less than 2.4

(B) At least 2.4, but less than 2.6

(C) At least 2.6, but less than 2.8

(D) At least 2.8, but less than 3.0

(E) At least 3.0

Page 422 **Solutions of Problem 21.**

Change the last seven lines to:

$$\frac{1}{\beta}\times\left(\sum_{i=1}^{n} x_i\right) - \frac{1}{1+\beta}\times\left(\sum_{i=1}^{n} x_i\right) - n\times\frac{2+2\beta}{2\beta+\beta^2} = 0.$$

For the given data, this equation is $\frac{47}{\beta} - \frac{47}{1+\beta} - \frac{16+16\beta}{2\beta+\beta^2} = 0.$

With common denominator $\beta(1+\beta)(2+\beta)$, this equation becomes

$$\frac{47(1+\beta)(2+\beta)}{\beta(1+\beta)(2+\beta)} - \frac{47\beta(2+\beta)}{\beta(1+\beta)(2+\beta)} - \frac{(16+16\beta)(1+\beta)}{\beta(1+\beta)(2+\beta)} = 0.$$

This reduces to the quadratic equation $16\beta^2 - 15\beta - 78 = 0$.

The equation has one negative solution, which is rejected, and the positive solution is 2.73, which is the mle of β.

Page 438 **Example 26.11, 5th line from the bottom.**

$r = .05$ should be $k = .05$.

Page 442 **Problem 11(b). First line.**

Change "readability: to "credibility".

Page 442 **Problem 12. Sixth and eighth lines.**

Change "readability: to "credibility".

Page 447 **Problem 29. Sixth line.**

Change "readability: to "credibility".

Page 462 **10th line from the top.**

Change "an IRA" to "a tax-deferred Individual Retirement Account (IRA)".

Page 462 **15th line from the top.**

Typo "dugs" should be "drugs"

Page 471 **9th line from the bottom.**

This line should read: "Collision and Other-than-Collision Coverage"

Page 477 **Solution of Problem 1. Sixth line.**

Change (i) to (ii).

Page 477 **Solution of Problem 1. Tenth line.**

Change "tow" to "two".

Page 481 **Table 29.2.**

Change "Incremental Loss Payments" to "Cumulative Loss Payments".

Page 483 **First line of equation (29.5)**, where the right side of the first equation is missing a term. This line should read:

$$\text{Estimated Loss Reserve}_{CL} = \text{Losses Paid-To-Date} \times (f_{\text{ult}} - 1)$$

Page 514 **13th line from the top.** Typo "Lost Cost" should be "Loss Cost"

Page 514 **On the bottom right of the figure,** $1.2234P$ should be $1.2243P$.

Page 515 At the end of Section 30.6. Add a new section:

30.7 Rate Classification Differentials

An example of a risk calculation variable is the geographical location of the property for property insurance. There may be several different locations in that classification. The classification may be refined further as to residential versus commercial property. A particular risk classification would be chosen as the **base cell**, and other related risk classifications are assigned **differentials**, with the base classification having a differential of 1.000.

Suppose that there are two risk classes, A and B, and A is the base cell or class. Under the **Lost Cost Method** for differentials, the differential for class B is

$$(\text{Indicated Differential}_B) = \frac{\text{Loss Cost}_{\boldsymbol{B}}}{\text{Base Loss Cost}},$$

where Base Loss Cost is the Class A loss cost (per unit of exposure). This can be extended to any other class.

Under the **Loss Ratio Method**, if a class has an existing differential, it can be updated based on new experience period loss information using

$$(\text{Indicated Differential})_B = (\text{Existing Differential})_B \times \frac{\text{Experience Period Loss Ratio}_B}{\text{Experience Period Loss Ratio}_A}.$$

These two methods may give different results and there may be some judgment needed to determine the new differential for a risk class. We could apply the **credibility approach**

$$\text{New Differential} = Z \times (\text{Indicated Differential}) + (1 - Z)(\text{Existing Differential})$$

where $0 \leq Z \leq 1$ is a **credibility factor** that measures the credibility of the existing loss data for that risk class.

Balancing Back

Suppose that there is a target overall rate change of $k\%$ to the base premium rate, so target premiums will be $1 + .01k$ times existing premiums. Suppose that an independent assessment has been made to reclassify risk differentials. Suppose that we wish to apply this to a portfolio of risks made up of three risk classes with the following characteristics.

Class	A	B	C
Earned Exposure	EU_A	EU_B	EU_C
Existing Differential	1.00	d_B	d_C
Proposed Differential	1.00	d_B^P	d_C^P

If the current **base premium rate** for Class A is $\text{BP}_A = 1.0$, then the current premium is

$$P = \text{BP}_A \times [\text{EU}_A + d_B \times \text{EU}_B + d_C \times \text{EU}_C] = 1.0 \times \text{EU}_A + d_B \times \text{EU}_B + d_C \times \text{EU}_C.$$

We wish for the new premium to be $P_{\text{new}} = (1 + .01k) \times P$.

It would seem that we should adjust the base premium rate for Class A from the current base rate of 1.00 to a new base rate of $1 + .01k$. If we do that and also use the new differential factors, then since the new differential factors apply to the new base rate for Class A, the new premium rate would be

$$(1 + .01k) \times [\text{EU}_A + d_B^P \times \text{EU}_B + d_C^P \times \text{EU}_C].$$

The ratio of this to the existing premium rate is

$$(1+.01k)\times\frac{\text{EU}_A+d_B^P\times\text{EU}_B+d_C^P\times\text{EU}_C}{\text{EU}_A+d_B\times\text{EU}_B+d_C\times\text{EU}_C}=(1+.01k)\times\frac{\text{New Average Differential}}{\text{Old Average Differential}}.$$

This only results in the target overall rate change of $k\%$ if $\frac{\text{New Average Differential}}{\text{Old Average Differential}}=1$, which is not necessarily true. We deal with this by applying a **balance-back factor** of $\frac{\text{Old Average Differential}}{\text{New Average Differential}}$ to the base premium rate for Class A, so that the new base premium rate for Class A is

$$\text{BP}_A^{\text{new}}=(1+.01k)\times\frac{\text{Old Average Differential}}{\text{New Average Differential}}.$$

Now the new overall premium is

$$P_{\text{new}}=\text{BP}_A^{\text{new}}\times[\text{EU}_A+d_B^P\times\text{EU}_B+d_C^P\times\text{EU}_C]=(1+.01k)\times P.$$

Note that this required changing the base premium rate for Class A by a different factor than the overall target of $1+.01k$.

Example 30.2. You are given the following information for two risk classes in a risk portfolio. Class A is the base class.

Class	Earned Exposure Units	Existing Rate	Loss Cost	Existing Class Differentials
A	3200	400	320	1.00
B	1600	650	480	1.625

The insurer wants to implement an average 10% rate change. Determine the change in the base (Class A) rate needed based on updated rate classification differentials under the two methods (loss cost method and loss ratio method). Calculate the new rates per exposure unit for Classes A and B

Solution.

The existing average differential is

$$\frac{3200}{4800}\times 1.00+\frac{1600}{4800}\times 1.625=1.2083.$$

Under the loss cost method, the indicated differential for Class B is $\frac{480}{320}=1.50$.

Under the loss ratio method, the indicated differential for Class B is $1.625\times\frac{480/650}{320/400}=1.50$.

The balance-back factor is

$$\frac{1.2083}{\frac{3200}{4800}\times 1.00+\frac{1600}{4800}\times 1.5}=\frac{1.2083}{1.1667}=1.0357$$

under either method. In order to have an overall average 10% rate change, the change in the Class A rate increase ratio should be $1.1\times 1.0357=1.139$ (13.9% rate increase) and the new rate for Class A will be $400\times 1.139=456$ and for Class B it will be $456\times 1.5=684$. Note that both methods resulted in the same indicated differential. This will be the case if the existing differential is the ratio of Class B to Class A existing rates. □

As outlined in the text, the three steps in a **rate change** process are

1 - determine the overall rate change

2 - determine the new differentials

3 - balance back to the overall indicated rate change

Step 1 may require applying trend and development factors to determine the new average loss cost. A **permissible loss ratio** would also be needed.

Page 521 **8th line from the top.** After "... by the ceding company." Add

The share covered by the reinsurer may be different in different loss intervals. A loss interval under this arrangement may be called a **treaty layer**.

Page 523 **Table in Problem 2.**

Change the Ammount Paid on Claim associated with Claim File ID 2 from 900,000 to 1,000,000.

Page 530 **Solution of Example 32.1. First line.**

Change $\max\{100 - S_1\ ,\ 0\}$ to $\max\{90 - S_1\ ,\ 0\}$.

Page 534 **Second graph**

Change the upper-upper node from 66.75 to 66.25.

Page 543 **Last line.**

Change the last formula to

$$100,000 \times 0.0696 = 6,960.$$

Page 547 **6th line from the bottom.**

Change $r - \frac{\sigma^2}{2} = 0.145$ to $r - \frac{\sigma^2}{2} = 0.055$.

Page 547 **4th line from the bottom.**

Change the display formula to:

$$P\left(\frac{S_1}{S_0} \geq 1.20\right) = P\left(\ln\ \frac{S_1}{S_0} \geq \ln\ 1.20\right) = P\left(\frac{\ln\ \frac{S_1}{S_0} - .055}{.3} \geq \frac{\ln\ 1.20 - .055}{.3}\right) = 1 - \Phi(.424) = 0.336.$$

Page 557 **Solution to Problem 1.** Change the calculations for individual Y and Z to:

For individual Y: $E[u_Y(W)] = \int_0^{10,000} \frac{w}{100} \times \frac{1}{10,000}\ dw = 50 = u_Y(C_Y) = \frac{C_Y}{100}$.
Solving for C_Y results in $C_Y = 5,000$

For individual Z: $E[u_Z(W)] = \int_0^{10,000} \left(\frac{w}{100}\right)^2 \times \frac{1}{10,000}\ dw = \frac{10,000}{3} = u_Z(C_Z) = \left(\frac{C_Z}{100}\right)^2$.

Solving for C_Z results in $C_Z = 100 \times \sqrt{\frac{10,000}{3}} = 5,773.50$

Page 562 **Solution to Problem 20.** Change the last two lines to:

The growth factor from CY3 to CY4 is $\frac{1.30592}{1.1871} = 1.1000$.
Approximate earned premium for CY3 at current rates is $1.1000 \times 4700 = 5170$.

Page 563 **Choice D of Problem 3.** Change $\dfrac{1 - .5e^{-r}}{1.5}$ to $\dfrac{2e^{-r} - 1}{1.5}$.

Page 570 Change the **solution of Problem 3** to:

The risk neutral probability of the stock price dropping to $0.5S_0$ is $q = \frac{2 - e^{-r}}{1.5}$. The price of the option is the expected present value

$$e^{-r} \times [q \times 1 + (1 - q) \times 0] = \frac{2e^{-r} - 1}{1.5}.$$

Answer D