Individual Health Insurance
April 30, 2008

Pages 167- 170

We have received feedback that this section of the text is confusing because some of the defined notation is inconsistent with comparable life insurance reserve notation. The author has, therefore provided replacement wording for this section. He has also corrected 2 errors. Replacement wording is noted in red in the attached.

$$
\begin{aligned}
{}_t^zV_x &= PV\{\text{Future Claims}\} - PV\{\text{Future Net Premiums}\} \\
&= \sum_{i=t}^{\omega}\left\{{}_i\,p_x \times v^{i-t} \times {}_i^zC_x\right\} - \sum_{i=t}^{\omega}\left\{{}_i\,p_x \times v^{i-t} \times {}_i^zP_x\right\},
\end{aligned} \tag{6.1}
$$

which can also be seen to be

$$
\begin{aligned}
&= PV\left\{\text{Future annual differences between claims and net premiums}\right\} \\
&= \sum_{i=t}^{\omega}\left\{{}_i\,p_x \times v^{i-t} \times [{}_i^zC_x - {}_i^zP_x]\right\}
\end{aligned} \tag{6.2}
$$

where

$$
\begin{aligned}
{}_i\,p_x &= \text{probability of survival to } i \text{ of policy issued at age } x, \\
v^t &= (1 + \text{annual interest rate})^{-t} \\
&= \text{present value factor}, \\
{}_i^zC_x &= \text{claim cost for someone age } x \text{ at duration } i, \text{issued in year } z, \text{ and} \\
{}_i^zP_x &= \text{net premium for someone age } x \text{ at duration } i, \text{issued in year } z.
\end{aligned}
$$

Note in the above formulae the addition of z, the calendar year of issue. This is a departure from the annotation of most actuarial texts, but is a necessary distinction for policies like major medical, subject to calendar-based claim trends. Such trends cause the year i claim cost, for a given issue age x, to vary from year to year. This is generally not needed for coverages where claims do not vary significantly because of calendar year.

For ease of understanding, this and other formulae have assumed both premiums and claims occur at the beginning of the year. A rigorous development would account for this, and is relatively easy to include in the formulas.

Formula 6.1 calculates a reserve at time t *per original issued policy*. This is the reserve calculation one might use for asset share purposes, or to project future aggregate reserves. If one wanted to calculate a reserve *per remaining* or *per surviving* policy, which I will call ${}_t^zV_x^s$ then the formula would be

$$
{}_t^zV_x^s = \sum_{i=t}^{\omega}\left\{{}_{i-t}\,p_{x+t} \times v^{i-t} \times {}_i^zC_x\right\} - \sum_{i=t}^{\omega}\left\{{}_{i-t}\,p_{x+t} \times v^{i-t} \times {}_i^zP_x\right\} \tag{6.1a}
$$

Formula 6.1a might be used to calculate reserve *factors*, which are applied to surviving policies each future year, to calculate statutory reserves. These methods are most useful in non-inflationary coverages such as DI and LTC.

These formulae are further complicated if p, C, or P are on a durational basis, as will happen, for example, if either mortality or lapse rates vary by duration.

When $t = 0$, Formula 6.1 is

$$ {}_0^zV_x = \sum_{i=0}^{\omega} \left\{ {}_i p_x \times v^i \times {}_i^zC_x \right\} - \sum_{i=0}^{\omega} \left\{ {}_i p_x \times v^i \times {}_i^zP_x \right\} \qquad (6.3) $$

But, by definition, ${}_0^zV_x = 0$ (reserves start at zero), so

$$ \sum_{i=0}^{\omega} \left\{ {}_i p_x \times v^i \times {}_i^zC_x \right\} = \sum_{i=0}^{\omega} \left\{ {}_i p_x \times v^i \times {}_i^zP_x \right\} \qquad (6.4) $$

which happens to be the defining condition of any net premium ${}_0^zP_x$: that its present value equals the present value of claims at time zero.

Formula 6.1 is the prospective formula for net level policy reserves, where the net premiums are assumed to be a constant percentage of the gross premiums. (This is the usual assumption in such calculations.)

From basic actuarial mathematics, we know there are equivalent prospective formulae (like Formula 6.1) and retrospective formulae. The retrospective formula corresponding to 6.1a is:

$$ {}_t^zV_x{}^s = \sum_{i=0}^{t-1} \left\{ \frac{1}{{}_{t-i}\, p_{x+i} \times v^{t-i}} \times {}_i^zP_x \right\} - \sum_{i=0}^{t-1} \left\{ \frac{1}{{}_{t-i}\, p_{x+i} \times v^{t-i}} \times {}_i^zC_x \right\} \qquad (6.5a) $$

Retrospective formulas can be helpful for valuation actuaries and auditors, in calculating and checking reserve factors.[3] They are not often used for other purposes.

[3] Jordan, Chester Wallace Jr.. *Life Contingencies*. pp 115-116.

The aggregate reserves held at time t are the product of (1) reserve factors shown in 6.5a, multiplied by (2) the number of surviving policies (which, in the context of calculations for a single policy, is ${}_t p_x$). Since:

$$ {}_{t-i} p_{x+i} \times {}_i p_x = {}_t p_x \tag{6.5b} $$

Then the aggregate reserve is

$$ {}_t^z V_x = \sum_{i=0}^{t-1} \left\{ {}_i p_x \times v^{i-t} \times \left[{}_i^z P_x - {}_i^z C_x \right] \right\} \tag{6.5} $$

EXERCISE 6.1: Using the above formulae, the claim cost stream included in *File 12 - Chapter 6 Data for Exercises.xls* worksheet, tab “6.1 Data” of the CD-ROM included with this text, and the discount rates included in the same spreadsheet, construct the reserve stream shown in that spreadsheet. For the purpose of this exercise, treat claims as occurring at the end of the year and premiums at the beginning of the year. ☐

COMPLICATIONS AND VARIATIONS TO THE RESERVE FORMULA

The basic formula can be complicated in a number of ways. The major complication, however, is how expenses are reflected in reserve calculations. Since early expenses (mostly at issue) are typically higher than later expenses, the double whammy of having to pay those expenses plus start holding policy reserves can put a large strain on the company’s bottom line that year. Statutory and GAAP accounting handle this in different ways.

In statutory accounting, there is not an explicit recognition of expenses in policy reserves. However, by allowing the use of modified reserve methods, there is an implicit recognition. The NAIC’s model law[4] setting reserve bases allows for “two year full preliminary term” (2YFPT) reserves for coverages other than LTC, and 1YFPT for LTC. Conceptually, the 2YFPT basis can be thought of as allowing the insurer, as they start holding reserves, to treat the policy as if it was actually issued two years later than it was, with the policyholder actually two years older. Algebraically, this translates to having no policy reserves for the first two years of the policy. In formulaic notation:

[4] op. cit., Health Insurance Reserves Model Regulation.

$$\begin{aligned} {}^{z}_{t}V^{2PT}_{x} &= {}_{t}p_{x}\left[\sum_{i=t-2}^{\omega}\left\{{}_{i-t+2}p_{x+t}\times v^{i-t+2}\times {}^{z}_{i}C_{x+2}\right\}\right. \\ &\qquad\qquad \left. -\sum_{i=t-2}^{\omega}\left\{{}_{i-t+2}p_{x+t}\times v^{i-t+2}\times {}^{z}_{i}P_{x+2}\right\}\right] \\ &\quad \text{for } t\geq 2\text{, and} \\ &= 0 \quad \text{for } t=0,1 \end{aligned} \tag{6.6}$$

If we are defining V as a reserve *per surviving policy*, similarly to Formula 6.1a, then 6.6 becomes:

$$\begin{aligned} {}^{z}_{t}V^{2PT,s}_{x} &= \sum_{i=t-2}^{\omega}\left\{{}_{i-t+2}p_{x+t}\times v^{i-t+2}\times {}^{z}_{i}C_{x+2}\right\} \\ &\qquad -\sum_{i=t-2}^{\omega}\left\{{}_{i-t+2}p_{x+t}\times v^{i-t+2}\times {}^{z}_{i}P_{x+2}\right\} \end{aligned} \tag{6.6a}$$

This articulation of the reserve also lends itself to durational calculations;

$$\begin{aligned} {}^{z}_{t}V^{2PT,s}_{[x]} &= \sum_{i=t-2}^{\omega}\left\{{}_{i-t+2}p_{[x]+t}\times v^{i-t+2}\times {}^{z}_{i}C_{[x]+2}\right\} \\ &\qquad -\sum_{i=t-2}^{\omega}\left\{{}_{i-t+2}p_{[x]+t}\times v^{i-t+2}\times {}^{z}_{i}P_{[x]+2}\right\} \end{aligned} \tag{6.6b}$$

The 1YFPT basis has a similar conceptual basis.

EXERCISE 6.2: Construct the two year preliminary term reserve stream corresponding to the net level reserves of Exercise 6.1. Assume a 5% increase in the net level premium for a 37-year old male compared to a 35 year-old male. ☐

Many times in practice, especially for medical coverages, calculations are done directly on electronic spreadsheets, without reference to commutation functions or published tables. DI and LTC reserves, on the other hand, are often based on published or collected data sources, and may rely on formulae in their development.

In GAAP accounting, expenses are explicitly reflected. The policy reserve is calculated using net premiums as previously described. If the gross premium structure has future rate changes built into it, net premiums are assumed to change proportionately to gross premiums, and a factor reflecting this change would be included in the stream of ${}^{z}_{i}P_{x}$. (This doesn't mean that future claim trends are assumed beyond the current rating period—they are usually not. The growth in gross premiums is generally that which occurs without recognition of future increases in the rate schedule itself. It might actually be a better theoretical treatment of this to make reasonable assumptions of future claim growth and premium growth, however this need is now being met through gross premium reserve calculations, discussed later.) This reserve is then referred to as the *benefit reserve*, and notation for the variables related to it are typically endowed with a post-superscripted "B," such as ${}^{z}_{t}V_{x}^{B}$.
To recognize expenses, a parallel calculation is done using expenses rather than benefits. This expense reserve is actually an asset, but is conceptually equivalent to a *negative* reserve, in that it performs the opposite function of benefit reserves. Benefit reserves cause a company to set aside funds which would otherwise become profit, so that those funds will (appropriately) be used later in the policy lifetime to subsidize later costs. Expense reserves allow a company to postpone recognition of certain expenses, and thus allow funds to flow through to profits earlier than they would if all expenses were fully reflected at the time they are incurred. The equivalent of the reserve in expense terms is called the *deferred acquisition cost*, or *DAC* asset.

The DAC is composed of deferrable expenses, which are those incurred to acquire the business. Typically it includes the cost of selling, underwriting, and issuing the policy. (There is sometimes also an expense reserve for maintenance expenses, which might increase over time, so are similar in nature to the benefit reserves. When recognized, they are often included as a loading on the benefit reserves.) If we denote the deferrable expenses at time t, from a policy in year z to a policyholder aged x as ${}^{z}_{t}E_{x}$, then the DAC reserve at time t can be described as in Formula 6.7. Note that "AV" is a term used to represent the accumulated value of past values, in this case with interest and terminations. It is the retrospective equivalent of the prospective case's PV, on a "per surviving policy" basis (in order to calculate factors).

$$
\begin{aligned}
{}^{z}_{t}DAC^{s}_{x} &= AV\{\text{Deferrable Expense}\} - AV\{\text{Net Expense Premiums}\} \\
&= \sum_{i=0}^{t-1}\left\{\frac{1}{{}_{t-i}p_{x+i}} \times v^{i-t} \times {}^{z}_{i}E_{x}\right\} - \sum_{i=0}^{t-1}\left\{\frac{1}{{}_{t-i}p_{x+i}} \times v^{i-t} \times {}^{z}_{i}P^{E}_{x}\right\} \qquad (6.7)
\end{aligned}
$$

This is the expense analogue of Formula 6.1a's benefit reserve. Formulas 6.1 and 6.2 also have analogous expense reserve formulae.

In working with reserves, it is helpful to understand how one year flows into the next. To start this, it is important to keep in mind the difference between the year-end reserve for a year t and the beginning reserve for year $t+1$. The end of year reserve is called a "terminal reserve." It is fairly standard, in doing financial statement valuations, to aggregate policies of a given issue age and duration, and assume a uniform distribution of issues throughout the year. This results in the average reserve (typically in the form of a "reserve factor" to be multiplied by the appropriate exposure value) being the arithmetic mean of the two terminal reserves. This is called a "mid-terminal reserve" methodology.
In that magical moment between the end of one year and the beginning of the next, our methods assume the net premium has been paid, and the reserve jumps by the value of that net premium. Then, during year $t+1$, the reserve accumulates with interest (raising the value of the reserve), and claims all occur at the end of the year (lowering the value of the reserve). The reserve that results from these calculations will match the reserve recalculated through the present value formulae, because they are algebraically equivalent.

EXERCISE 6.3: Show how formula 6.1 at time t, when adjusted by the accumulation described in the prior paragraph, is equivalent to formula 6.1 at time $t+1$. ☐

EXERCISE 6.4: Using the expense and other data from the corresponding worksheet in the CD included with this text, calculate the stream of DAC shown in the results tab of the spreadsheet. ☐

PURCHASE GAAP

When a company is acquired, the GAAP effect with respect to an existing block of policies is that the DAC asset, which represents the amortized value of the selling company's original costs, is released. (This makes sense, because the DAC is used to align the timing of acquisition expenses with future premium or profit. When the business is sold, future profit is capitalized into the sales price, so the DAC is as well.) In its place, the acquiring company creates a VOBA (*value of business acquired*). The VOBA is the pre-tax value of the acquired business. (To the extent there are deferrable expenses incurred after the acquisition, such as second year commissions higher than those in years 3+, there will be a DAC asset generated by this, independently of the selling company's DAC.)

VOBAs are usually calculated as the present value of cash profits over the future lifetime of the business assumed, discounted at a risk rate of return. (This is often based on the models used in the appraisal work done to determine purchase price for the business.) The VOBA is then amortized over the future lifetime of the business, using principles similar to those described earlier.

PRE-FUNDING TRENDS

Another complication to the basic policy reserve calculation is caused by the various sources of increasing claim costs. In medical insurance there are three sources for increasing claim costs over time: inflationary and similar secular trends, aging of the insured, and durational trends. The inflationary and related secular trends can be thought of as "environmental" trends – those which would cause increases in costs from year to year with an identical and unchanging insured risk from year to year.

In today's market and with ongoing sizeable claim trends for the foreseeable future, it is not feasible to completely pre-fund the three sources of claim cost increases. Therefore, those insurers who pre-fund have funded only some part of that increase. Most typically, they pre-fund aging of the insured and a few years of durational deterioration, but no claim trend. Future rate increases are thus assumed to be sufficient to cover the increased cost due to claim trend, further durational deterioration, and the multiplier/cross product leveraging of those elements of cost increases.

In recent work, an American Academy of Actuaries task force indicated that it would be unrealistic to assume carriers' pricing methods implicitly use more than the first five years of durational deterioration. (Some members believed the actual number to be less than five, for most carriers.)

Most major medical premiums do not pre-fund trends, but instead use premiums calculated to exactly match that year's claims. Most DI and LTC products are priced on a level premium basis, recognizing aging and durational effects.

A decade or so ago, in a paper proposing a new regulatory reserve standard[5], a new modified reserve methodology was suggested, which created a modified net premium stream. That net premium stream could be considered as replacing a realistically projected future stream of claim costs with a net premium stream that increases with secular trend plus x%. With $x = 0\%$, the method fully prefunds aging, duration, and the multiplicative compounding element of the combination of the three sources of increasing costs. Premiums are assumed to grow each year proportionately to secular trend. As x increases in value, the level of pre-funding goes down, and more of the growth in costs is borne by future premium increases. The reserves for such a formula would be calculated similarly to Formula 6.1, but perhaps where both the future claims and premium grow annually by some constant value j:

$$ {}^{z}_{t}V_x = \sum_{i=t}^{\omega}\left\{ {}_{i}p_x \times v^{i-t} \times {}^{z}_{i}C_x \times (1+j)^{i} \right\} - \sum_{i=t}^{\omega}\left\{ {}_{i}p_x \times v^{i-t} \times {}^{z}_{i}P_x \times (1+j)^{i} \right\} \qquad (6.8) $$

Formula 6.8 is, once again, a calculation of reserves per issued policy, rather than a reserve per surviving policy, which would be

$$ {}^{z}_{t}V^{s}_{x} = \sum_{i=t}^{\omega}\left\{ {}_{i-t}p_{x+t} \times v^{i-t} \times {}^{z}_{i}C_x \times (1+j)^{i} \right\} - \sum_{i=t}^{\omega}\left\{ {}_{i-t}p_{x+t} \times v^{i-t} \times {}^{z}_{i}P_x \times (1+j)^{i} \right\} \qquad (6.8a) $$

A further assumption in that paper was that assumed lapses be recognized in ${}_{i}p_x$. When actual lapses are in excess of that amount, the policy reserves per remaining policy could be increased to recognize the implicit antiselection in that added lapsation.

This is only one of many such schemes which might be constructed for such pre-funding, each based on a differing pattern for the premium stream. Another scheme was proposed by Bob Cumming and Leigh Wachenheim in their article, "A Simplified Method for Calculating Contract Reserves."[6]

[5] Bluhm, William F. "Duration Based Policy Reserves," *Transactions of the Society of Actuaries*, 1993, pp 11-31

[6] Society of Actuaries, *Health Section News*, Issue No. 35, June, 1998.