S. BROVERMAN EXAM MLC STUDY GUIDE - VOLUME 1

LIFE CONTINGENCIES

NOTES, EXAMPLES AND PROBLEM SETS

Introductory Note

TABLE OF CONTENTS - PART A

TABLE OF CONTENTS - PART B

INTRODUCTORY NOTE

This study guide is designed to help in the preparation for Exam MLC of the Society of Actuaries.

The study guide is divided into two volumes. Volume 1 consists of review notes, examples and problem sets. Volume 2 contains 4 practice exams of 20 multiple choice questions each and written questions with a total point value of 56 points. Volume 2 also has 15 multiple choice tests of 20 questions each. In Volume 1, there are close to 200 examples in the notes, over 375 multiple choice problems and almost 30 written answer problems in the problem sets. All of these (about 1000) questions have detailed solutions. The notes are broken up into 42 sections Each section has a suggested approximate time frame.

Many of the examples in the notes and about half of the problems in the problem sets are from older SOA or CAS exams on the relevant topics. Some of the questions on the practice exams are variations on actual exam questions. The SOA has posted on its website a sample question file for Exam MLC with solutions. Many of those questions are from old exams, and there may be some overlap in the questions found in this study guide and those found in the SOA files, but I have attempted to limit that duplication. The SOA and CAS questions are copyrighted by the SOA and CAS, and I gratefully acknowledge that I have been permitted to include them in this study guide.

Because of the time constraint on the exam, a crucial aspect of exam taking is the ability to work quickly. I believe that working through many problems and examples is a good way to build up the speed at which you work. It can also be worthwhile to work through problems that have been done before, as this helps to reinforce familiarity, understanding and confidence. Working many problems will also help in being able to more quickly identify topic and question types. I have attempted, wherever possible, to emphasize shortcuts and efficient and systematic ways of setting up solutions. There are also occasional comments on interpretation of the language used in some exam questions. While the focus of the study guide is on exam preparation, from time to time there will be comments on underlying theory in places that I feel those comments may provide useful insight into a topic.

It has been my intention to make this study guide self-contained and comprehensive for all Exam MLC topics, but there are may be occasional references to the book listed in the SOA exam catalog. While the ability to derive formulas used on the exam is usually not the focus of an exam question, it is useful in enhancing the understanding of the material and may be helpful in memorizing formulas. There may be an occasional reference in the review notes to a derivation, but you are encouraged to review the official reference material for more detail on formula derivations.

In order for the review notes in this study guide to be most effective, you should have some background at the junior or senior college level in probability and statistics. It will be assumed that you are reasonably familiar with differential and integral calculus.

INTRODUCTORY NOTE

Of the various calculators that are allowed for use on the exam, I think that the BA II PLUS is probably the best choice. It has several memories and has good financial functions. I think that the TI-30X IIS would be the second best choice.

There is a set of tables that has been provided with the exam in past sittings. These tables consist of a standard normal distribution probability table and a life table with insurance and annuity values at a 6% rate of interest. The tables are available for download from the Society of Actuaries website, but are included at the ends of both Volumes 1 and 2 for convenience.

If you have any questions, comments, criticisms or compliments regarding this study guide, you may contact me at the address below. I apologize in advance for any errors, typographical or otherwise, that you might find, and it would be greatly appreciated if you would bring them to my attention. I will be maintaining a website for errata that can be accessed from www.sambroverman.com . It is my sincere hope that you find this study guide helpful and useful in your preparation for the exam. I wish you the best of luck on the exam.

Samuel A. Broverman
Department of Statistics
University of Toronto
100 St. George Street
Toronto, Ontario CANADA M5S 3G3
E-mail: sam@utstat.toronto.edu or 2brove@rogers.com
Internet: www.sambroverman.com

January, 2015

MLC SECTION 14 - DISCRETE WHOLE LIFE ANNUITY-DUE

The suggested time frame for covering this section is 2 hours.

A life annuity is an ongoing series of payments made while someone remains alive. A life annuity-due is an annuity with annual payments paid at the start of each year and payable over some period related to an individual's lifetime. In the formulations that follow, we will make use of the compound interest annuity-due,

$$\ddot{a}_{\overline{m}|} = 1 + v + v^2 + \cdots + v^{m-1} = \frac{1-v^m}{1-v} = \frac{1-v^m}{d} \ ,$$

where d is the annual effective rate of discount.

Whole Life Annuity-Due of 1 Per Year

We can describe the actuarial present value of a life annuity-due by considering the present value of the "average" amount an insurer will have to pay. Suppose an insurer issues a life annuity-due of 1 per year to ℓ_x individuals. Based on the expected numbers of survivors from year to year, the amount that the insurer expects to pay can be described by the following time diagram.

Age/time	$x/0$	$x+1/1$	$x+2/2$	$\cdots$	$x+k/k$	$\cdots$
Insurer pays 1 to each of	ℓ_x	ℓ_{x+1}	ℓ_{x+2}	$\cdots$	ℓ_{x+k}	$\cdots$

The present value of the total expected payment by the insurer is

$$\ell_x + v\ell_{x+1} + v^2\ell_{x+2} + \cdots + v^k\ell_{x+k} + \cdots = \sum_{k=0}^{\infty} v^k\,\ell_{x+k} \ .$$

The present value per individual annuity is

$$\left(\frac{1}{\ell_x}\right)(\ell_x + v\ell_{x+1} + v^2\ell_{x+2} + \cdots + v^k\ell_{x+k} + \cdots) = \sum_{k=0}^{\infty} v^k\ \frac{\ell_{x+k}}{\ell_x} = \sum_{k=0}^{\infty} v^k\ {}_kp_x.$$

This is the actuarial present value of the whole life annuity-due, and it is denoted $\ddot{a}_x$.

The formulation $\boldsymbol{\ddot{a}_x = \sum_{k=0}^{\infty} v^k\ {}_kp_x}$ may be referred to as the current payment form of the annuity. This life annuity is a sum of a series of pure endowments, one for each integer year.

The Exam MLC Illustrative Life Table provides a column of whole-life annuity-due values at integer ages.

The annuity can also be considered from a stochastic point of view, with $\ddot{a}_x$ being the expected value of a present value random variable. This annuity pays 1 at the start of each year as long as (x) lives. If (x) survives K complete years and dies in the $K+1$-st year, then there would be $K+1$ payments made (payments at times $0, 1, 2, \ldots, K$).

The PVRV is $\boldsymbol{Y = 1 + v + v^2 + \cdots + v^K = \ddot{a}_{\overline{K+1}|} = \frac{1-v^{K+1}}{d} = \frac{1-e^{-\delta(K+1)}}{d}}$.

Note that Y is a discrete random variable that takes on the following possible values with corresponding probabilities based on the listed events:

Completed No. Years until Death, K	PV of annuity, Y	Probability
$K=0$	$Y=\ddot{a}_{\overline{1}|}$	$P[K=0]=q_x$
$K=1$	$Y=\ddot{a}_{\overline{2}|}$	$P[K=1]={}_{1|}q_x$
$K=2$	$Y=\ddot{a}_{\overline{3}|}$	$P[K=2]={}_{2|}q_x$
$\vdots$	$\vdots$	$\vdots$
$K=k$	$Y=\ddot{a}_{\overline{k+1}|}$	$P[K=k]={}_{k|}q_x$
$\vdots$	$\vdots$	$\vdots$

$$\textbf{APV} = \boldsymbol{E[Y] = \ddot{a}_x} = \ddot{a}_{\overline{1}|}\cdot q_x + \ddot{a}_{\overline{2}|}\cdot {}_{1|}q_x + \ddot{a}_{\overline{3}|}\cdot {}_{2|}q_x + \cdots + \ddot{a}_{\overline{k+1}|}\cdot {}_{k|}q_x + \cdots$$

$$\boldsymbol{= E[\ddot{a}_{\overline{K+1}|}] = \sum_{k=0}^{\infty} \ddot{a}_{\overline{k+1}|}\, {}_kp_x\, q_{x+k}} \quad \text{(aggregate payment form)}$$

$$\boldsymbol{= \sum_{k=0}^{\infty} v^k\, {}_kp_x = \sum_{k=0}^{\infty} {}_kE_x = 1 + vp_x + v^2\, {}_2p_x + \cdots} \quad \text{(current payment form).}$$

In the following diagram, each row represents the payments made based on death occurring in a particular year. The first row corresponds to death in the first year, $K=0$, so only the payment at age x is made, and the PV of the amount paid is $\ddot{a}_{\overline{1}|}$ (the first row in the table above, which occurs with probability q_x). The second row in the diagram corresponds to death in the second year, $K=1$, so only payments at ages x and $x+1$ are made, and the PV of the amount paid is $\ddot{a}_{\overline{2}|}$ (the second row of the table above, which occurs with probability ${}_{1|}q_x$), etc. Each row represents one possible outcome related to the PVRV Y.

PV	prob.	K	x	$x+1$	$x+2$	$x+3$	$\cdots$	$x+k-1$	$x+k$	$\cdots$
$\ddot{a}_{\overline{1}|}$	q_x	0	1							
$\ddot{a}_{\overline{2}|}$	${}_{1|}q_x$	1	1	1						
$\ddot{a}_{\overline{3}|}$	${}_{2|}q_x$	2	1	1	1					
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$						
$\ddot{a}_{\overline{k}|}$	${}_{k-1|}q_x$	$k-1$	1	1	1	1	$\cdots$	1		
$\ddot{a}_{\overline{k+1}|}$	${}_{k|}q_x$	k	1	1	1	1	$\cdots$	1	1	

Current payment form is an expression for a life annuity APV as a sum of pure endowments, and aggregate payment form is an expression using the basic formulation for the expected value $E[Y]$. The current payment form is usually more convenient to use for calculations.
The current payment form can be derived from the aggregate payment form by using the relationship ${}_{k|}q_x = {}_kp_x\, q_{x+k} = {}_kp_x - {}_{k+1}p_x$ (this results in a "summation by parts").
In general, the APV of a discrete life annuity may be formulated as the sum of a collection of pure endowments, with each pure endowment associated with a payment amount to be made at a specific point in time contingent on survival to that point in time.

As k gets larger, so does $\ddot{a}_{\overline{k+1}|}$, and as $k \to \infty$, the limit of $\ddot{a}_{\overline{k+1}|}$ is $\ddot{a}_{\overline{\infty}|} = \frac{1}{d}$.
For instance, if $i = .08$, then $d = .0741$. The possible values of the PVRV Y are
$\ddot{a}_{\overline{1}|} = 1$, $\ddot{a}_{\overline{2}|} = 1 + v = 1.9259$, $\ddot{a}_{\overline{3}|} = 2.7833$, $\ldots$. The insurer will have to make at least the first payment of 1, but under no circumstances will the present value of the amount paid by the insurer be any more than $\ddot{a}_{\overline{\infty}|} = \frac{1}{d} = 13.5$.

In the notes on life insurance, it was pointed out that in order to find probabilities for an event related to a present value random variable, it is usually most efficient to translate the event into one involving the time until death random variable. The same is true for life annuities. This is illustrated in the following example.

Example 59: Survival is assumed to be based on the Illustrative Life Table. Y is the present value random variable for a whole life annuity-due of 1 per year starting at age 50. The annual effective rate of interest is .08.
Find (a) $P[Y < 3]$, (b) $P[Y \geq 10]$, and (c) $P[Y \leq 15]$.
Solution: (a) From calculations above, we see that $\ddot{a}_{\overline{3}|} = 2.7833$ and $\ddot{a}_{\overline{4}|} = 3.5771$.
Y denotes the present value of the annuity payments, and therefore, $P[Y < 3]$ is the probability that (50) receives at most 3 payments. This occurs if (50) dies before receiving the 4th payment, which is scheduled to be paid at age 53. Therefore,
$P[Y < 3] = {}_3q_{50} = 1 - \frac{\ell_{53}}{\ell_{50}} = .0192$ (this is also equal to $P[K + 1 \leq 3] = P[K \leq 2]$; note that $K = 2$ corresponds to 3 payments being made).

(b) We can proceed as we did in part (a) and calculate values of $\ddot{a}_{\overline{n}|}$, until we see that $\ddot{a}_{\overline{17}|} = 9.85 < 10 < 10.12 = \ddot{a}_{\overline{18}|}$. Therefore, $P[Y \geq 10]$ is the probability that (50) receives at least 18 payments. Since the 18th payment is made at time 17, (50) must survive at least 17 years in order to have $Y \geq 10$. $P[Y \geq 10] = {}_{17}p_{50} = \frac{\ell_{67}}{\ell_{50}} = .8046$.

Note that we can solve for n in the equation $\ddot{a}_{\overline{n}|} = \frac{1-v^n}{.0741} = 10$. We get $n = \frac{ln\,.259}{ln\,v} = 17.5$. Since n must be an integer, we must have that n is at least 18 as before.

(c) The present value of the annuity payments can be no larger than $\ddot{a}_{\overline{\infty}|} = \frac{1}{d} = 13.5$, so Y must be ≤ 15 ; therefore, $P[Y \leq 15] = 1$.

In (a) and (b), the event involving the annuity present value was translated into an event involving the survival of (50). Note that "small" values of Y correspond to "small" values of K. Also, note that $P[K+1 \leq k] = {}_kq_x$ and $P[K+1 \geq j] = {}_{j-1}p_x$. □

IMPORTANT NOTE: There is an algebraic relationship linking the PVRV of the discrete whole life annuity-due and the PVRV of the whole life insurance payable at the end of the year of death. Recall that the PVRV for the whole life insurance of 1 payable at the end of the year of death is $Z = v^{K+1}$.

$Y = \ddot{a}_{\overline{K+1}|} = \frac{1-v^{K+1}}{d} = \frac{1-Z}{d}$, $\rightarrow \ddot{a}_x = E[Y] = E[\frac{1-Z}{d}] = \frac{1-A_x}{d} \rightarrow \boldsymbol{A_x = 1 - d\ddot{a}_x}$,
and $\boldsymbol{Var[Y] = Var[\frac{1-Z}{d}] = \frac{1}{d^2}\, Var[Z] = \frac{1}{d^2}\, [{}^2A_x - (A_x)^2].}$

Note that if the force of interest is doubled, the relationship $A_x = 1 - d\ddot{a}_x$ becomes $\boldsymbol{{}^2A_x = 1 - {}^2d \cdot {}^2\ddot{a}_x}$, where ${}^2d = 2d - d^2$ is the annual effective discount rate that results when the force of interest is doubled. The variance of Y can be written in terms of annuity functions:

$$Var[Y] = \frac{1}{d^2}\,[\,{}^2A_x - (A_x)^2] = \frac{1}{d^2}\,[1 - (2d-d^2)\,{}^2\ddot{a}_x - (1 - d\ddot{a}_x)^2]$$
$$= \frac{2}{d}[\ddot{a}_x - {}^2\ddot{a}_x] + {}^2\ddot{a}_x - \ddot{a}_x^2 = E[Y^2] - (E[Y])^2 ,$$

where ${}^2\ddot{a}_x = \sum_{k=0}^{\infty} v^{2k}\,{}_kp_x = \sum_{k=0}^{\infty} e^{-2\delta k}\,{}_kp_x$.

Note also that $E[Y^2] = \frac{2}{d}[\ddot{a}_x - {}^2\ddot{a}_x] + {}^2\ddot{a}_x$.

The Exam MLC Illustrative Life Table insurance and annuity values are based on $i = .06$.
For example, for $x = 50$, we have
$Var[Y] = \frac{1}{d^2}[{}^2A_x - (A_x)^2] = \frac{1}{(.0566)^2}[{}^2A_{50} - A_{50}^2] = \frac{1}{(.0566)^2}[.09476 - (.24905)^2] = 10.22.$

Whole life annuity-due of 1 per year

$$Y = 1 + v + v^2 + \cdots + v^K = \ddot{a}_{\overline{K+1}|} = \frac{1-v^{K+1}}{d} = \frac{1-Z}{d}$$

$E[Y] = \ddot{a}_x = \sum_{k=0}^{\infty} v^k \,{}_kp_x$ (current payment form)

$= \sum_{k=0}^{\infty} \ddot{a}_{\overline{k+1}|} \,{}_kp_x \, q_{x+k}$ (aggregate payment form)

$$Var[Y] = \frac{1}{d^2} Var[Z] = \frac{1}{d^2}[{}^2A_x - (A_x)^2]$$

The current payment representation is most convenient when considering a life annuity with payments that are varying. Suppose a life annuity-due issued to (x) has payment c_k at age $x+k$, if alive. The actuarial present value in current payment form is
$c_0 + c_1 v p_x + c_2 v^2 \,{}_2p_x + \cdots = \sum_{k=0}^{\infty} c_k \, v^k \,{}_kp_x$.

MLC SECTION 14 - EXERCISES

1. Suppose the survival model is $S_0(t) = 1 - \frac{t}{100}$ for $0 \le t \le 100$, and $i = .08$.
Find $\ddot{a}_{90}$, ${}^2\ddot{a}_{90}$ and $Var[Y]$, where Y is the PVRV for a whole life annuity-due of 1 issued to (90).

2. Y is the PVRV for a whole life annuity-due of 1 per year issued to (50). Use the Exam MLC Illustrative Life Table at 6% to find the probability $P[Y > \ddot{a}_{50}]$.

MLC SECTION 14 - SOLUTIONS TO EXERCISES

1. This is a DeMoivre model, so that ${}_kp_{90} = \frac{100-90-k}{100-90} = \frac{10-k}{10}$.

$$\ddot{a}_{90} = \sum_{k=0}^{\infty} v^k \, {}_kp_{90} = \sum_{k=0}^{9} v^k (\tfrac{10-k}{10}) = (\tfrac{1}{10})(10 + 9v + 8v^2 + \cdots + v^9)$$
$$= (\tfrac{1}{10})(D\ddot{a})_{\overline{10}|.08} = (\tfrac{1}{10})(\tfrac{10 - a_{\overline{10}|.08}}{d}) = (\tfrac{1}{10})(\tfrac{10-6.710}{.0741}) = 4.44.$$

$${}^2\ddot{a}_{90} = \sum_{k=0}^{\infty} v^{2k} \, {}_kp_{90} = \sum_{k=0}^{9} v^{2k} (\tfrac{10-k}{10}) = (\tfrac{1}{10})[10 + 9v^2 + 8(v^2)^2 + \cdots + (v^2)^9]$$
$$= (\tfrac{1}{10})({}^2D\ddot{a})_{\overline{10}|.08} = (\tfrac{1}{10})(\tfrac{10 - {}^2a_{\overline{10}|}}{2d-d^2}) = (\tfrac{1}{10})(\tfrac{10-4.720}{.1427}) = 3.70.$$

$Var[Y] = \frac{2}{d}[\ddot{a}_{90} - {}^2\ddot{a}_{90}] + {}^2\ddot{a}_{90} - \ddot{a}_{90}^2 = 3.97$, or
$Var[Y] = \frac{1}{d^2}[{}^2A_{90} - A_{90}^2] = (\frac{1}{.0741})^2[\frac{{}^2a_{\overline{10}|}}{10} - (\frac{a_{\overline{10}|}}{10})^2] = 3.97$.

2. From the table we have $\ddot{a}_{50} = 13.2668$. We solve the following equation for k:
$\ddot{a}_{\overline{k}|} = \frac{1-v^k}{d} = 13.2668$; $k = \frac{ln(1-13.2668d)}{ln\, v} = 23.8$. Therefore, $P[Y > 13.2668]$
is equal to the probability that (50) receives at least 24 payments. This is the probability that (50) survives at least 23 years (the 24th payment is at age $50 + 23 = 73$). Thus,
$P[Y > \ddot{a}_{50}] = P[Y > 13.2668] = {}_{23}p_{50} = \frac{\ell_{73}}{\ell_{50}} = .661.$

MLC - PROBLEM SET 4

PROBLEM SET 4 -MULTIPLE CHOICE PROBLEMS

1. If $\overline{a}_{50} = 12.76$ when $i = .06$, and $\overline{a}_{50} = 7.72$ when $i = .1226$, what is the variance (nearest \$1,000) of the present value random variable for a continuous life annuity of \$100 per year issued to (50) at $i = .06$?
A) 100,000 B) 102,000 C) 104,000 D) 106,000 E) 108,000

2. If $\ddot{a}_{50} = 13.267$, $\ddot{a}_{70} = 8.569$ and $a_{50:\overline{20}|} = 45.655$, then ${}_{20}E_{50} =$
A) .23 B) .24 C) .25 D) .26 E) .27

3. A discrete 20 year temporary life annuity-due of \$20,000 per year is issued to (60) . The annual effective interest rate is 4% and mortality is on the basis of the MLC Illustrative Table. What is the probability that the present value random variable (of the annuity-due) is greater than \$150,000 ?
A) .82 B) .83 C) .84 D) .85 E) .86

4. A special continuous life annuity issued to (50) is designed so that the payments continue until 10 years after the death of (50) . If $\delta = .08$ and mortality follows the pattern $\mu_x = .04$ for all x , what is the net single premium ?
A) 10.0 B) 10.2 C) 10.4 D) 10.6 E) 10.8

5. For the three quantities $R = (I\ddot{a})_{x:\overline{n}|}$, $S = (D\ddot{a})_{x:\overline{n}|}$ and $T = \left(\frac{n+1}{2}\right) \cdot \ddot{a}_{x:\overline{n}|}$, which of the following is the correct ranking regarding their sizes (assuming $i \geq 0$)?
A) R $\leq$ S $\leq$ T B) R $\leq$ T $\leq$ S C) S $\leq$ R $\leq$ T D) T $\leq$ R $\leq$ S E) T $\leq$ S $\leq$ R

6. For a certain survival distribution, it is found that if $\delta = .10$, then $\overline{a}_x = \frac{100}{x+1}$ for $x \geq 10$. Find μ_{50}.
A) .33 B) .36 C) .39 D) .42 E) .45

7. If $a_{20:\overline{40}|} = 22.0112$ and $\ddot{s}_{20:\overline{40}|} = 93.2535$, then the value of $\ddot{s}_{21:\overline{39}|}$ is nearest

A) 89.00 B) 89.15 C) 89.30 D) 89.45 E) 89.60

8. Given $i = .08,\ s^{(12)}_{\overline{1}|.08} = 1.03616,\ \ddot{a}_x = 10.6123$ and ${}^2\ddot{a}_x = 5.8922$, what is $Var[Z]$, where Z is the present value random variable for a whole life insurance benefit of 1 payable at the end of the month of (x)'s death? Assume UDD in each year of age.

A) .10 B) .11 C) .12 D) .13 E) .14

9. Which of the following expressions is equal to 1 ?

A) $i^{(m)} \cdot \ddot{a}^{(m)}_{x:\overline{n}|} + A^{(m)}_{x:\overline{n}|}$ B) $i^{(m)} \cdot \ddot{a}^{(m)}_{x:\overline{n}|} + (1 + \frac{i^{(m)}}{m}) \cdot A^{(m)}_{x:\overline{n}|}$ C) $i^{(m)} \cdot \left(\ddot{a}^{(m)}_{x:\overline{n}|} - \frac{1}{m}\right) + A^{(m)}_{x:\overline{n}|}$

D) $i^{(m)} \cdot \left(\ddot{a}^{(m)}_{x:\overline{n}|} - \frac{1}{m}\right) + (1 + \frac{i^{(m)}}{m}) \cdot A^{(m)}_{x:\overline{n}|}$ E) None of A, B, C or D

10. You are given:

(i) $\overline{A}_x$ and $\overline{a}_x$ are based on force of interest δ and force of mortality μ_{x+t}.

(ii) $\overline{A}'_x$ and $\overline{a}'_x$ are based on force of interest $k + \delta$ and force of mortality μ_{x+t}.

(iii) $\overline{A}''_x$ and $\overline{a}''_x$ are based on force of interest δ and force of mortality $k + \mu_{x+t}$.

Determine $\overline{A}''_x - \overline{A}_x$.

A) $k\overline{a}_x$ B) $\overline{A}'_x - \overline{A}_x$ C) $\overline{A}'_x + k\overline{A}_x$ D) $(k-\delta)\overline{a}'_x + \delta\overline{a}_x$ E) $\delta(\overline{a}_x - \overline{a}'_x)$

11. T is the random variable for future lifetime of (x). Determine $Cov[\overline{a}_{\overline{T}|}\,,\ v^T\,]$.

A) $\frac{\overline{A}_x^2 - {}^2\overline{A}_x}{\delta}$ B) $\overline{A}_x^2 - {}^2\overline{A}_x$ C) 0 D) ${}^2\overline{A}_x - \overline{A}_x^2$ E) $\frac{{}^2\overline{A}_x - \overline{A}_x^2}{\delta}$

12. For (x) you are given:

(i) $\mu_{x+t} = \frac{-0.024}{ln(0.4)}$, for $t \geq 0$ (ii) $\delta = 0.03$

Calculate the probability that $\overline{a}_{\overline{T_x}|}$ will exceed 20.

A) 0.45 B) 0.55 C) 0.67 D) 0.74 E) 0.82

13. You are given: (i) $Var[\overline{a}_{\overline{T}|}] = \frac{100}{9}$ (ii) $\mu_{x+t} = k$ for all t (iii) $\delta = 4k$

Calculate k.

A) .005 B) .010 C) .015 D) .020 E) .025

14. You are given:

(i) $\mu_{x+t} = \mu$ for $t \geq 0$ (ii) force of interest is δ

Which of the following is a correct expression for $\overline{a}_{x:\overline{1}|}$?

I. $\frac{1-e^{-(\mu+\delta)}}{(\mu+\delta)^2}$ II. $\frac{\ddot{a}_{x:\overline{1}|}+a_{x:\overline{1}|}}{2}$ III. $\frac{\ddot{a}_{x:\overline{1}|}-a_{x:\overline{1}|}}{-ln\, a_{x:\overline{1}|}}$ IV. $\frac{1-e^{-(\mu+\delta)}}{ln\, a_{x:\overline{1}|}}$

A) None B) I only C) II only D) III only E) IV only

15. Y is the present value random variable for a continuous life annuity of 1 on (x).

You are given:

(i) $\mu_{x+t} = \mu$ (ii) $\overline{a}_x = 5$ (iii) ${}^2\overline{a}_x = 4$

Calculate $Var(Y)$.

A) 15 B) 25 C) 35 D) 45 E) 55

16. For a 10-year deferred life annuity-due of 1 per year on (60), you are given:

(i) $S_0(t) = 1 - \frac{t}{100}$ for $0 \leq t \leq 100$ (ii) $i = 0$

Calculate the probability that the sum of the payments made under the annuity will exceed the actuarial present value, at issue, of the annuity.

A) 0.475 B) 0.500 C) 0.525 D) 0.550 E) 0.575

17. You are given:

(i) $\ddot{a}_x = 8$ for all integral x (ii) $i = 0.08$

Calculate ${}_8q_{30}$.

A) 0.263 B) 0.364 C) 0.537 D) 0.636 E) 0.737

18. You are given

k	$\ddot{a}_{\overline{k}|}$	$_{k-1|}q_x$
1	1.00	0.33
2	1.93	0.24
3	2.80	0.16
4	3.62	0.11

Calculate $\ddot{a}_{x:\overline{4}|}$.

A) 1.6 B) 1.8 C) 2.0 D) 2.2 E) 2.4

19. Consider the following present value random variables, where K is the curtate future lifetime of (x): $Y = \ddot{a}_{\overline{K_x+1}|}$ for $K \geq 0$, $Z = \begin{cases} \ddot{a}_{\overline{K_x+1}|} & 0 \leq K < n \\ \ddot{a}_{\overline{n}|} & K \geq n \end{cases}$.

You are given (i) $i = .06$ (ii) $A_x = .20755$ (iii) $a_{x:\overline{n-1}|} = 6$

Calculate $E[Y] - E[Z]$.

A) 4 B) 5 C) 6 D) 7 E) 8

20. Y is the present value random variable for a 30-year temporary life annuity of 1 payable at the beginning of each year while (x) survives. You are given:

(i) $i = 0.05$ (ii) $_{30}p_x = 0.7$ (iii) $^2A_{\overset{1}{x}:\overline{30}|} = 0.0694$ (iv) $A_{\overset{1}{x}:\overline{30}|} = 0.1443$

Calculate $E[Y^2]$.

A) 35.6 B) 47.1 C) 206.4 D) 218.0 E) 233.6

21. You are given:

(i) $_{10}E_{30} = 0.35$ (ii) $a_{30:\overline{9}|} = 5.6$ (iii) $i = 0.10$

Calculate $A_{\overset{1}{30}:\overline{10}|}$.

A) 0.05 B) 0.10 C) 0.15 D) 0.20 E) 0.25

22. You are given:

(i) ${}_{10}E_x = 0.40$ (ii) ${}_{10|}a_x = 7$ (iii) $\ddot{s}_{x:\overline{10}|} = 15$

Calculate $\ddot{a}_x$.

A) 12.0 B) 12.5 C) 12.9 D) 13.0 E) 13.4

23. Which of the following expression is equivalent to $ia_{x:\overline{n}|} + (1+i)A_{x:\overline{n}|} - 1$?

A) 0 B) $i \cdot {}_nE_x$ C) i D) ${}_nE_x$ E) 1

24. You are given:

(i) $1000A_{x:\overline{n}|} = 563$ (ii) $1000A_x = 129$ (iii) $d = 0.057$ (iv) $1000\,{}_nE_x = 543$

Calculate ${}_{n|}a_x$.

A) 7.07 B) 7.34 C) 7.61 D) 7.78 E) 7.94

25. T_x is the random variable for future lifetime of (x). You are given:

(i) $\mu_{x+t} = 0.04$, $t \geq 0$ (ii) $\delta = 0.06$

(a) Find $\frac{\partial}{\partial n}\,{}_nE_x$ (b) Find $\overline{a}_{x:\overline{n}|}$ (c) Find the standard deviation of $\overline{a}_{\overline{T_x}|}$ (d) Find $\frac{\partial}{\partial x}\,\overline{a}_{x:\overline{n}|}$

26. Which of the following is true, regardless of the assumption about deaths within each year of age?

I. $\overline{A}_x = \frac{i}{\delta}A_x$ II. $\overline{a}_x = \overline{a}_{x:\overline{1}|} + \delta p_x \overline{a}_{x+1}$ III. $\overline{a}_x = \frac{id}{\delta^2} \cdot \ddot{a}_x - \frac{i-\delta}{\delta^2}$ IV. $\overline{A}_x = 1 - \delta\overline{a}_x$

A) None B) I only C) II only D) III only E) IV only

27. $(\overline{I}_{\overline{n}|}\overline{a})_x$ is equal to $E(Y)$ where $Y = \begin{cases} (\overline{I}\,\overline{a})_{\overline{T_x}|} & 0 \leq T_x < n \\ (\overline{I}\,\overline{a})_{\overline{n}|} + n(\,{}_{n|}\overline{a}_{\overline{T_x - n}|}) & T_x \geq n \end{cases}$.

You are given: (i) $\mu_x = .04$ for all x (ii) $\delta = .06$.

Calculate $\frac{d}{dn}\,(\overline{I}_{\overline{n}|}\overline{a})_x$.

A) $ne^{-.1n}$ B) $10e^{-.1n}$ C) $-e^{-.1n}$ D) $e^{-.1n}$ E) 10

28. Which of the following identities are correct?

I. ${}_{n|}\ddot{a}_x = a_x - a_{x:\overline{n-1}|}$ II. $1 + a_{x:\overline{n-1}|} = (1+i)a_{x:\overline{n}|}$ III. $A_{x:\overline{n}|} = v\ddot{a}_{x:\overline{n}|} - a_{x:\overline{n-1}|}$

A) I B) I, II C) I, III D) II, III E) I, II, III

29. A company has just assumed responsibility for paying pensions to a group of 100 people who have just turned 65. The pension will pay each of them $25,000 annually, payable continuously while they are alive. Assume that all the lives are independent, and that the same mortality table applies to all of them. In addition, you are given:

(i) $\delta = 0.06$ (ii) ${}^2\overline{A}_{65} = 0.2$

(iii) The actuarial present value of the portfolio of 100 pensions is $30,000,000.

Calculate the standard deviation of the total present value of the payments for those 100 pensions.

A) Less than $1,000,000 B) At least $1,000,000 but less than $1,250,000

C) At least $1,250,000 but less than $1,500,000

D) At least $1,500,000 but less than $1,750,000 E) At least $1,750,000

30. Ia and $I\ddot{a}$ represent the standard increasing annuities. A person aged 20 buys a special five-year temporary life annuity-due, with payments of 1, 3, 5, 7, and 9.

Given:

i) $\ddot{a}_{20:\overline{4}|} = 3.41$ ii) $a_{20:\overline{4}|} = 3.04$

iii) $(I\ddot{a})_{20:\overline{4}|} = 8.05$ iv) $(Ia)_{20:\overline{4}|} = 7.17$

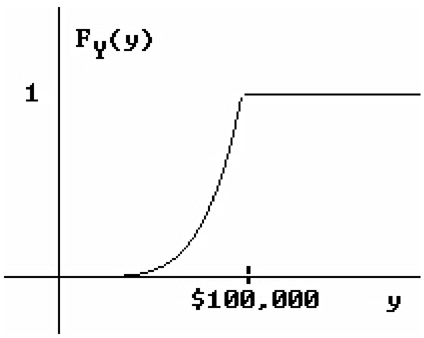
Calculate the net single premium.

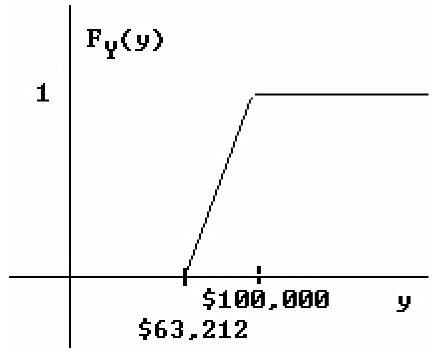
A) Less than 18.0 B) At least 18.0, but less than 18.5

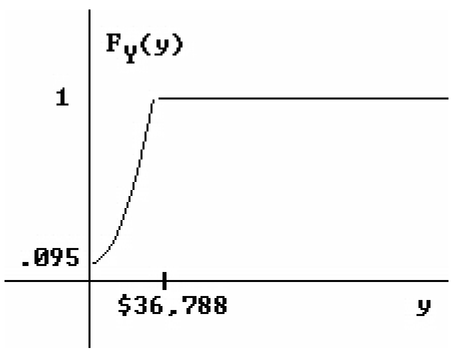
C) At least 18.5, but less than 19.0 D) At least 19.0, but less than 19.5

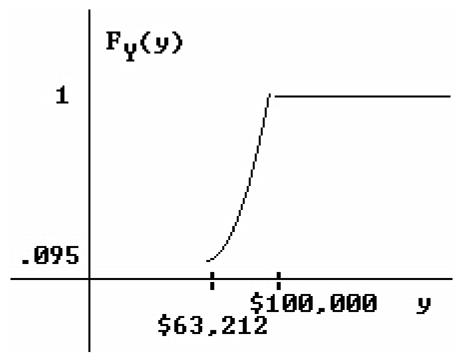
E) At least 19.5

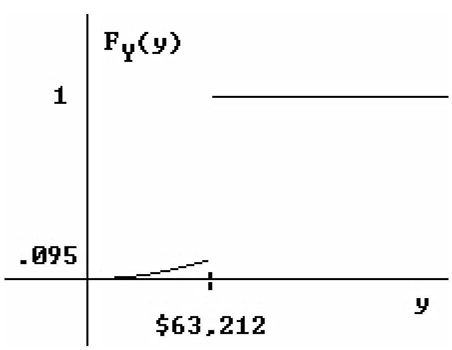
31. A 10-year certain and whole life annuity of amount \$10,000, payable continuously, is issued to (65). The force of mortality is constant at .01, and the force of interest is constant at .1 . Let Y be the random variable representing the present value of the benefits paid. Which of the following graphs correctly shows the cumulative probability distribution function, $F_Y(y)$, for the random variable?

A) 

B) 

C) 

D) 

E) 

32. Given: $v = 0.95$, ${}_{10}p_{25} = 0.87$, $\ddot{a}_{25:\overline{15}|} = 9.868$, $\ddot{a}_{35:\overline{5}|} = 4.392$

Find $A^{\;1}_{25:\overline{10}|}$.

A) Less than .16 B) At least .16, but less than .32

C) At least .32, but less than .48 D) At least .48, but less than .64

E) At least .64

33. You are given the following:

The probability that a newborn lives to be 25 is 70%.

The probability that a newborn lives to be 35 is 50%.

The following annuities-due each have actuarial present value equal to 60,000:

a life annuity-due of 7,500 on (25)

a life annuity-due of 12,300 on (35)

a life annuity-due of 9,400 on (25) that makes at most 10 payments

What is the interest rate?

A) 8.0% B) 8.1% C) 8.2% D) 8.3% E) 8.4%

34. A special 30 year annuity-due on a person age 30 pays 10 for the first 10 years, 20 for the next 10 years, and 30 for the last 10 years. You are given:

${}_{20}E_{30} = m$, $\ddot{a}_{30:\overline{10}|} = u$, $\ddot{a}_{30:\overline{30}|} = v$, $\ddot{a}_{50:\overline{10}|} = w$

Which of the following represents the actuarial present value of this annuity?

A) $20v + 10w(1 - m)$ B) $10u + 20v + 10w$ C) $10u + 20v + 10mw$

D) $-10u + 20v + 10mw$ E) $-20u + 20v + 10w$

35. "Actuary Today", the magazine, offers 1-year subscriptions for $25, payable at the start of the year. Given a renewal rate of 90% per year, what is the actuarial present value of a new subscriber? (Assume a discount factor of 0.95).

A) Less than $150 B) At least $150, but less than $160

C) At least $160, but less than $170 D) At least $170, but less than $180

E) $180 or more

36. A 60-year-old lottery winner has the choice of receiving (i) a single lump sum payment of P_1 in 10 years if she is still alive or (ii) a 20-year life annuity-due of P_2 per year beginning today. Given:

- The payouts in (i) and (ii) are actuarially equivalent.
- $d = 6\%$
- $A_{\substack{\ \ 1\\60:\overline{10}|}} = 0.4188$
- ${}^2A_{\substack{\ \ 1\\60:\overline{10}|}} = 0.2339$
- $A_{60:\overline{20}|} = 0.4417$
- ${}^2A_{60:\overline{20}|} = 0.2312$

Calculate the difference in the variances of the payouts in (i) and (ii).

A) $0.04P_1^2$ B) $0.12P_1^2$ C) $0.20P_1^2$ D) $0.33P_1^2$ E) $0.41P_1^2$

37. You are given:

- The force of mortality, μ, is constant.
- The force of interest, δ, is constant.

Which of the following represents the probability that $\overline{a}_{\overline{T}|}$ will exceed $\overline{a}_x$ for a life age x?

A) $[\mu/(\delta+\mu)]^{(\delta/\mu)}$ B) $[\mu/(\delta+\mu)]^{(\mu/\delta)}$ C) $[\delta/(\delta+\mu)]^{(\delta/\mu)}$
D) $[\delta/(\delta+\mu)]^{(\mu/\delta)}$ E) $1/(\delta+\mu)$

38. The distribution of Jack's future lifetime is a two-point mixture:
(i) With probability 0.60, Jack's future lifetime follows the Illustrative Life Table, with deaths uniformly distributed over each year of age.
(ii) With probability 0.40, Jack's future lifetime follows a constant force of mortality $\mu = 0.02$.
(iii) A fully continuous whole life insurance of 1000 is issued on Jack at age 62.
Calculate the APV for a continuous whole life annuity of 1 per year for Jack if $i = .06$.

39. The force of mortality is constant at μ and the force interest is constant at δ and you are given that $\overline{a}_x = 12.5$. Use the Woolhouse approximation to 3 terms to find $\ddot{a}_x^{(12)}$.

A) Less than 12.20 B) At least 12.20 but less than 12.40
C) At least 12.40 but less than 12.60 D) At least 12.60 but less than 12.80
E) At least 12.80

40. For a group of individuals all of age x, of which 30% are smokers and 70% are non-smokers, you are given:
(i) $\delta = 0.10$ (ii) $\overline{A}_x^{\text{smoker}} = 0.444$ (iii) $\overline{A}_x^{\text{non-smoker}} = 0.286$
(iv) T_x is the future lifetime of (x)
(v) $Var[\overline{a}_{\overline{T_x}|}^{\text{smoker}}] = 8.818$ (vi) $Var[\overline{a}_{\overline{T_x}|}^{\text{non-smoker}}] = 8.503$
Calculate $Var[\overline{a}_{\overline{T}|}]$ for an individual chosen at random from this group.
A) 8.5 B) 8.6 C) 8.8 D) 9.0 E) 9.1

41. You are given:
(i) $\mu_{x+t} = c\,,\ t \geq 0$ (ii) $\delta = .08$ (iii) $\overline{A}_x = .3443$
(iv) T_x is the future lifetime random variable for (x).
Calculate $Var(\overline{a}_{\overline{T_x}|})$.
A) 12 B) 14 C) 16 D) 18 E) 20

42. For a three-year temporary life annuity due of 100 on (75), you are given:
(i) $\int_0^x \mu_t\, dt = 0.01x^{1.2}\,,\ x > 0$ (ii) $i = 0.11$
Calculate the actuarial present value of this annuity.
A) 264 B) 266 C) 268 D) 270 E) 272

43. For a special fully discrete, 30-year deferred, annual life annuity-due of 200 on (30), you are given:
(i) The net single premium is refunded without interest at the end of the year of death
if death occurs during the deferral period.
(ii) Mortality follows the Illustrative Life Table (iii) $i = 0.06$
Calculate the net single premium for this annuity.
A) 350 B) 360 C) 370 D) 380 E) 390

44. For a special 30-year deferred annual whole life annuity-due of 1 on (35):
(i) If death occurs during the deferral period, then C% net single premium is refunded without interest at the end of the year of death.
(ii) $\ddot{a}_{65} = 9.90$
(iii) $A_{35:\overline{30|}} = 0.21$
(iv) $A^{1}_{35:\overline{30|}} = 0.07$
(v) The net single premium for this special deferred annuity is 1.4.
Calculate C.
A) Less than 10 B) At least 10 but less than 12 C) At least 12 but less than 14
D) At least 14 but less than 16 E) At least 16

45. You are given the following for every integer age x:
Assuming UDD over each year of age, calculate $\bar{a}_{20:\overline{10|}}$.
A) 6.0 B) 6.1 C) 6.2 D) 6.3 E) 6.4

46. An insurance company has agreed to make payments to a worker age x who was injured at work.
(i) The payments are 150,000 per year, paid annually, starting immediately and continuing for the remainder of the worker's life.
(ii) After the first 500,000 is paid by the insurance company, the remainder will be paid by a reinsurance company.
(iii) ${}_tp_x = \begin{cases} (.7)^t, & 0 \le t \le 5.5 \\ 0, & 5.5 < t \end{cases}$
(iv) $i = 0.05$
Calculate the actuarial present value of the payments to be made by the reinsurer.
A) Less than 50,000 B) At least 50,000, but less than 100,000
C) At least 100,000, but less than 150,000 D) At least 150,000, but less than 200,000
E) At least 200,000

47. For a whole life annuity-due of 1 on (x), payable annually:
(i) $q_x = 0.01$
(ii) $q_{x+1} = 0.05$
(iii) $i = 0.05$
(iv) $\ddot{a}_{x+1} = 6.951$
Calculate the change in the actuarial present value of this annuity-due if p_{x+1} is increased by 0.03.
A) 0.16 B) 0.17 C) 0.18 D) 0.19 E) 0.20

PROBLEM SET 4 - WRITTEN ANSWER PROBLEMS

1. Z_1 is the present value random variable for a 10-year deferred continuous life-annuity of \$10,000 per year issued to (40). Z_2 is the present value random variable for a 25-year continuous certain and whole life annuity issued to (40).
(a) (2 points) Show that the variance of Z_1 is the same as that of Z_2. Formulate (but do not calculate) the covariance between Z_1 and Z_2.
(b) (2 points) Suppose that the force of interest is $\delta = .05$, and the force of mortality is constant at .01 for all ages. Calculate $E[Z_1]$ and $E[Z_2]$.
(c) (4 points) With $\delta = .05$ and $\mu_{40+t} = .01$ for $t \geq 0$, formulate algebraically and sketch the graph of the distribution function of Z_1, ($F_{Z_1}(y) = P(Z_1 \leq y)$.
(d) (1 point) Describe in words the difference between the graph of F_{Z_1} and F_{Z_2} .

2. Z is the present value random variable for a 20-year deferred life annuity-due of 1 per year issued to (40). Survival follows the model $S(t) = 1 - \frac{t}{120}$, $t \geq 0$ (survival from birth). Interest is at annual effective rate 4%.
(a) (3 points) Calculate $E[Z]$, the net single premium for this annuity.
(b) (2 points) Formulate the random variable Z in terms of K_{60} , the curtate future lifetime of the annuitant,
(c) (3) Calculate the median of Z.

3. For a group of individuals all of age x, of which 30% are smokers and 70% are non-smokers, you are given:

(i) $\delta = 0.10$ (ii) $\mu_{x+t}^{\text{smoker}} = .02$ for $t \geq 0$ (iii) $\mu_{x+t}^{\text{non-smoker}} = .01$ for $t \geq 0$

(iv) T_x is the future lifetime of (x)

An individual is chosen at random from this group. Y denotes the present value random variable of the total payment made to the individual.

(a) (2 points) Calculate $E[Y]$ for this individual.

(b) (2 points) Calculate $Var[Y]$ for this individual.

(c) (2 points) Calculate $P[Y \leq 5.0]$

LIFE CONTINGENCIES - PROBLEM SET 4 SOLUTIONS

MULTIPLE CHOICE

1. $Y = \overline{a}_{\overline{T_{50}}|} = \frac{1-v^{T_{50}}}{\delta} \to Var[Y] = \frac{Var[v^{T_{50}}]}{\delta^2} = \frac{{}^2\overline{A}_{50} - (\overline{A}_{50})^2}{\delta^2}$.
But ${}^2\overline{A}_{50} = 1-(2\delta)\cdot {}^2\overline{a}_{50} = .10031$
and $\overline{A}_{50} = 1-\delta\cdot\overline{a}_{50} = .25647 \to Var[Y] = 10.17 \to Var[100Y] = 101,700$. Answer: B.

2. $a_{50} = a_{50:\overline{20}|} + {}_{20|}a_{50} = a_{50:\overline{20}|} + {}_{20}E_{50}\cdot a_{70} \to \frac{a_{50}}{{}_{20}\mathrm{E}_{50}} = s_{50:\overline{20}|} + a_{70}$
$\to {}_{20}E_{50} = \frac{a_{50}}{s_{50:\overline{20}|}+a_{70}} = \frac{\ddot{a}_{50}-1}{s_{50:\overline{20}|}+\ddot{a}_{70}-1} = \frac{12.267}{45.655+7.569} = .23$. Answer: A.

3. $P[20,000\,\ddot{a}_{\overline{K_{60}+1}|} \geq 150,000] = P[\ddot{a}_{\overline{K_{60}+1}|} \geq 7.5] = P\left[\frac{1-v^{K_{60}+1}}{d} \geq 7.5\right]$
$= P[v^{K_{60}+1} \leq .712] = P[(K_{60}+1)\,ln(v) \leq ln\,.712]$
$= P[K_{60} \geq 7.7] = P[K \geq 8] = {}_8p_{60} = \frac{\ell_{68}}{\ell_{60}} = \frac{7,018,432}{8,188,074} = .857$. Answer: E.

4. SBP $= E\left[\overline{a}_{\overline{T+10}|}\right] = E\left[\overline{a}_{\overline{T}|} + v^T\cdot\overline{a}_{\overline{10}|}\right] = \overline{a}_{50} + \overline{A}_{50}\cdot\overline{a}_{\overline{10}|}$. But ${}_tp_{50} = e^{-.04t}$
$\to \overline{a}_{50} = \int_0^\infty e^{-.08t}\cdot e^{-.04t}\,dt = \frac{1}{.12} = 8.3333 \to \overline{A}_{50} = 1-\delta\cdot\overline{a}_{50} = .3333$, and
$\overline{a}_{\overline{10}|} = \frac{1-e^{-10\delta}}{\delta} = \frac{1-e^{-.8}}{.08} = 6.68834 \to$ SBP $= 10.63$. Answer D.

5. Since all three series have the same total paid (i.e., $n \times \frac{n+1}{2}$), the sooner the payments are made, the larger the present value of the series. The decreasing annuity has the largest payments first and thus has the largest present value, and the increasing annuity has the smallest payments first, and has the smallest present value. Thus, R $\leq$ T $\leq$ S. Answer: B.

6. $\frac{d}{dx}\,\overline{a}_x = [\mu_x + \delta]\,\overline{a}_x - 1 \;\to\; \mu_x = [1 + \frac{d}{dx}\,\overline{a}_x]/\overline{a}_x - \delta = [1 - \frac{100}{(x+1)^2}]/[\frac{100}{x+1}] - \delta$
$\to\; \mu_x = \frac{x+1}{100} - \frac{1}{x+1} - \delta \;\to\; \mu_{50} = .51 - \frac{1}{51} - .10 = .390$. Answer: C.

7. $\ddot{s}_{21:\overline{39}|} = \ddot{s}_{20:\overline{40}|} - \frac{1}{{}_{40}E_{20}}$. But $a_{20:\overline{40}|} = \ddot{a}_{20:\overline{40}|} - 1 + {}_{40}E_{20} = {}_{40}E_{20} \cdot \ddot{s}_{20:\overline{40}|} - 1 + {}_{40}E_{20}$
$\rightarrow {}_{40}E_{20} = \frac{a_{20:\overline{40}|}+1}{\ddot{s}_{20:\overline{40}|}+1} = .24414 \rightarrow \ddot{s}_{21:\overline{39}|} = 93.2535 - \frac{1}{.24414} = 89.16$. Answer: B.

8. $Var[Z] = {}^2A_x^{(12)} - (A_x^{(12)})^2$.
Under UDD $A_x^{(12)} = \frac{i}{i^{(12)}} \cdot A_x = s_{\overline{1}|}^{(12)} \cdot (1 - d \cdot \ddot{a}_{\text{x}}) = .22163$.
${}^2A_x^{(12)} = \frac{{}^2i}{{}^2i^{(12)}} \cdot {}^2A_x$, where ${}^2i = (1+i)^2 - 1 = 21 + i^2 = .1664$,
${}^2i^{(12)} = 12 \cdot [\,(1 + \frac{i^{(12)}}{12})^2 - 1] = .15942$, since $i^{(12)} = \frac{i}{s_{\overline{1}|}^{(12)}} = .077208$,
and ${}^2A_x = 1 - {}^2d \cdot {}^2\ddot{a}_{\text{x}} = .15942$, since ${}^2d = \frac{{}^2i}{1+{}^2i} = .14266$.
Thus, ${}^2A_x^{(12)} = .17123$ and then $Var[Z] = .17123 - (,22163)^2 = .122$. Answer: C.

9. $A_{x:\overline{n}|}^{(m)} = 1 - d^{(m)} \cdot \ddot{s}_{x:\overline{n}|}^{(m)} \rightarrow (1 + \frac{i^{(m)}}{m}) \cdot A_{x:\overline{n}|}^{(m)} + i^{(m)} \cdot \ddot{a}_{x:\overline{n}|}^{(m)} = 1 + \frac{i^{(m)}}{m}$
$\rightarrow (1 + \frac{i^{(m)}}{m}) \cdot A_{x:\overline{n}|}^{(m)} + i^{(m)} \cdot \left(\ddot{a}_{x:\overline{n}|}^{(m)} - \frac{1}{m}\right) = 1$. Answer: D.

10. $\overline{A}_x'' - \overline{A}_x = 1 - \delta\overline{a}_x'' - (1 - \delta\overline{a}_x) = \delta(\overline{a}_x - \overline{a}_x'')$.
$\overline{a}_x'' = \int_0^\infty e^{-\delta t}\, {}_tp_x'' \cdot dt$, and
${}_tp_x'' = exp[-\int_0^t \mu_{x+s}''\, ds] = exp[-\int_0^t (k + \mu_{x+s})\, ds]$
$= exp[-kt - \int_0^t \mu_{x+s}\, ds] = e^{-kt} \cdot exp[-\int_0^t \mu_{x+s}\, ds] = e^{-kt} \cdot {}_tp_x$.
Then $\overline{a}_x''$ becomes
$\overline{a}_x'' = \int_0^\infty e^{-\delta t} \cdot {}_tp_x'' dt = \int_0^\infty e^{-\delta t} e^{-kt} \cdot {}_tp_x\, dt = \int_0^\infty e^{-(k+\delta)t} \cdot {}_tp_x\, dt = \overline{a}_x'$.
Therefore, $\overline{A}_x'' - \overline{A}_x = \delta(\overline{a}_x - \overline{a}_x'') = \delta(\overline{a}_x - \overline{a}_x')$. Answer: E

11. $Cov[\overline{a}_{\overline{T}|}\,,\, v^T] = Cov[\frac{1-v^T}{\delta}\,,\, v^T] = \frac{-1}{\delta} Cov[v^T\,,\, v^T] = \frac{-1}{\delta} Var[v^T]$
$= \frac{-1}{\delta} [\,{}^2\overline{A}_x - \overline{A}_x^2\,]$. Answer: A

12. $P[T > t] = {}_tp_x = e^{-\mu t} = exp[-(\frac{-0.024}{ln(0.4)})t]$.
$P[\overline{a}_{\overline{T(x)}|} > 20] = P[\frac{1-e^{-\delta T}}{\delta} > 20] = P[e^{-.03T} < .4] = P[T > \frac{ln(.4)}{-.03}]$
$= exp[-(\frac{-0.024}{ln(0.4)})(\frac{ln(.4)}{-.03})] = e^{-.8} = .45$. Answer: A

13. $Var[\overline{a}_{\overline{T}|}] = \frac{1}{\delta^2}[{}^2\overline{A}_x - (\overline{A}_x)^2]$. The force of mortality is constant, so we have
$\overline{A}_x = \frac{\mu}{\mu+\delta}$ and ${}^2\overline{A}_x = \frac{\mu}{\mu+2\delta}$.
Then $Var[\overline{a}_{\overline{T}|}] = \frac{1}{\delta^2} \cdot [\frac{\mu}{\mu+2\delta} - (\frac{\mu}{\mu+\delta})^2] = \frac{1}{(4k)^2} \cdot [\frac{k}{k+2(4k)} - (\frac{k}{k+4k})^2]$
$= \frac{1}{16k^2} \cdot [\frac{1}{9} - (\frac{1}{5})^2] = \frac{1}{16k^2} \cdot \frac{16}{225} = \frac{100}{9}$.
Solving for k results in $k = .02$. Answer: D

14. $\overline{a}_{x:\overline{1}|} = \int_0^1 e^{-\delta t} \cdot {}_tp_x \, dt = \int_0^1 e^{-\delta t} \cdot e^{-\mu t} \, dt = \int_0^1 e^{-(\delta+\mu)t} \, dt = \frac{1-e^{-(\delta+\mu)}}{\delta+\mu}$.
$\ddot{a}_{x:\overline{1}|} = 1$, $a_{x:\overline{1}|} = vp_x = e^{-\delta} \cdot e^{-\mu} = e^{-(\delta+\mu)}$, $-\ln a_{x:\overline{1}|} = \delta + \mu$.
III is correct. Answer: D

15. $Var[Y] = \frac{1}{\delta^2}[{}^2\overline{A}_x - (\overline{A}_x)^2]$. Since the force of mortality is a constant μ for all ages, we have $\overline{A}_x = \frac{\mu}{\mu+\delta}$ and ${}^2\overline{A}_x = \frac{\mu}{\mu+2\delta}$.
Also, $\overline{a}_x = \frac{1}{\mu+\delta} = 5$ and ${}^2\overline{a}_x = \frac{1}{\mu+2\delta} = 4$, so that $\mu + \delta = .2$, $\mu + 2\delta = .25$.
Using the two equations we can solve for μ and δ; $\mu = .15$, $\delta = .05$.
Then, $\overline{A}_x = \frac{\mu}{\mu+\delta} = \frac{.15}{.15+.05} = .75$ and ${}^2\overline{A}_x = \frac{\mu}{\mu+2\delta} = \frac{.15}{.15+.1} = .6$, so that
$Var[Y] = \frac{1}{\delta^2}[{}^2\overline{A}_x - (\overline{A}_x)^2] = \frac{1}{(.05)^2} \cdot [.6 - (.75)^2] = 15$.
Note that it is also possible to formulate $Var[Y]$ directly in terms of annuities:
$Var[Y] = \frac{2}{\delta}(\overline{a}_x - {}^2\overline{a}_x) - (\overline{a}_x)^2 = \frac{2}{\delta}(5-4) - (5)^2 = \frac{2}{\delta} - 25$. We still must find δ by solving the two equations in μ and δ that were given above, $\mu + \delta = .2$, $\mu + 2\delta = .25$.
$Var[Y] = \; = \frac{2}{\delta} - 25 = \frac{2}{.05} - 25 = 15$. Answer: A

16. The APV is ${}_{10|}\ddot{a}_{60} = \sum_{k=10}^{\omega-60-1} v^k \, {}_kp_{60} = \sum_{k=10}^{39} \frac{100-60-k}{100-60} = \frac{1}{40}(30 + 29 + \cdots + 1)$
$= \frac{1}{40} \cdot \frac{(30)(31)}{2} = 11.625$.
P[sum of payments exceeds 11.625] $= P$[at least 12 payments].
The 12th payment is made at age 81 (1st payment is at age 70), so that
P[at least 12 payments] $= P$[survival to at least age 81] $= {}_{21}p_{60} = \frac{100-60-21}{100-60} = .475$.
Answer: A

17. $\ddot{a}_y = 1 + vp_y\ddot{a}_{y+1} \rightarrow 8 = 1 + \frac{1}{1.08} \cdot p_y \cdot (8) \rightarrow p_y = .945$ (for any integral y).
Then, ${}_8p_{30} = p_{30} \cdot p_{31} \cdots p_{37} = (.945)^8 = .636 \rightarrow {}_8q_{30} = .364$. Answer: B

18. The information is given in a form which appears to be more convenient for the application of the aggregate payment formulation of the annuity:

$\ddot{a}_{x:\overline{4}|} = \sum_{j=0}^{3} \ddot{a}_{\overline{j+1}|} \cdot {}_{j|}q_x + \ddot{a}_{\overline{4}|} \cdot {}_4p_x = \ddot{a}_{\overline{1}|} \cdot q_x + \ddot{a}_{\overline{2}|} \cdot {}_{1|}q_x + \ddot{a}_{\overline{3}|} \cdot {}_{2|}q_x + \ddot{a}_{\overline{4}|} \cdot {}_{3|}q_x + \ddot{a}_{\overline{4}|} \cdot {}_4p_x$

$= (1)(.33) + (1.93)(.24) + (2.80)(.16) + (3.62)(.11) + (3.62) \cdot {}_4p_x.$

We use the relationship $\sum_{j=0}^{n-1} {}_{j|}q_x = {}_nq_x$, so that $\sum_{j=0}^{3} {}_{j|}q_x = q_x + {}_{1|}q_x + {}_{2|}q_x + {}_{3|}q_x = .84 = {}_4q_x.$

Then ${}_4p_x = .16$ and

$\ddot{a}_{x:\overline{4}|} = (1)(.33) + (1.93)(.24) + (2.80)(.16) + (3.62)(.11) + (3.62) \cdot (.16) = 2.22.$

Answer: D

19. Y is the PVRV for a whole life annuity-due of 1 per year, so that

$E[Y] = \ddot{a}_x = \frac{1-A_x}{d} = \frac{1-A_x}{i/(1+i)} = \frac{1-.20755}{.06/1.06} = 14$.

Z is the PVRV for an n-year temporary life annuity-due of 1 per year, so that

$E[Z] = \ddot{a}_{x:\overline{n}|} = 1 + a_{x:\overline{n-1}|} = 1 + 6 = 7.$

Then, $E[Y] - E[Z] = 14 - 7 = 7.$ Answer: D

20. $Var[Y] = E[Y^2] - (E[Y])^2 \rightarrow E[Y^2] = Var[Y] + (E[Y])^2.$

For an n-year temporary annuity due of 1 per year, $Var[Y] = \frac{1}{d^2}[\,{}^2A_{x:\overline{n}|} - (A_{x:\overline{n}|})^2]$

Therefore, $Var[Y] = \frac{1}{d^2}[\,{}^2A_{x:\overline{30}|} - (A_{x:\overline{30}|})^2].$

$d = \frac{i}{1+i} = .047619$, $A_{x:\overline{30}|} = A^{1}_{x:\overline{30}|} + A_{x:\overline{30}|}^{\;\;\;\,1} = .1443 + v^{30}\,{}_{30}p_x = .3063$.

${}^2A_{x:\overline{30}|} = {}^2A^{1}_{x:\overline{30}|} + {}^2A_{x:\overline{30}|}^{\;\;\;\,1} = .0694 + v^{60}\,{}_{30}p_x = .1069$,

$\rightarrow Var[Y] = \frac{1}{(.047619)^2}[.1069 - (.3063)^2] = 5.768$.

$E[Y] = \ddot{a}_{x:\overline{30}|} = \frac{1-A_{x:\overline{30}|}}{d} = \frac{1-.3063}{.047619} = 14.57.$

$E[Y^2] = Var[Y] + (E[Y])^2 = 5.77 + (14.57)^2 = 218.$ Answer: D

21. $A_{30:\overline{10}|}^{\;\;\;\;\;1} = A_{30:\overline{10}|} - {}_{10}E_{30} = 1 - d\ddot{a}_{30:\overline{10}|} - {}_{10}E_{30} = 1 - d(1 + a_{30:\overline{9}|}) - {}_{10}E_{30} = .05.$

Answer: A

22. ${}_{10|}a_x = {}_{10}E_x \cdot a_{x+10}$
$\rightarrow {}_{10|}\ddot{a}_x = {}_{10}E_x \cdot \ddot{a}_{x+10} = {}_{10}E_x \cdot (1 + a_{x+10}) = {}_{10}E_x + {}_{10|}a_x = .4 + 7 = 7.4$
$\rightarrow \ddot{a}_x = \ddot{a}_{x:\overline{10}|} + {}_{10|}\ddot{a}_x = {}_{10}E_x \cdot \ddot{s}_{x:\overline{10}|} + {}_{10|}\ddot{a}_x = (.4)(15) + 7.4 = 13.4.$ Answer: E

23. $ia_{x:\overline{n}|} + (1+i)A_{x:\overline{n}|} - 1 = i \cdot [\ddot{a}_{x:\overline{n}|} - 1 + {}_nE_x] + (1+i) \cdot (1 - d \cdot \ddot{a}_{x:\overline{n}|}) - 1$
$= i \cdot \ddot{a}_{x:\overline{n}|} - i + i \cdot {}_nE_x + 1 + i - i \cdot \ddot{a}_{x:\overline{n}|} - 1 = i \cdot {}_nE_x.$ Answer: B

24. ${}_{n|}a_x = {}_nE_x \cdot a_{x+n} = {}_nE_x \cdot (\ddot{a}_{x+n} - 1) = {}_nE_x \cdot (\frac{1 - A_{x+n}}{d} - 1).$
From $A_{x:\overline{n}|} = A_{\overset{1}{x}:\overline{n}|} + {}_nE_x$, we get $.563 = A_{\overset{1}{x}:\overline{n}|} + .543 \rightarrow A_{\overset{1}{x}:\overline{n}|} = .02.$
Then $A_x = A_{\overset{1}{x}:\overline{n}|} + {}_nE_x \cdot A_{x+n} \rightarrow .129 = .02 + (.543)A_{x+n} \rightarrow A_{x+n} = .2007$.
Finally, ${}_{n|}a_x = {}_nE_x \cdot (\frac{1 - A_{x+n}}{d} - 1) = (.543)(\frac{1 - .2007}{.057} - 1) = 7.07.$ Answer: A

25.(a) ${}_nE_x = e^{-\delta n}e^{-\mu n} = e^{-.1n} \rightarrow \frac{\partial}{\partial n}\, {}_nE_x = \frac{\partial}{\partial n}\, e^{-.1n} = -.1e^{-.1n}$.
(b) $\overline{a}_{x:\overline{n}|} = \int_0^n v^t\, {}_tp_x\, dt = \int_0^n e^{-\delta t}e^{-\mu t}\, dt = \frac{1 - e^{-.1n}}{.1}.$
(c) $Var[\overline{a}_{\overline{T}|}] = \frac{1}{\delta^2}[\, {}^2\overline{A}_{x|} - \overline{A}_x^2] = \frac{1}{\delta^2}[\frac{\mu}{2\delta + \mu} - (\frac{\mu}{\delta + \mu})^2] = \frac{1}{(.06)^2}[\frac{.04}{.16} - (\frac{.04}{.1})^2] = 25$
so standard deviation is 5.
(d) $\frac{\partial}{\partial x}\ \overline{a}_{x:\overline{n}|} = \frac{\partial}{\partial x}\, \frac{1 - e^{-.1n}}{.1} = 0.$

26. I and III require UDD. II is false (a corrected version of II is $\overline{a}_x = \overline{a}_{x:\overline{1}|} + vp_x\overline{a}_{x+1}$).
IV is always true. Answer: E

27. $(\overline{I}_{\overline{n}|}\overline{a})_x = (\overline{I}\,\overline{a})_{x:\overline{n}|} + n \cdot {}_{n|}\overline{a}_x = \int_0^n te^{-\delta t} \cdot {}_tp_x\, dt + ne^{-\delta n} \cdot {}_np_x \cdot \overline{a}_{x+n}$.
Since the force of mortality is constant at μ, we have ${}_np_x = e^{-n\mu}$, and $\overline{a}_{x+n} = \frac{1}{\delta + \mu}$.
Then, $(\overline{I}_{\overline{n}|}\overline{a})_x = \int_0^n te^{-(\delta+\mu)t}dt + ne^{-n(\delta+\mu)} \cdot \frac{1}{\delta + \mu}$
$\rightarrow \frac{d}{dn}(\overline{I}_{\overline{n}|}\overline{a})_x = \frac{d}{dn}[\int_0^n te^{-(\delta+\mu)t}\, dt + ne^{-n(\delta+\mu)} \cdot \frac{1}{\delta + \mu}]$
$= ne^{-(\delta+\mu)n} + e^{-n(\delta+\mu)} \cdot \frac{1}{\delta+\mu} - ne^{-n(\delta+\mu)} = e^{-n(\delta+\mu)} \cdot \frac{1}{\delta+\mu} = 10e^{-.1n}.$
We have used the calculus rule $\frac{d}{dx}\int_a^x f(t)\, dt = f(x).$ Answer: B

28. I. ${}_{n|}\ddot{a}_x = {}_{n-1|}a_x = a_x - a_{x:\overline{n-1}|}$. True.
II. $1 + a_{x:\overline{n-1}|} = \ddot{a}_{x:\overline{n}|} \neq (1+i)a_{x:\overline{n}|}$. False
III. $A_{x:\overline{n}|} = 1 - d\ddot{a}_{x:\overline{n}|} = 1 - (1-v)\ddot{a}_{x:\overline{n}|} = v\ddot{a}_{x:\overline{n}|} - (\ddot{a}_{x:\overline{n}|} - 1) = v\ddot{a}_{x:\overline{n}|} - a_{x:\overline{n-1}|}$. True.
Answer: C

29. The variance of an annuity with annual payment of 1 is $\frac{1}{\delta^2}[\,{}^2\overline{A}_{65} - \overline{A}_{65}^2]$.
We are given $(100)(25,000\overline{a}_{65}) = 30,000,000$, so that $\overline{a}_{65} = 12$, and
$\overline{A}_{65} = 1 - \delta\overline{a}_{65} = .28$. The variance of an annuity with annual payment of 1 is then
$\frac{1}{(.06)^2}[\,.2 - (.28)^2] = 33.78$. The standard deviation of the present value of 100 annuities each with annual payment of 25,000 is $\sqrt{100(25,000)^2(33.78)} = 1,453,000$. Answer: C

30. The schedule of payments is

Age	20	21	22	23	24
Pmt.	1	3	5	7	9

The schedule of payments can be expressed as

Age	20	21	22	23	24
Pmt.	0	2	4	6	8
$+$	1	1	1	1	1

The net single premium is the combination of the two actuarial present values in the second representation. The APV is $2(Ia)_{20:\overline{4}|} + \ddot{a}_{20:\overline{5}|}$.
We use the relationship $\ddot{a}_{20:\overline{k}|} = 1 + a_{20:\overline{k-1}|}$, so that $\ddot{a}_{20:\overline{5}|} = 1 + a_{20:\overline{4}|} = 4.04$.
The net single premium is $2(7.17) + 4.04 = 18.38$. Answer: B

31. The present value random variable for the annuity is $Y = \begin{cases} 10{,}000\,\overline{a}_{\overline{10}|} & \text{for } 0<T\leq 10 \\ 10{,}000\,\overline{a}_{\overline{T}|} & \text{for } T>10 \end{cases}$.
The 10-year certain period guarantees that the pv is at least $10,000\,\overline{a}_{\overline{10}|} = 63,212$, and the minimum payment is made if death occurs within 10 years; this has probability
${}_{10}q_{65} = 1 - e^{-10(.01)} = .095$. Therefore, $F_Y(y) = 0$ for $y < 63,212$, and
$F_Y(63,212) = .095$. There is a probability mass of .095 at $y = 63,212$. This corresponds to a "jump" in the graph of $F_Y(y)$ from 0 to .095 at that point. This occurs only in graph D.
Answer: D

32. We use the relationship $A_{\substack{1\\x:\overline{n}|}} = A_{x:\overline{n}|} - v^n\,{}_np_x$ to get

$A_{\substack{1\\25:\overline{10}|}} = A_{25:\overline{10}|} - v^{10}\,{}_{10}p_{25} = A_{25:\overline{10}|} - (.95)^{10}(.87) = A_{25:\overline{10}|} - .5209$.

We then use the relationships $A_{25:\overline{10}|} = 1 - d\,\ddot{a}_{25:\overline{10}|}$ and

$\ddot{a}_{25:\overline{15}|} = \ddot{a}_{25:\overline{10}|} + v^{10}\,{}_{10}p_{25}\,\ddot{a}_{35:\overline{5}|}$.

We get $\ddot{a}_{25:\overline{10}|} = 9.868 - (.95)^{10}(.87)(4.392) = 7.5802$,

and then $A_{25:\overline{10}|} = 1 - (1-v)(7.5802) = 1 - (.05)(7.5802) = .6210$.

Finally, $A_{\substack{1\\25:\overline{10}|}} = A_{25:\overline{10}|} - .5209 = .6210 - .5209 = .10$. Answer: A

33. We are given: ${}_{25}p_0 = .70$, ${}_{35}p_0 = .50$,

$7,500\ddot{a}_{25} = 60,000$, $12,300\ddot{a}_{35} = 60,000$, $9,400\ddot{a}_{25:\overline{10}|} = 60,000$

We use the relationships $\ddot{a}_{25} = \ddot{a}_{25:\overline{10}|} + v^{10}\,{}_{10}p_{25}\,\ddot{a}_{35}$ and ${}_{35}p_0 = {}_{25}p_0 \cdot {}_{10}p_{25}$.

We first get ${}_{10}p_{25} = \frac{.5}{.7} = \frac{5}{7}$, and then $8.0 = 6.3830 + v^{10} \times \frac{5}{7} \times 4.8780$.

Solving for v^{10} results in $v^{10} = .4641$. Then. $v = \frac{1}{1+i} = (.4641)^{.1} = .926$, so that $i = .080$.

Answer: A

34. The series of payments is

Age	30	...	39	40	...	49	50	...	59	60
Pmt	10	...	10	20	...	20	30	...	30	

There are various ways in which we can regroup the payments.

From the information given, we have the actuarial pv of a 10 year annuity-due starting at age 30.

We can also use the relationship $\ddot{a}_{30:\overline{30}|} = \ddot{a}_{30:\overline{20}|} + {}_{20}E_{30} \cdot \ddot{a}_{50:\overline{10}|}$ to solve for $\ddot{a}_{30:\overline{20}|}$.

$\ddot{a}_{30:\overline{20}|} = \ddot{a}_{30:\overline{30}|} - {}_{20}E_{30} \cdot \ddot{a}_{50:\overline{10}|} = v - mw$.

The series of payments can be formed in the following combination of payments:

Age	30	...	39	40	...	49	50 ...	59	60
Series 1	20	...	20	20	...	20			
Series 2	-10	...	-10						
Series 3							30 ...	30	
Net Total	10	...	10	20	...	20	30 ...	30	

34. continued

The APV of series 1 is $20\ddot{a}_{30:\overline{20}|} = 20(v - mw)$.

The APV of series 2 is $-10\ddot{a}_{30:\overline{10}|} = -10u$.

The APV of series 3 is $30 \cdot {}_{20}E_{30} \cdot \ddot{a}_{50:\overline{10}|} = 30mw$.

The total APV of the three series of payments is

$20(v - mw) - 10u + 30mw = -10u + 20v + 10mw$. Answer: D

35. The actuarial present value of a subscriber is the actuarial present value of all future subscription payments. This forms a life annuity, with constant probability $p_x = p = .90$.

The probability of continual renewal to k years from now is ${}_kp_y = p^k$.

The APV is $25[1 + vp + v^2p^2 + \cdots] = \frac{25}{1-vp} = \frac{25}{1-(.95)(.9)} = 172.41$. Answer: D

36. The actuarial present value of payout (i) is $P_1 \cdot A_{\substack{1\\60:\overline{10}|}} = .4188P_1$.

The variance of the payout in (i) is $P_1^2 \cdot [\ {}^2A_{\substack{1\\60:\overline{10}|}} - (A_{\substack{1\\60:\overline{10}|}})^2] = .0585P_1^2$.

Since (i) and (ii) are actuarially equivalent, we have $P_1 \cdot A_{\substack{1\\60:\overline{10}|}} = P_2 \cdot \ddot{a}_{60:\overline{20}|}$.

We use the relationship $\ddot{a}_{60:\overline{20}|} = \frac{1-A_{60:\overline{20}|}}{d}$ to get $\ddot{a}_{60:\overline{20}|} = 9.2883$.

Then, $.4188P_1 = 9.2883P_2 \to P_2 = .0451P_1$.

The variance of an n-year life annuity-due of 1 per year is $\frac{1}{d^2} \cdot [{}^2A_{60:\overline{20}|} - (A_{60:\overline{20}|})^2]$,

so the variance of payout (ii) is the variance of a 20 year life annuity-due of P_2 per year, which is

$\frac{P_2^2}{d^2} \cdot [{}^2A_{60:\overline{20}|} - (A_{60:\overline{20}|})^2] = (.0451P_1)^2 \cdot \frac{1}{.0036} \cdot [.2312 - (.4427)^2] = .0199P_1^2$.

The difference in the two variances is $.0585P_1^2 - .0199P_1^2 = .039P_1^2$. Answer: A

37. With constant force of mortality μ, we have $\overline{a}_x = \frac{1}{\delta+\mu}$.

$P[\overline{a}_{\overline{T}|} > \overline{a}_x] = P[\overline{a}_{\overline{T}|} > \frac{1}{\delta+\mu}] = P[\frac{1-e^{-\delta T}}{\delta} > \frac{1}{\delta+\mu}] = P[e^{-\delta T} < \frac{\mu}{\delta+\mu}]$.

With constant force of mortality μ, we also have ${}_np_x = e^{-\mu n} = P[T > n]$.

Then, $P[e^{-\delta T} < \frac{\mu}{\delta+\mu}] = P[-\delta T < ln(\frac{\mu}{\delta+\mu})]$

$= P[T > -\frac{1}{\delta} \cdot ln(\frac{\mu}{\delta+\mu})] = exp\Big[-\mu \cdot [-\frac{1}{\delta} \cdot ln(\frac{\mu}{\delta+\mu})]\Big] = (\frac{\mu}{\delta+\mu})^{\mu/\delta}$. Answer: B

38. The present value random variable for Jack's insurance is $1000e^{-\delta T}$, and the actuarial present value is
$E[1000e^{-\delta T}] = E[1000e^{-\delta T}|\text{Illustrative Table}] \cdot P[\text{Illustrative Table}]$
$\qquad + E[1000e^{-\delta T}|\text{constant force}] \cdot P[\text{constant force}]$
$= 1000[\frac{i}{\delta} A_{62}^{IT} \cdot (.6) + \frac{\mu}{\mu+\delta} \cdot (.4)] = 1000[\frac{.06}{ln\, 1.06} \cdot (.3967)(.6) + \frac{.02}{.02+ln\, 1.06} \cdot (.4)]$
$= 347.3.$
$\overline{a}_{62} = \frac{1-\overline{A}_{62}}{\delta} = \frac{1-.3473}{ln\, 1.06} = 11.2.$

39. With constant force of interest δ and constant force of mortality μ, we have
$\overline{a}_x = \frac{1}{\delta+\mu} = 12.5$. It follows that $\delta + \mu = .08$. Then, $\ddot{a}_x = \frac{1}{1-vp} = \frac{1}{1-e^{-\delta} \cdot e^{-\mu}} = 13.00667.$
Using Woolhouse's approximation with $m = 12$ we have
$\ddot{a}_x^{(12)} = \ddot{a}_x - \frac{12-1}{2\times 12} - \frac{12^2-1}{12\times 12^2}(\mu+\delta) = 12.542.$ Answer: C

40. We use the relationship for the variance of a continuous whole life annuity of 1 per year:
$Var[\overline{a}_{\overline{T}|}] = \frac{1}{\delta^2} \cdot [{}^2\overline{A}_x - (\overline{A}_x)^2].$
The survival distribution is a mixture of the survival distributions for smokers and non-smokers.
$\overline{A}_x = .3\overline{A}_x^{\text{smoker}} + .7\overline{A}_x^{\text{non-smoker}} = (.3)(.444) + (.7)(.286) = .3334.$
${}^2\overline{A}_x = .3\,{}^2\overline{A}_x^{\text{smoker}} + .7\,{}^2\overline{A}_x^{\text{non-smoker}}.$

We find ${}^2\overline{A}_x^{\text{smoker}}$ and ${}^2\overline{A}_x^{\text{non-smoker}}$ from the given annuity variances.
$8.818 = Var[\overline{a}_{\overline{T}|}^{\text{smoker}}] = \frac{1}{\delta^2} \cdot [{}^2\overline{A}_x^{\text{smoker}} - (\overline{A}_x)^2] = \frac{1}{(.1)^2} \cdot [{}^2\overline{A}_x^{\text{smoker}} - (.444)^2]$,
from which we get ${}^2\overline{A}_x^{\text{smoker}} = .2853.$

Similarly, $8.503 = Var[\overline{a}_{\overline{T}|}^{\text{non-smoker}}] = \frac{1}{\delta^2} \cdot [{}^2\overline{A}_x^{\text{non-smoker}} - (\overline{A}_x)^2] = \frac{1}{(.1)^2} \cdot [{}^2\overline{A}_x^{\text{smoker}} - (.286)^2]$,
from which we get ${}^2\overline{A}_x^{\text{non-smoker}} = .1668.$
Then, ${}^2\overline{A}_x = .3\,{}^2\overline{A}_x^{\text{smoker}} + .7\,{}^2\overline{A}_x^{\text{non-smoker}} = .2024$, and finally,
$Var[\overline{a}_{\overline{T}|}] = \frac{1}{\delta^2} \cdot [{}^2\overline{A}_x - (\overline{A}_x)^2] = \frac{1}{(.1)^2} \cdot [.2024 - (.3334)^2] = 9.1$. Answer: E □

41. $Var(\overline{a}_{\overline{T_x}|}) = \frac{1}{\delta^2} \cdot ({}^2\overline{A}_x - \overline{A}_x^2)$
Since the force of mortality is constant at c, we have $\overline{A}_x = \frac{c}{\delta+c}$ and ${}^2\overline{A}_x = \frac{c}{2\delta+c}$.
Therefore, from $\overline{A}_x = .3443 = \frac{c}{.08+c}$, we get $c = .042$,
and then ${}^2\overline{A}_x = \frac{.042}{2(.08)+.042} = .2079$.
$Var(\overline{a}_{\overline{T(x)}|}) = \frac{1}{(.08)^2} \cdot [.2079 - (.3443)^2] = 13.96$. Answer: B

42. $\ddot{a}_{75:\overline{3}|} = 1 + vp_{75} + v^2\,{}_2p_{75}$.
From (i) we get ${}_np_{75} = e^{-\int_{75}^{75+n} \mu_t\,)\,dt} = e^{-.01[(75+n)^{1.2} - 75^{1.2}]}$.
Therefore, $p_{75} = e^{-.01[76^{1.2} - 75^{1.2}]} = .9719$ and
${}_2p_{75} = e^{-.01[77^{1.2} - 75^{1.2}]} = .9445$.
The APV of the annuity is $1 + \frac{.9719}{1.11} + \frac{.9445}{(1.11)^2} = 2.64$. Answer: A

43. The equivalence principle equation is $Q = 200\,{}_{30|}\ddot{a}_{30} + QA_{\substack{1\\30:\overline{30}|}}$.
${}_{30|}\ddot{a}_{30} = {}_{20}E_{30} \cdot {}_{10}E_{50} \cdot \ddot{a}_{60} = (.29374)(.51081)(11.1454) = 1.672$.
$A_{30} = A_{\substack{1\\30:\overline{30}|}} + {}_{20}E_{30} \cdot {}_{10}E_{50} \cdot A_{60}$, so that
$A_{\substack{1\\30:\overline{30}|}} = .10248 - (.29374)(.51081)(.36913) = .04709$.
Solving for Q results in $Q = \frac{200(1.672)}{1-.04709} = 351$. Answer: A

44. $Q = .01C \cdot QA_{\substack{1\\35:\overline{30}|}} + {}_{30|}\ddot{a}_{35} \rightarrow Q = \frac{{}_{30|}\ddot{a}_{35}}{1-.01C \cdot A_{\substack{1\\35:\overline{30}|}}}$.
We are given $A_{\substack{1\\35:\overline{30}|}} = 0.07$.
${}_{30|}\ddot{a}_{35} = v^{30}\,{}_{30}p_{35} \cdot \ddot{a}_{65}$. We are given $\ddot{a}_{65} = 9.90$.
We also have $v^{30}\,{}_{30}p_{35} = A_{35:\overline{30}|} - A_{\substack{1\\35:\overline{30}|}} = .21 - .07 = .14$.
Then, $1.4 = \frac{(.14)(9.90)}{1-(.01C)(.07)}$. Then $C = 14.3$ Answer: D

45. We use the relationship $\overline{a}_x = \overline{a}_{x:\overline{n}|} + v^n \, {}_np_x \, \overline{a}_{x+n}$:

$$\overline{a}_{20} = \overline{a}_{20:\overline{10}|} + v^{10} \, {}_{10}p_{20} \, \overline{a}_{30} \; .$$

From UDD we have $\overline{A}_x = \frac{i}{\delta} \, A_x = \frac{.05}{ln\,1.05} \cdot \frac{11}{21} = .5368$ for all x ,
and then $\overline{a}_x = \frac{1-\overline{A}_x}{\delta} = 9.494$ for all x.
Therefore $9.494 = \overline{a}_{20:\overline{10}|} + v^{10} \, {}_{10}p_x \cdot (9.494) = \overline{a}_{20:\overline{10}|} + \frac{.5680}{(1.05)^{10}} \cdot (9.494)$
and then $\overline{a}_{20:\overline{10}|} = 6.2.$
Note that since $v = \frac{20}{21}$ and $p_x = .945$, we have $vp_x = .9$, so that
$v^{10} \, {}_{10}p_x = (.9)^{10} = .348678.$ Answer: C

46. The reinsurer's first potential payment is 100,000 and will be made 3 years from now (the primary insurer would pay 150,000 now and for the next 2 years, and 50,000 in three years to reach the limit of 500,000). Starting 4 years from now, the reinsurer would 150,000 per year, as long as the worker survives. The actuarial present value of the reinsurer's payments is

$$100,000v^3 \, {}_3p_x + 150,000[v^4 \, {}_4p_x + v^5 \, {}_5p_x + v^6 \, {}_6p_x + \cdots]$$

Since ${}_tp_x = 0$ for $t > 5.5$, and ${}_tp_x = (.7)^t$ for $t \leq 5.5$, this becomes
$100,000\frac{(.7)^3}{(1.05)^3} + 150,000[(\frac{.7}{1.05})^4 + (\frac{.7}{1.05})^5] = 79,012$. Answer: B

47. $\ddot{a}_x = 1 + vp_x \, \ddot{a}_{x+1} = 7.5538$,
$\ddot{a}_{x+1} = 1 + vp_{x+1}\ddot{a}_{x+2} \rightarrow \ddot{a}_{x+2} = \frac{6.951-1}{vp_{x+1}} = 6.57742$
The revised values (superscript N) are found as follows:
$\ddot{a}_x^N = 1 + vp_x^N \, \ddot{a}_{x+1}^N$, $\ddot{a}_{x+1}^N = 1 + vp_{x+1}^N \ddot{a}_{x+2}$
$p_x^N = p_x = .99$, $p_{x+1}^N = .98$, $\ddot{a}_{x+2}^N = \ddot{a}_{x+2}$ (only change is at age $x+1$)
$\rightarrow \ddot{a}_{x+1}^N = 1 + vp_{x+1}^N \ddot{a}_{x+2} = 1 + \frac{.98}{1.05}(6.57742) = 7.1389$
$\rightarrow \ddot{a}_x^N = 1 + vp_x^N \, \ddot{a}_{x+1}^N = 1 + \frac{.99}{1.05}(7.1389) = 7.731.$
Change is $7.731 - 7.554 = .177.$ Answer: C

WRITTEN ANSWER

1.(a) $Z_2 = Z_1 + 10,000\,\overline{a}_{\overline{10|}}$. Since Z_1 and Z_2 differ by a constant, they must have the same variance.
$Cov[Z_1, Z_2] = Cov[Z_1, Z_1 + \overline{a}_{\overline{25|}}] = Cov[Z_1, Z_1] = Var[Z_1]$.

(b) $E[Z_1] = 10,000\,{}_{10|}\overline{a}_{40} = 10,000e^{-10\delta}\,{}_{10}p_{40} \times \overline{a}_{50}$.
${}_{10}p_{40} = e^{-10\times .01}$, and $\overline{a}_{50} = \int_0^\infty e^{-\delta t}\,{}_tp_{50}\,dt = \int_0^\infty e^{-.05t} \times e^{-.01t}\,dt = \frac{1}{.06}$.
$E[Z_1] = 914.69$.

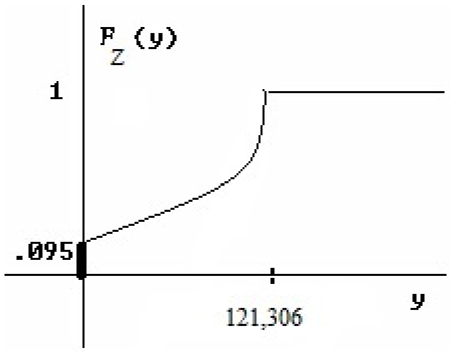
$\overline{a}_{\overline{10|}} = \frac{1-e^{10\delta}}{\delta} = 7.869$, so that $E[Z_2] = 914.69 + 10,000 \times 7.869 = 79,609$.

(c) $Z_1 = \begin{cases} 0 & T_{40} \le 10 \\ 10,000\,e^{-.5}\,\overline{a}_{\overline{T_{40}-10|}} & T_{40} > 10 \end{cases} = \begin{cases} 0 & T_{40} \le 10 \\ 10,000\,e^{-.5}\,\frac{1-e^{-.05(T_{40}-10)}}{.05} & T_{40} > 10 \end{cases}$

$= \begin{cases} 0 & T_{40} \le 10 \\ 200,000\,(e^{-.5} - e^{-.05T_{40}}) & T_{40} > 10 \end{cases}$
$P(Z_1 = 0) = P(T_{40} \le 10) = {}_{10}q_{40} = 1 - {}_{10}p_{40} = 1 - e^{-10\times .01} = .0952$.
For $w > 0$, $P(Z_1 \le y) = P(Z_1 = 0) + P(0 < Z_1 \le y)$
$= .0952 + P[0 < 200,000\,(e^{-.5} - e^{-.05T_{40}}) \le y]$
We can rewrite the inequality $0 < 200,000\,(e^{-.5} - e^{-.05T_{40}}) \le y$ in the form
$10 < T_{40} \le -20\,ln(\,e^{-.5} - \frac{y}{200,000}\,)$.
From $P(u < T_{40} \le v) = {}_up_{40} - {}_vp_{40}$, we get
$P[0 < 200,000\,(e^{-.5} - e^{-.05T_{40}}) \le y] = P[10 < T_{40} \le -20\,ln(\,e^{-.5} - \frac{y}{200,000}\,)]$,
so $u = 10$, and $v = -20\,ln(\,e^{-.5} - \frac{y}{200,000}\,)$.
Also, with contstant force of mortality .01, we have ${}_tp_x = e^{-.01t}$.
Then $P[0 < 200,000\,(e^{-.5} - e^{-.05T_{40}}) \le y] = e^{-.1} - (\,e^{-.5} - \frac{y}{200,000}\,)^{.2}$ for $w > 0$.
The maximum value of w is $10,000\,{}_{10|}\overline{a}_{\overline{\infty|}} = 200,000e^{-.5} = 121,306$.

Finally, for $w > 0$, $F_{Z_1}(y) = P(Z_1 = 0) + P(0 < Z_1 \le y)$
$= 1 - (\,e^{-.5} - \frac{y}{200,000}\,)^{.2}$.

The graph of the distribution function is



(d) Since $Z_2 = Z_1 + 10,000\,\overline{a}_{\overline{10|}}$, the graph of F_{Z_2} is the same shape as that of F_{Z_1} , but it is shifted to the right by an amount of $10,000 \times \overline{a}_{\overline{10|}} = 78,690$.

2.(a) The APV is ${}_{20|}\ddot{a}_{60} = \sum_{k=20}^{39} v^k\,{}_kp_{60} = \sum_{k=20}^{59} \frac{120-60-k}{120-60}\,v^k$

$= \frac{1}{60} \times [40v^{20} + 39v^{21} + \cdots + 2v^{58} + v^{59}] = \frac{1}{60} \times v^{19} \times (Da)_{\overline{40|}.04}$

$= \frac{1}{60} \times v^{19}_{.04} \times \frac{40-a_{\overline{40|}.04}}{.04} = 3.9963$.

(b) $Z = \begin{cases} 0 & K_{60} < 20 \\ v^{20}\,\ddot{a}_{\overline{K_{60}+1-20|}} & K_{60} \geq 20 \end{cases}$

(c) The median of Z is the smallest c for which $P[Z \leq c] \geq .5$,
From the nature of the distribution, we see that ${}_kq_x = 1 - \frac{S(x+k)}{S(x)} = \frac{k}{120-x}$,
so that $P[K_{60} \leq k-1] = {}_kq_{60} = \frac{k}{60} \geq .5$ is equivalent to $k \geq 30$ or $k-1 \geq 29$,
Since $Z = v^{20}\,\ddot{a}_{\overline{K_{60}+1-20|}}$ it follows that
$P[Z \leq c] = P[v^{20}\,\ddot{a}_{\overline{K_{60}+1-20|}} \leq c] = P[K_{60} \leq k-1]$, where
$v^{20}\,\ddot{a}_{\overline{k+1-20|}} = c$. But since $k = 30$ is the smallest k for which $P[K_{60} \leq k-1] \geq .5$
it follows that $c = v^{20}\,\ddot{a}_{\overline{29+1-20|}} = 3.8497$ is the smallest value of c for which
$P[Z \leq c] \geq .5$. The median value of Z is 3.8497 ,

3. Y is a mixture of Y^s and Y^{ns}, with mixing weights .3 and .7, respectively.
(a) $E[Y] = .3 \times E[Y^s] + .7 \times E[Y^{ns}]$
For a mortality model with constant force of mortality μ with constant force of interest δ, the net single premium for a continuous whole life annuity is $\frac{1}{\mu+\delta}$, so
$E[Y^s] = \frac{1}{.02+.1} = 8.3333$ and $E[Y^{ns}] = \frac{1}{.01+.1} = 9.0909$
and $E[Y] = .3 \times 8.333 + .7 \times 9.0909 = 8.864$.

(b) Since $Y = \frac{1-Z}{\delta}$, where Z is the PVRV for a continuous whole life insurance of 1, it follows that $Var[Y] = \frac{1}{\delta^2} \times Var[Z] = 100 \times Var[Z]$., and $Var[Z] = E[Z^2] - E[Z]^2$.
From the mixture distribution and the fact that $\overline{A}_x = \frac{\mu}{\mu+\delta}$ for the constant force model, we get
$E[Z] = .3 \times E[Z^s] + .7 \times E[Z^{ns}] = .3 \times \frac{.02}{.02+.1} + .7 \times \frac{.01}{.01+.1} = .1136$
(note that this is consistent with $E[Y] = \frac{1-E[Z]}{\delta}$: $8.864 = \frac{1-.1136}{.1}$) .
From the mixture distribution and the fact that ${}^2\overline{A}_x = \frac{\mu}{\mu+2\delta}$ for the constant force model, we get
$E[Z^2] = .3 \times E[(Z^s)^2] + .7 \times E[(Z^{ns})^2] = .3 \times \frac{.02}{.02+.2} + .7 \times \frac{.01}{.01+.2} = .0606$.
Then $Var[Z] = E[Z^2] - E[Z]^2 = .0606 - (.1136)^2 = .0477$.
Finally, $Var[Y] = \frac{1}{\delta^2} \times Var[Z] = 100 \times .0477 = 4.77$.
Note that we could formulate $Var[Y]$ as $E[Y^2] - E[Y]^2$, and it continues to be true for the mixture distribution that $E[Y^2] = .3 \times E[(Y^s)^2] + .7 \times E[(Y^{ns})^2]$, but it is easier to find $E[(Z^s)^2]$ directly than to find $E[(Y^s)^2]$ directly.

(c) For the mixture distribution we can use conditioning to find probability.
$P[Y \leq 5] = .3 \times P[Y^s \leq 5] + .7 \times P[Y^{ns} \leq 5]$.
$P[Y^s \leq 5] = P[\frac{1-e^{-\delta T_x^s}}{\delta} \leq 5] = P[T_x^s \leq -10\, ln\, .5] = P[T_x^s \leq 6.93]$
$= {}_{6.93}q_x^s = 1 - {}_{6.93}p_x = 1 - e^{-6.93\mu_x^s} = 1 - e^{-6.93 \times .02} = .129$.
In a similar way, $P[Y^{ns} \leq 5] = P[T_x^{ns} \leq 6.93] = 1 - e^{-6.93 \times .01} = .067$.
Then $P[Y \leq 5] = .3 \times .129 + .7 \times .067 = .0856$.

S. BROVERMAN MLC STUDY GUIDE
MULTIPLE CHOICE TEST 6

1. You are given:
   (i) The survival function for males is $S_0(x) = 1 - \frac{x}{\omega}, \quad 0 < x < \omega$.
   (ii) Female mortality follows the survival model $S_0(t) = 1 - \frac{t}{85}$ (De Moivre).
   (iii) At age 60, the female force of mortality is 60% of the male force of mortality.

For two independent lives, a male age 65 and a female age 60, calculate the expected time until the second death.

A) 4.33 B) 5.63 C) 7.23 D) 11.88 E) 13.17

2. Mr. Ucci has only 3 hairs left on his head and he won't be growing any more.
(i) The future mortality of each hair follows ${}_{k|}q_x = 0.1(k+1), \quad k = 0, 1, 2, 3$
and x is Mr. Ucci's age
(ii) Hair loss follows the hyperbolic assumption at fractional ages.
(iii) The future lifetimes of the 3 hairs are independent.
Calculate the probability that Mr. Ucci is bald (has no hair left) at age $x + 2.5$.

A) 0.098 B) 0.103 C) 0.108 D) 0.113 E) 0.118

3. A government creates a fund to pay this year's lottery winners.
You are given:
(i) There are 100 winners each age 40.
(ii) Each winner receives payments of 10 per year for life, payable annually, beginning immediately.
(iii) Mortality follows the Illustrative Life Table.
(iv) The lifetimes are independent.
(v) $i = 0.06$.
(vi) The amount of the fund is 15,092.
Using the normal approximation, find the probability that the fund is sufficient to make all payments.

A) 82% B) 85% C) 88% D) 91% E) 95%

4. You are given:

(i) $P_x = 0.090$ (ii) $P_{\overset{\,1}{x:\overline{n}|}} = .085$ (iii) $P^{\;1}_{x:\overline{n}|} = 0.00864$

Calculate ${}_nV_x$.

A) 0.50 B) 0.52 C) 0.54 D) 0.56 E) 0.58

5. Don, age 50, is an actuarial science professor. His career is subject to two decrements:

(i) Decrement 1 is mortality. The associated single decrement table follows the model $S_0(t) = 1 - \frac{t}{100}$ (DeMoivre).

(ii) Decrement 2 is leaving academic employment, with $\mu^{(2)}_{50+t} = 0.05, \quad t \geq 0$

Calculate the probability that Don dies between ages 55 and 60 while still employed as a professor.

A) 0.05 B) 0.06 C) 0.07 D) 0.08 E) 0.09

6. For a 20-year term life insurance on (x), you are given:

- $i = 0$
- $\mu^{(1)}_{x+t} = \frac{t}{20}$, the force of mortality due to accident
- $\mu^{(2)}_{x+t} = \frac{t}{10}$, the force of mortality due to other causes
- The benefit is paid at the moment of death.
- The benefit of 2 is paid if death occurs by accident, and a benefit of 1 is paid if death occurs by other causes.

Calculate the actuarial present value of this insurance.

A) $\frac{2}{3}(1 - e^{-20})$ B) $1 - e^{-20}$ C) $1 - e^{-30}$ D) $\frac{2}{3}(1 - e^{-30})$ E) $\frac{4}{3}(1 - e^{-30})$

7. Lee, age 63, considers the purchase of a single premium whole life insurance of 10,000 with death benefit payable at the end of the year of death.

The company calculates net premiums using:

(i) Mortality based on the Illustrative Life Table,

(ii) $i = 0.05$

The company calculates contract premiums as 112% of net premiums. The single contract premium at age 63 is Q. Lee decides to delay the purchase for two years and invests the Q. The annual rate of return that the investment earns is 3.0% and two years later it provides exactly the amount equal to the single contract premium at age 65. Calculate Q.

A) Less than 5000 B) At least 5000 but less than 5100

C) At least 5100 but less than 5200 D) At least 5200 but less than 5300

E) At least 5300

8. For a special fully discrete 20-year endowment insurance on (55):
(i) Death benefits in year k are given by $b_k = (21 - k)$, $k = 1, 2, \ldots, 20$.
(ii) The maturity benefit is 1.
(iii) Annual net premiums are level.
(iv) ${}_kV$ denotes the net reserve at the end of year k, $k = 1, 2, \ldots, 20$.
(v) ${}_{10}V = 5.0$ (vi) ${}_{19}V = 0.6$ (vii) ${}_{11}V = 5.3$ (viii) $i = 0.08$
Calculate q_{55} .
A) Less than .06 B) At least .06 but less than .07 C) At least .07 but less than .08
D) At least .08 but less than .09 E) At least .09

9. Mortality follows Gompertz's law with $B = 0.0001$ and $c > 0$.
You are given that for a pair of independent lives both aged x, the joint life mortality random variable T_{xx} has a distribution that is identical to that of T_{x+4}, the single life mortality random variable an individual aged $x + 4$. Determine the value of p_{50}.
A) Less than .50 B) At least .50 but less than .60 C) At least .60 but less than .70
C) At least .70 but less than .80 C) At least .80

10. For a special fully discrete 35-payment whole life insurance on (30):
(i) The death benefit is C for the first 20 years and is 5 thereafter.
(ii) The initial net premium paid during the each of the first 20 years is one fifth of the net premium paid during each of the 15 subsequent years.
(iii) Mortality follows the Illustrative Life Table.
(iv) $i = 0.06$.
(v) $A_{30:\overline{20|}} = 0.32307$
(vi) $\ddot{a}_{30:\overline{35|}} = 14.835$
(vii) The initial annual net premium is .016.
Calculate C.
A) 1.5 B) 1.7 C) 1.5 D) 1.7 E) 1.9

11. The Boiler Room Sales company ranks each of its sales people at the end of each month according to the sales made by that sales person. There are three rankings:
Excellent , Good , Poor.
A salesperson who had poor sales for the month is immediately fired and never again rehired. Some sales people die during the month (perhaps because of the high pressure under which they have to work). The BRS company has created a homogeneous Markov Chain model to describe transitions in a salesperson's ranking from one month to the next. The model has three states:

E - excellent sales for the month just ended,
G - good sales for the month just ended, and
P - poor sales or died in the month just ended.

The one-step transition matrix is $\begin{bmatrix} p_{E,E} & p_{E,G} & p_{E,P} \\ p_{G,E} & p_{G,G} & p_{G,P} \\ p_{P,E} & p_{P,G} & p_{P,P} \end{bmatrix} = \begin{bmatrix} .2 & .6 & .2 \\ .1 & .5 & .4 \\ 0 & 0 & 1 \end{bmatrix}$.

A salesperson has just received a ranking of G.
BRS company pays a salesperson a bonus of \$1000 whenever that person has two consecutive months with a ranking of E. At a monthly interest rate of 1%, find the combined actuarial present values of the bonus payments to be made at the end of the next three months to a salesperson whose current rating is G, and a salesperson whose current rating is E .

A) Less than 300 B) At least 300 but less than 320 C) At least 320 but less than 340
D) At least 340 but less than 360 E) At least 360

12. Under the assumption of UDD in the multiple decrement table, what is the formulation for the APV of a disability income benefit of \$C per month payable to age 75, issued at age 40, with a 6-month waiting period, and with coverage expiring at age 60. Assume that disability occurs at mid-year.

A) $12C\sum_{k=0}^{19} v^{k+\frac{1}{2}}\, {}_kp_{40}^{(\tau)}\, q_{40+k}^{(i)}\, \ddot{a}_{41+k:\overline{34-k}|}^{(12)i}$ B) $C\sum_{k=0}^{19} v^{k+\frac{1}{2}}\, {}_kp_{40}^{(\tau)}\, q_{40+k}^{(i)}\, \ddot{a}_{41+k:\overline{34-k}|}^{(12)i}$

C) $12C\sum_{k=0}^{19} v^{k+1}\, {}_kp_{40}^{(\tau)}\, q_{40+k}^{(i)}\, \ddot{a}_{41+k:\overline{34-k}|}^{(12)i}$ D) $C\sum_{k=0}^{19} v^{k+1}\, {}_kp_{40}^{(\tau)}\, q_{40+k}^{(i)}\, \ddot{a}_{41+k:\overline{34-k}|}^{(12)i}$

E) $12C\sum_{k=0}^{19} v^{k+1}\, {}_kp_{40}^{(\tau)}\, q_{40+k}^{(i)}\, \ddot{a}_{41+k:\overline{34\frac{1}{2}-k}|}^{(12)i}$

13. You have calculated the actuarial present value of a last-survivor whole life insurance of 1 on (x) and (y). You assumed:
(i) The death benefit is payable at the moment of death.
(ii) You assume that the future lifetimes of (x) and (y) follow a common shock model in which each life has a constant force of mortality with $\mu = 0.06$ and with a common shock component with constant force 0.02.
(iii) $\delta = 0.05$.
Your supervisor points out that these are independent future lifetimes, and each mortality assumption for (x) and for (y) is correct.
Calculate the change in the actuarial present value over what you originally calculated.
A) .039 larger B) .039 smaller C) .093 larger D) .093 smaller E) no change

14. A decreasing term life insurance on (80) pays $(20 - k)$ at the end of the year of death if (80) dies in year $k+1$, for $k = 0, 1, 2, \ldots, 19$.
You are given:
(i) $i = 0.06$
(ii) For a certain mortality table with $q_{80} = 0.2$, the single net premium for this insurance is 13.
(iii) For this same mortality table, except that $q_{80} = C$, the single net premium for this insurance is 12.30.
Calculate C.
A) Less than .10 B) At least .10 but less than .11 C) At least .11 but less than .12
D) At least .12 but less than .13 E) At least .13

15. Mortality for Audra, 25 years old, follows a constant force of mortality of $\mu = .01$ at all ages. If she takes up hot air ballooning for the coming year, her assumed mortality will be adjusted so that for the coming year only, she will have a mortality probability of 0.1 based on uniform distribution of death with that year. Calculate the decrease in the 11-year temporary complete life expectancy for Audra if she takes up hot air ballooning.
A) 0.60 B) 0.70 C) 0.80 D) 0.90 E) 1.00

16. For a special 20-year term life insurance on (40), you are given:
(i) The death and endowment benefits are 10,000.
(ii) The death benefit is payable at the moment of death.
(iii) During the 5th year the gross premium is 750 paid continuously at a constant rate
(iv) The force of mortality follows Gompertz's law with $B = 0.00004$ and $c = 1.1$
(v) The force of interest is 4%.
(vi) Expenses are 5% of premium payable continuously and 100 payable at the moment of death.
(vii) At the end of the 5th year the expected value of the present value of future losses random variable is 3000.
Euler's method with steps of $h = 0.25$ years is used to calculate a numerical solution to Thiele's differential equation. Calculate the expected value of the present value of future losses random variable at the end of 4.5 years.
A) Less than 2450 B) At least 2450 but less than 2550
C) At least 2550 but less than 2650 D) At least 2650 but less than 2750
E) At least 2750

17. (x) and (y) are two lives with identical expected mortality. You are given:
$P_x = P_y = 0.1$,
$P_{xy} = 0.18$, where P_{xy} is the annual net premium for a fully discrete insurance of 1 on (xy).
$d = 0.06$
Calculate the premium $P_{\overline{xy}}$, the annual net premium for a fully discrete insurance of 1 on $(\overline{xy})$.
A) 0.04 B) 0.06 C) 0.08 D) 0.10 E) 0.12

18. For a multiple decrement model on (60):
(i) $\mu_{60+t}^{(1)}$, $t \geq 0$, follows the Illustrative Life Table.
(ii) $\mu_{60+t}^{(\tau)} = c \cdot \mu_{60+t}^{(1)}$, $t \geq 0$
(iii) ${}_{10}q_{60}^{(\tau)} = .57372$
Calculate c.
A) 2.0 B) 2.5 C) 3.0 D) 3.5 E) 4.0

19. For a special fully discrete 3-year term insurance on (x):

(i) Level net premiums are paid at the beginning of each year.

(ii)

k	b_{k+1}	q_{x+k}
0	200,000	0.03
1	150,000	0.06
2	100,000	0.09

(iii) $i = 0.06$

Calculate the initial net reserve for year 3 (nearest 100).

A) 6,500 B) 7,500 C) 8,500 D) 9,500 E) 10,500

20. For a special fully discrete whole life insurance on (x):

(i) The level premium is determined using the equivalence principle.

(ii) Death benefits are given by $b_{k+1} = (1+i)^{k+1}$ where i is the interest rate.

(iii) L is the loss random variable at $t = 0$ for the insurance.

(iv) K is the future curtate lifetime random variable of (x).

Which of the following expressions is equal to L?

A) $\frac{(v^{K+1} - A_x)}{(1 - A_x)}$ B) $(v^{K+1} - A_x)(1 + A_x)$ C) $\frac{(v^{K+1} - A_x)}{(1 + A_x)}$

D) $(v^{K+1} - A_x)(1 - A_x)$ E) $\frac{(v^{K+1} + A_x)}{(1 + A_x)}$

S. BROVERMAN MLC STUDY GUIDE
MULTIPLE CHOICE TEST 6 SOLUTIONS

1. The male survival distribution is De Moivre's Law with ω. The force of mortality for a male at age 60 is $\frac{1}{\omega-60}$. Female mortality follows De Moivre's Law with upper age limit 85.
Female force of mortality at age 60 is $\frac{1}{85-60} = \frac{1}{25} = .6(\frac{1}{\omega-60}) \rightarrow \omega = 75$.
$\overset{\circ}{e}_{\overline{65M:60F}} = \overset{\circ}{e}_{65M} + \overset{\circ}{e}_{60F} - \overset{\circ}{e}_{65M:60F} = \frac{75-65}{2} + \frac{85-60}{2} - \int_0^{10} ({}_tp_{65M})({}_tp_{60F})\, dt$.
${}_tp_{65M} = \frac{75-65-t}{75-65} = \frac{10-t}{10}$, ${}_tp_{60F} = \frac{85-60-t}{85-60} = \frac{25-t}{25}$
$\rightarrow \int_0^{10} ({}_tp_{65M})({}_tp_{60F})\, dt = \int_0^{10} (\frac{10-t}{10})(\frac{25-t}{25})\, dt = \frac{1}{(10)(25)} \int_0^{10} (250 - 35t + t^2)\, dt$
$= \frac{1}{(10)(25)}[2500 - 35(\frac{100}{2}) + \frac{1000}{3}] = 4.333 \rightarrow \overset{\circ}{e}_{\overline{65M:60F}} = 5 + 12.5 - 4.333 = 13.16$.
Answer: E

2. For a particular hair, the probability that this hair has been lost by time 2.5 is ${}_{2.5}q_x$. The probability of being bald by time 2.5 is $P[\text{(lose hair 1)} \cap \text{(lose hair 2)} \cap \text{(lose hair 3)}]$.
Since the hairs are independent of one another, this probability is
$P[\text{(lose hair 1)}] \cdot P[\text{(lose hair 2)}] \cdot P[\text{(lose hair 3)}] = ({}_{2.5}q_x)^3$
(we use the rule for independent events A and B that says $P[A \cap B] = P[A] \cdot P[B]$).
From the given information we have $q_x = {}_{0|}q_x = .1$,
${}_2q_x = q_x + {}_{1|}q_x = .1 + .2 = .3$, ${}_3q_x = q_x + {}_{1|}q_x + {}_{2|}q_x = .6$.
Then ${}_2p_x = .7$ so that $q_{x+2} = \frac{{}_{2|}q_x}{{}_2p_x} = \frac{3}{7}$.
${}_{2.5}q_x = 1 - {}_{2.5}p_x = 1 - {}_2p_x \cdot {}_{.5}p_{x+2} = 1 - (.7)(1 - {}_{.5}q_{x+2})$.
From the hyperbolic assumption, we have ${}_{.5}q_{x+2} = \frac{.5 \cdot q_{x+2}}{1-(1-.5)\cdot q_{x+2}} = .2727$.
Then, ${}_{2.5}q_x = \ = 1 - (.7)(1 - .2727) = .491$, and $({}_{2.5}q_x)^3 = (.49)^3 = .118$.
Answer: E

3. This problem involves the variance of a life annuity present value random variable and the normal approximation. The expected present value of payment to the winners is $E[S] = (100)(10)(\ddot{a}_{40}) = 14,816.6$ (for 100 annuitants). Since the prize winners are independent, the combined variance of the present value random variables is the sum of the variances of the separate present value random variables. For one prize winner, the variance of the present value random variable is

$Var[10Y] = 100Var[Y] = 100(\frac{1}{d^2})[^2A_{40} - (A_{40})^2] = 705.55$.
The variance of the total present value paid to the winners is $Var[S] = 100(705.55) = 70,555.$
We assume that S is approximately normal to find the probability that $P[S \leq 15,092]$.
We standardize the probability to get
$P[S \leq 15,092] = P[\frac{S-E[S]}{\sqrt{Var[S]}} \leq \frac{15,092-E[S]}{\sqrt{Var[S]}}] = P[Z \leq \frac{15,092-14,817}{\sqrt{70,555}}]$
$= \Phi(1.035) = .85$ (from the normal table). Answer: B

4. ${}_nV_x = \frac{P_x - P_{\overset{1}{x}:\overline{n}|}}{P_{x:\overset{1}{\overline{n}|}}} \to {}_nV_x = \frac{.09-.085}{.00864} = .5787$. Answer: E

5. We wish to find ${}_{5|5}q_{50}^{(1)} = {}_5p_{50}^{(\tau)} \times {}_5p_{55}^{(1)} = {}_5p_{50}^{\prime(1)} \times {}_5p_{50}^{\prime(2)} \times \int_0^5 {}_tp_{55}^{(\tau)} \cdot \mu_{55+t}^{(1)}\,dt$.
Under De Moivre's Law with $\omega = 100$ we have ${}_5p_{50}^{\prime(1)} = \frac{100-50-5}{100-50} = .9$ and
and under constant force of decrement ${}_5p_{50}^{\prime(2)} = e^{-5(.05)} = e^{-.25}$.
Under De Moivre's Law we also have ${}_tp_{55}^{\prime(1)} = \frac{100-55-t}{100-55}$ and $\mu_{55+t}^{(1)} = \frac{1}{100-55-t}$, so
${}_tp_{55}^{(\tau)} \cdot \mu_{55+t}^{(1)} = {}_tp_{55}^{\prime(1)} \times {}_tp_{55}^{\prime(2)} \cdot \mu_{55+t}^{(1)} = \frac{100-55-t}{100-55} \times e^{-.05t} \times \frac{1}{100-55-t}$
$= \frac{e^{-.05t}}{45}$.
Then ${}_5p_{55}^{(1)} = \int_0^5 {}_tp_{55}^{(\tau)} \cdot \mu_{55+t}^{(1)}\,dt = \int_0^5 \frac{e^{-.05t}}{45}\,dt = \frac{1-e^{-.25}}{.05\times 45}$.
Finally, ${}_{5|5}q_{50}^{(1)} = {}_5p_{50}^{(\tau)} \times {}_5p_{55}^{(1)} = .9 \times e^{-.25} \times \frac{1-e^{-.25}}{.05\times 45} = .0689$. Answer: C

6. ${}_np_x^{(\tau)} = exp[-\int_0^n \mu_{x+t}^{(\tau)}\,dt] = exp[-\int_0^n .15t\,dt] = e^{-.075n^2}$.
APV of accidental death benefit is $2\int_0^{20} {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(1)}\,dt = 2\int_0^{20} e^{-.075t^2} \cdot \frac{t}{20}\,dt$
Using the substitution, $s = .075t^2$, $ds = .15t\,dt$, and the integral becomes
$\int e^{-s} \frac{1}{20(.15)}\,ds$, so the APV is $2 \times \frac{1}{3}(-e^{-.075t^2}\Big|_{t=0}^{t=20}) = \frac{2}{3}(1-e^{-30})$.
The APV of the benefit for death due to other causes is
$\int_0^{20} {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(2)}\,dt = \int_0^{20} e^{-.075t^2} \cdot \frac{t}{10}\,dt$. We see that this integral is twice as large as
$\int_0^{20} e^{-.075t^2} \cdot \frac{t}{20}\,dt$, so $\int_0^{20} e^{-.075t^2} \cdot \frac{t}{10}\,dt = \frac{2}{3}(1-e^{-30})$.
The total APV is $\frac{4}{3}(1-e^{-30})$. Answer: E

7. This problem involves single net premium for life insurance with death benefit payable at the end of the year of death, and it also involves the recursive relationship for insurance valuation. The accumulated value of the Q invested for 2 years at 3.0% is $(1.03)^2Q$. The single net premium for the insurance at age 65 (2 years after age 63) is $10,000A_{65}$, and the contract premium is $(1.12)(10,000)A_{65}$. We cannot use the A_x values in the Illustrative Life Table, since they are based on an interest rate of 6%. The single net premium at age 63 is $Q = 10,000A_{63}$. We use the relationship $A_x = vq_x + v^2 \cdot {}_{1|}q_x + v^2 \cdot {}_2p_x \cdot A_{x+2}$. From the Illustrative Life Table, we have $q_{63} = .01788$, $q_{64} = .01952$. The single net premium at age 63 is $\frac{Q}{1.12}$. Therefore,

$$\frac{Q}{1.12} = \frac{10{,}000}{1.05} \cdot (.01788) + \frac{10{,}000}{(1.05)^2} \cdot (.98212)(.01952) + \frac{10{,}000}{(1.05)^2} \cdot (.98212)(.98048) \cdot A_{65}$$

$\rightarrow 10,000A_{65} = 1.022250Q - 394.05$

The contract premium for the insurance at age 65 will be

$(1.12)(10,000)A_{65} = (1.12)(1.022250Q - 394.05) = (1.03)^2Q$.

Solving for Q results in $Q = 5252$. Answer: D

8. Since we are given ${}_{10}V$ we use the relationship

$({}_{10}V + P)(1+i) - b_{11} \cdot q_{x+10} = p_{x+10} \cdot {}_{11}V$,

where P is the annual net premium and b_{11} is the death benefit payable at the end of the 11th year for death in the 19-th year. The insurance is a 20-year decreasing insurance, so that

$b_{11} = 21 - 11 = 10$. Also, the issue age is $x = 55$, so that the equation becomes

$(5.0 + P)(1.08) - 10q_{65} = p_{65} \cdot {}_{11}V$, which is $(5.0 + P)(1.08) - 10q_{65} = p_{65}(5.3)$.

In order to find q_{65} we need to know P. We can find P by using the recursive relationship for the final (20th) year: $({}_{19}V + P)(1+i) - b_{20} \cdot q_{x+19} = p_{x+19} \cdot {}_{20}V$.

When a policy matures, the maturity amount is the reserve at the end of the policy term. Therefore, ${}_{20}V = 1$, since we are told that this is a 20-year policy with maturity value 1. The death benefit is also $b_{20} = 1$ in the 20th year. The recursive relationship becomes

$(0.6 + P)(1.08) - q_{64} = p_{64} \cdot 1$, which becomes $(.6 + P)(1.08) = 1$, since

$q_x + p_x = 1$ for any x. We then get $P = .3259$, and then $q_{65} = .10$. Answer: E

9. $\mu_x = Bc^x \rightarrow \mu_{x:x} = \mu_x + \mu_x = Bc^x + Bc^x = \mu_{x+4} = Bc^{x+4}$.
It follows that $2 = c^4$, from which we get $c = 1.1892$.
Under Gompertz law,
$p_{50} = exp[-\int_0^1 \mu_{50+t}\,dt] = exp[-\int_0^1 .0001 \cdot 1.1892^{50+t}\,dt]$
$= exp[-.0001 \cdot \frac{1.1892^{51} - 1.1892^{50}}{ln\,1.1892}] = .53$. Answer: B

10. This problem involves the equivalence principle for finding a net premium.
For the first 20 years, the net premium is P, and for the next 15 years it is $5P$. The death benefit is C for the first 20 years and then 5 after that. The equivalence principle equation is
$(.016)[\ddot{a}_{30:\overline{20}|} + 5v^{20} \cdot {}_{20}p_{30} \cdot \ddot{a}_{50:\overline{15}|}] = CA_{\overset{1}{30}:\overline{20}|} + 5v^{20} \cdot {}_{20}p_{30} \cdot A_{50}$.
We use the relationships $v^{20} \cdot {}_{20}p_{30} \cdot \ddot{a}_{50:\overline{15}|} = \ddot{a}_{30:\overline{35}|} - \ddot{a}_{30:\overline{20}|}$, and
$\ddot{a}_{30:\overline{20}|} = \frac{1 - A_{30:\overline{20}|}}{d}$, and $A_{\overset{1}{30}:\overline{20}|} = A_{30:\overline{20}|} - v^{20} \cdot {}_{20}p_{30}$.
From the Illustrated Life Table, we get $v^{20} \cdot {}_{20}p_{30} = .29374$.
Then, $\ddot{a}_{30:\overline{20}|} = \frac{1 - A_{30:\overline{20}|}}{d} = 11.959$,
$v^{20} \cdot {}_{20}p_{30} \cdot \ddot{a}_{50:\overline{15}|} = \ddot{a}_{30:\overline{35}|} - \ddot{a}_{30:\overline{20}|} = 2.876$, and
$A_{\overset{1}{30}:\overline{20}|} = A_{30:\overline{20}|} - v^{20} \cdot {}_{20}p_{30} = .0293$. Then, $C = 1.9$. Answer: E

11. The following sequences of rankings for the next three months result in two consecutive E-E ratings (including the current ranking of G):
Sequence that pay bonus at time 2: $G - E - E$, prob. $(.1)(.2) = .02$
Sequences that pay bonus at time 3:
$G - G - E - E$, prob. $(.5)(.1)(.2) = .01$, $G - E - E - E$, prob. $(.1)(.2)(.2) = .004$,
total prob. of bonus payment at time 3 is .014.
Actuarial present value is $1000[.02v^2 + .014v^3] = 33.19$.
The following sequences of rankings for the next three months result in two consecutive E-E ratings (including the current ranking of E):
Sequence that pays bonus at time 1: $E - E$, prob. .2
Sequence that pay bonus at time 2: $E - E - E$, prob. $(.2)(.2) = .04$
Sequences that pay bonus at time 3:
$E - G - E - E$, prob. $(.6)(.1)(.2) = .012$, $E - E - E - E$, prob. $(.2)(.2)(.2) = .008$,
total prob. of bonus payment at time 3 is .02.
Actuarial present value is $1000[.2v + .04v^2 + .02v^3] = 256.64$.
Total APV is 289.82 . Answer: A

12. The annual rate of payment is $12C$, so answers B and D are incorrect. Since disability occurs at mid-year of age, and it begins $\frac{1}{2}$-year later, at the next year of age and continues to one month before age 75, and the valuation is on age 40. Answer: C

13. This problem involves the common shock model for dependence of two lives.
The insurance single net premium for each of (x) and (y) is $\overline{A}_x = \overline{A}_y = \frac{.06}{.06+.05} = \frac{6}{11}$.
Since $\mu_x = .06 = \mu_x^* + \lambda$ and $\lambda = .02$, it follows that (x)'s force of mortality is $\mu_x^* = .04$ before common shock, and for (y) it is similarly true that $\mu_y^* = .04$. The force of failure for the joint life status is $\mu_x^* + \mu_y^* + \lambda = .04 + .04 + .02 = .10$, so the single net premium for a joint life insurance is $A_{xy} = \frac{.10}{.10+.05} = \frac{10}{15}$. Then, the single net premium for the last survivor insurance, under the original common shock model is
$\overline{A}_{\overline{xy}} = \overline{A}_y + \overline{A}_y - \overline{A}_{xy} = \frac{6}{11} + \frac{6}{11} - \frac{10}{15} = .424$.
If it assumed that the lives are independent with $\mu_x = \mu_y = .06$, then the force of failure for the joint life status is $\mu_x + \mu_y = .12$, and the joint life insurance net premium is $A_{xy} = \frac{.12}{.12+.05} = \frac{12}{17}$. Then the single net premium for the last survivor insurance, under the independence model is $\overline{A}_{\overline{xy}} = \overline{A}_y + \overline{A}_y - \overline{A}_{xy} = \frac{6}{11} + \frac{6}{11} - \frac{12}{17} = .385$.
The new value is $.424 - .385 = .039$ smaller than the original value. Answer: B

14. The APV at age 80 is $(DA)^{\,1}_{80:\overline{20}|} = 20vq_{80} + vp_{80} \cdot (DA)^{\,1}_{81:\overline{19}|}$.
We are given that $i = .06$, $q_{80} = .2$ and $(DA)^{\,1}_{80:\overline{20}|} = 13$.
Therefore, $(DA)^{\,1}_{81:\overline{19}|} = \frac{13-20v(.2)}{v(.8)} = 12.225$.
If q_{80} is changed to C, then $(DA)^{\,1}_{81:\overline{19}|} = 12.225$ remains unchanged, and the revised value of the premium $(DA)^{new\,1}_{80:\overline{20}|}$ is $20vC + v(1-C) \cdot (12.225) = 12.30$.
Solving for C results in $C = .1046$. Answer: B

15. If she does not take up hot-air ballooning, Audra's 11-year temporary complete life expectancy will be $\overset{\circ}{e}_{25:\overline{11}|} = \int_0^{11} {}_tp_{25}\,dt = \int_0^{11} e^{-.01t}\,dt = 10.4166$. The expectation can also be written as

$\overset{\circ}{e}_{25:\overline{11}|} = \overset{\circ}{e}_{25:\overline{1}|} + p_{25} \cdot \overset{\circ}{e}_{26:\overline{10}|} = \int_0^1 {}_tp_{25}\,dt + p_{25} \cdot \int_0^{10} e^{-.01t}\,dt$

$= \int_0^1 {}_tp_{25}\,dt + p_{25} \cdot (9.5163).$

If Audra does take up hot-air ballooning for one year (year of age 25), her survival probability for year of age 25 will be ${}_tp_x^* = 1 - .1t$ for $0 \le t < 1$ (UDD for the year).

Her revised 11-year temporary complete life expectancy will be

$\overset{\circ}{e}{}^*_{25:\overline{11}|} = \overset{\circ}{e}{}^*_{25:\overline{1}|} + p_{25}^* \cdot \overset{\circ}{e}{}^*_{26:\overline{10}|} = \int_0^1 {}_tp_{25}^*\,dt + p_{25}^* \cdot (9.5163)$

$= .95 + (.9)(9.5163) = 9.51467$.

Her expectation would decrease $10.4166 - 9.51467 = .90$. Answer: D

16. From Gompertz's law we have $\mu_{45} = Bc^{45} = .00004 \times 1.1^{45} = .002916$, and $\mu_{44.75} = 002847.$ Then

${}_{4.75}\overline{V} = 3000 - .25(750(.95) + .04(3000) - .002916(20,100 - 3000)) = 2804.34$,

and

${}_{4.5}\overline{V} = 2804.34 - .25(750(.95) + .04(2804.34) - .002847(20,100 - 2804.34)) = 2610.48$

Answer: C .

17. We wish to find $P_{\overline{xy}} = \frac{A_{\overline{xy}}}{\ddot{a}_{\overline{xy}}} = \frac{1-d\ddot{a}_{\overline{xy}}}{\ddot{a}_{\overline{xy}}} = \frac{1}{\ddot{a}_{\overline{xy}}} - d$, so if we can find $\ddot{a}_{\overline{xy}}$, we can find $P_{\overline{xy}}$. From $P_x = \frac{1}{\ddot{a}_x} - d$, we get $\ddot{a}_x = \frac{1}{P_x+d} = \frac{1}{.16}$, and also

$\ddot{a}_y = \frac{1}{P_y+d} = \frac{1}{.16}$. Also, $P_{xy} = \frac{A_{xy}}{\ddot{a}_{xy}} = \frac{1-d\ddot{a}_{xy}}{\ddot{a}_{xy}} = \frac{1}{\ddot{a}_{xy}} - d$, so that

$\ddot{a}_{xy} = \frac{1}{P_{xy}+d} = \frac{1}{.24}$.

However, since $\ddot{a}_{\overline{xy}} = \ddot{a}_x + \ddot{a}_y - \ddot{a}_{xy}$, we get $\ddot{a}_{\overline{xy}} = \frac{1}{.16} + \frac{1}{.16} - \frac{1}{.24} = 8.333$

Then, $P_{\overline{xy}} = \frac{1}{\ddot{a}_{\overline{xy}}} - d = \frac{1}{8.333} - .06 = .06$. Answer: B

18. This problem relies on the following relationship ${}_np_x^{(\tau)} = e^{-\int_0^n \mu_{x+t}^{(\tau)}\,dt}$.
Since we are told that $\mu_{60+t}^{(1)}$ follows the Illustrative Life Table, it follows that
$e^{-\int_0^n \mu_{60+t}^{(1)}\,dt} = {}_np_{60}$ (from the Illustrative Life Table).
Then, since $\mu_{60+t}^{(\tau)} = c \cdot \mu_{60+t}^{(1)}$, we get
${}_np_x^{(\tau)} = e^{-\int_0^n \mu_{x+t}^{(\tau)}\,dt} = e^{-\int_0^n c\mu_{60+t}^{(1)}\,dt} = e^{-c\int_0^n \mu_{60+t}^{(1)}\,dt}$
$= [e^{-\int_0^n \mu_{60+t}^{(1)}\,dt}]^c = ({}_np_{60})^c$ (from the Illustrative Life Table).
From the Illustrative Table we have ${}_{10}p_{60} = \frac{\ell_{70}}{\ell_{60}} = \frac{6{,}616{,}155}{8{,}188{,}074} = .808023$.
It follows that $1 - .57372 = .42628 = (.808023)^c$, and solving for c results in
$c = \frac{ln(.42628)}{ln(.808023)} = 4.0$. Answer: E

19. This problem can be solved with the accumulation relationship for net reserves.
The initial net reserve for a particular year is the terminal reserve for the previous year just ended plus the annual net premium:
year $t+1$ initial net reserve $= {}_tV +$ net premium .
$({}_2V + P)(1+i) - b_3 \cdot q_{x+2} = p_{x+2} \cdot {}_3V$, and the initial net reserve for year 3 is
${}_2V + P$. It is always the case that ${}_3V = 0$ for a 3-year term insurance, and for this problem,
$i = .06$, $q_{x+2} = .03$, and $b_3 = 100,000$.
Then $({}_2V + P)(1+.06) - 100,000(.09) = 0$, so that ${}_2V + P = 8,491$. Answer: C

20. This problem involves the loss-at-issue random variable. The loss-at-issue is
$L =$ present value random variable of benefit
$-$ present value random value random variable of premiums .
The PVRV of benefit is $b_{K+1} \cdot v^{K+1} = (1+i)^{K+1}v^{K+1} = 1$ and the PVRV of premium is
$Q\ddot{a}_{\overline{K+1}|}$, where Q is the equivalence principle premium. Q is found from the equivalence principle equation APV benefit $=$ APV premium . This equation is $E[1] = Q \cdot \ddot{a}_x$, so that
$Q = \frac{1}{\ddot{a}_x}$. The loss random variable becomes
$L = 1 - Q\ddot{a}_{\overline{T}|} = 1 - \frac{\ddot{a}_{\overline{K+1}|}}{\ddot{a}_x} = 1 - \frac{(1-v^{K+1})/d}{(1-A_x)/d} = \frac{v^{K+1}-A_x}{1-A_x}$. Answer: A