S. BROVERMAN MLC STUDY GUIDE
PRACTICE EXAM 1
SECTION A - MULTIPLE CHOICE

1. For a fully discrete 5-payment 10-year decreasing term insurance on (60), you are given:
(i) $b_{k+1} = 1000(10-k)$, $k = 0,1,2,...,9$
(ii) $q_{60+k} = 0.02 + 0.001k$, $k = 0,1,2,...,9$
(iii) $i = 0.06$
(iv) The net reserve at the end of year 2 is ${}_2V = 77.66$.
Calculate the level net annual premium.
A) 188 B) 194 C) 202 D) 210 E) 218

2. For a double-decrement model:
(i) ${}_tp_{40}^{\prime(1)} = 1 - \frac{t^2}{1600}$, $0 \le t \le 40$
(ii) ${}_tp_{40}^{\prime(2)} = 1 - \frac{t^2}{900}$, $0 \le t \le 30$
Calculate $\mu_{40+20}^{(\tau)}$.
A) Less than .05 B) At least .05 but less than .10 C) At least .10 but less than .15
D) At least .15 but less than .20 E) At least .20

3. For independent lives (35) and (45):
(i) ${}_5p_{35} = 0.90$ (ii) ${}_5p_{45} = 0.80$ (iii) $q_{40} = 0.03$ (iv) $q_{50} = 0.05$
Calculate the probability that the first death of (35) and (45) occurs in the 6th year.
A) 0.0465 B) 0.0565 C) 0.0665 D) 0.0765 E) 0.0865

4. A pension plan provides a retirement annuity with annual rate of:
a - 2% of the first 25,000 of final year salary for each of the first 25 completed years of service
b - 2.5% of the amount of final year salary in excess of 25,000 for each year of the first 25 completed years of service
c - 3% of the first 25,000 of final year salary for each completed year of service in excess of 25 years
d - 3.5% of the amount of final salary in excess of 25,000 for each completed year of service in excess of 25 years.
If (45) has 10 years of service already and if $CAS_{45} = 35,000$ (current actual salary), then $PAB_{65.5}$ (projected annual benefit) can be expressed in the form $A + \frac{S_{65}}{S_{45}} \cdot B$. Find $A + B$.
A) 24,000 B) 24,250 C) 24,500 D) 24,750 E) 25,000

5. You are given: $q_x = \begin{cases} 0.05 & 50 \leq x < 60 \\ 0.04 & 60 \leq x < 70 \end{cases}$

Calculate ${}_{4|14}q_{50}$.

A) 0.34 B) 0.36 C) 0.38 D) 0.40 E) 0.42

6. For an increasing 10-year term insurance, you are given:

(i) $b_{k+1} = 100,000(1+k)$, $k = 0, 1, ..., 9$

(ii) Benefits are payable at the end of the year of death.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

(v) The net single premium for this insurance on (40) is $15,551$.

Calculate the net single premium for this insurance on (41).

A) 15,220 B) 15,780 C) 16,220 D) 16,780 E) 17,220

7. You are given that the force of interest is 5% and the force of mortality is constant at .01.

A is the exact value of $\overline{a}_x$ and B is the value of $\overline{a}_x$ using Woolhouse's formula to three terms.

Determine the ratio A/B .

A) Less than .97 B) At least .97 but less than .99 C) At least .99 but less than 1.01

D) At least 1.01 but less then 1.03 E) At least 1.03

8. For a fully discrete 3-year endowment insurance of 1000 on (x), you are given:

(i) ${}_kL$ is the prospective loss random variable at time k.

(ii) $i = 0.10$ (iii) $\ddot{a}_{x:\overline{3}|} = 2.70182$

(iv) Premiums are determined by the equivalence principle.

Calculate ${}_2L$, given that (x) survives to the end of the third year from issue.

A) 540 B) 630 C) 655 D) 720 E) 910

9. For a special 3-year term insurance on (30), you are given:

(i) Premiums are payable semiannually.

(ii) Premiums are payable only in the first year.

(iii) Benefits, payable at the end of the year of death, are:

k	b_{k+1}
0	1000
1	500
2	250

(iv) Mortality follows the Illustrative Life Table.

(v) Deaths are uniformly distributed within each year of age.

(vi) $i = 0.08$

Calculate the amount of each semiannual net premium for this insurance.

A) Less than 1.2 B) At least 1.2 but less than 1.3 C) At least 1.3 but less than 1.4

D) At least 1.4 but less than 1.5 E) At least 1.5

10. For a Markov model with three states, Healthy (0), Disabled (1), and Dead (2):

(i) The annual transition matrix is given by

	0	1	2
0	0.70	0.20	0.10
1	0.10	0.65	0.25
2	0	0	1

(ii) There are 100 lives at the start, all Healthy. Their future states are independent.

Calculate the variance of the number of the original 100 lives who die in the second year.

A) Less than 8 B) At least 8 but less than 10 C) At least 10 but less than 12

D) At least 12 but less than 14 E) At least 14

11. A bank offers the following choices for certificates of deposit:

Term (in years)	Nominal annual interest rate convertible quarterly
1	4.00%
3	5.00%
5	5.65%

The certificates mature at the end of the term. The bank does NOT permit early withdrawals. During the next 6 years the bank will continue to offer certificates of deposit with the same terms and interest rates. An investor initially deposits 10,000 in the bank and withdraws the principal and interest at the end of 6 years. Calculate the maximum annual effective rate of interest the investor can earn over the 6-year period.

A) 5.09% B) 5.22% C) 5.35% D) 5.48% E) 5.61%

12. For a life table with a one-year select period, you are given:

(i)

x	$\ell_{[x]}$	$d_{[x]}$	ℓ_{x+1}	$\mathring{e}_{[x]}$
80	1000	90	–	–
81	920	90	–	8.2

(ii) Deaths are uniformly distributed over each year of age.

Calculate $\mathring{e}_{[80]}$.

A) 8.1 B) 8.2 C) 8.3 D) 8.4 E) 8.5

13. A fully continuous whole life insurance of face amount 1 is based on the following assumptions:

- constant force of interest of 6%
- constant force of mortality of .015 at all ages
- annual contract premium of .016

An insurer determines that with a portfolio of n independent policies of this type, using the normal approximation, the probability of a positive loss on all policies combined is .05. How many additional independent policies of the same type would be needed to reduce the probability of a positive total loss to .025 (continuing to use the normal approximation)?

A) Less than 500 B) At least 500, but less than 600 C) At least 600, but less than 700
D) At least 700, but less than 800 E) At least 800

14. For a fully discrete whole life insurance of b on (x), you are given:

(i) $q_{x+9} = 0.02904$ (ii) $i = 0.03$

(iii) The net amount at risk for policy year 10 is 872.

(iv) $\ddot{a}_x = 14.65976$

(v) ${}_9V = 296.1$

Calculate the initial net reserve for policy year 10.

A) 295 B) 321 C) 343 D) 368 E) 388

15. A fully discrete 3-year term insurance of 10,000 on (40) is based on a double-decrement model, death and withdrawal:

(i) Decrement 1 is death. (ii) $q_{x+k}^{(1)} = 0.02$, $k \geq 0$

(iii) Decrement 2 is withdrawal, which occurs at the end of the year.

(iv) $q_{40+k}^{\prime(2)} = 0.04$, $k = 0, 1, 2$ (v) $v = 0.95$

Calculate the actuarial present value of the death benefits for this insurance.

A) 492 B) 502 C) 512 D) 522 E) 532

16. A 5-year endowment insurance policy with face amount 100,000 and discrete annual premiums has issue age 50. The policy has the following schedule of expenses: 40% of premium in the first year, 10% of premium in renewal years and a per policy expense of 250 in the first year and 50 in each renewal year. Mortality follows the Illustrative Life Table in the Exam MLC Tables found at the end of this study guide, with an annual interest rate of 6%. The policy has liabilities based on benefit-plus-expense reserves. Suppose that the mortality, expenses and interest experienced in the fourth year (time 3 to time 4) is the same as the assumptions, but the liability basis is changed to full preliminary term reserves. What is the gain in the fourth year on this policy?

A) Less than -1000 B) At least -1000 but less than 0

C) At least 0 but less than 1000 D) At least 1000 but less than 2000

E) At least 2000

17. You are pricing a special 3-year annuity-due on two independent lives, (x) and (y). The annuity pays 30,000 if both persons are alive, 20,000 if (x) dies first and 10,000 if (y) dies first. You are given:

(i)

k	${}_kp_x = {}_kp_y$
1	0.91
2	0.82
3	0.72

(ii) $i = 0.05$

Calculate the actuarial present value of this annuity (nearest 100).

A) 78,300 B) 80,400 C) 82,500 D) 84,700 E) 86,800

18. You are given:

(i) $\mu_{x+t} = 0.03$, $t \geq 0$ (ii) $\delta = 0.05$

(iii) T_x is the future lifetime random variable.

(iv) g is the standard deviation of $\overline{a}_{\overline{T_x|}}$.

Calculate $Pr\left(\overline{a}_x - g < \overline{a}_{\overline{T_x|}} < \overline{a}_x + g\right)$.

A) 0.50 B) 0.54 C) 0.58 D) 0.62 E) 0.66

19. For a fully discrete whole life insurance of 100,000 on (35) you are given:
(i) Percent of premium expenses are 10% per year.
(ii) Per policy expenses are 25 per year.
(iii) Per thousand expenses are 2.50 per year.
(iv) All expenses are paid at the beginning of the year.
(v) The level annual expense loaded premium is 1234.
Calculate $100,000P_{35}$.
A) Less than 800 B) At least 800 but less than 815 C) At least 815 but less than 830
D) At least 830 but less than 845 E) At least 845

20. For a fully discrete 3-year endowment insurance of 1000 on (x):
(i) $i = 0.05$ (ii) $p_x = p_{x+1} = p$
(iii) the second year net reserve is ${}_2V = 526$.
Calculate p.
A) .62 B) .64 C) .66 D) .68 E) .70

SECTION B - WRITTEN ANSWER

1. (7 points)
Z_1 denotes the present value random variable for a 10 year pure endowment benefit of 1 issued to (x). Z_2 denotes the present value random variable for a 20 year pure endowment benefit of 1 issued to (x). You are given:

$\mu_{x+t} = .01$ for all $t \geq 0$, $\delta =$ $.04$ (force of interest)

(a) (2 points) Formulate the present value random variables of Z_1 and Z_2.
(b) (4 points) Find the variances of Z_1 and Z_2 and find $Cov[Z_1, Z_2]$
(c) (3 points) Suppose that Z_3 is the present value random variable for a 20 year discrete endowment insurance benefit of 1. Formulate the covariance between Z_1 and Z_3 in terms of insurance single premium notation and compound interest. Give a brief explanation on whether this covariance is positive or negative and why.

2. (7 points)
Company ABC is using the following assumptions to set the contract premium for a 5 year deferred continuous life annuity of 1 per year to (x):

(i) $\delta = 0.03$ (ii) $\mu_{x+t} = \begin{cases} 0.02 & \text{for } t \leq 10 \\ 0.01 & \text{for } t > 10 \end{cases}$

(a) Calculate the contract premium in each of the following cases:
(i) (2 points) The premium is a single premium paid at the time the annuity is issued.
(ii) (1 points) Premiums are paid annually on a continuous basis during the deferral period.
(b) Another actuary uses the alternative mortality assumption that $\mu_{x+t} = .02$ for $t \geq 0$.
(i) (2 points) Calculate the expected gain at issue based on the alternative mortality assumption if the premium paid is the single premium calculated (a)(i).
(ii) (1 point) Calculate the expected gain at issue based on the alternative mortality assumption if the premium paid is the annual premium calculated (a)(ii).
(iii) (1 point) Explain the relationship between the outcomes in (b)(i) and (b)(ii).

3. (9 points)
A fully discrete whole life insurance policy issued at age 50 has face amount 1,000,000 .
Mortality is based on the Illustrative Table at annual effective interest rate 6%.
The policy expenses are:

	1st Year	Renewal Years
Percent of Premium	20%	5%
Face Amount	5 per 1000	1 per 1000
Per Policy	1000	200
Settlement	1000 (at the end of the year of death)	

(a) (2 points) Calculate both the net and the gross annual premium for this policy based on the equivalence principle.
(b) (3 points) Calculate the 9th year terminal expense augmented reserve using the prospective formulation. Apply the recursive reserve accumulation from time 9 to time 10 to find the 10th year terminal expense augmented reserve.
(c) (2 points) Calculate the 10th year terminal expense reserve. Give a brief explanation as to why the expense reserve is negative.
(d) (2 points) An actuary wishes to analyze possible profit in the 10th policy year. The actuary makes the following assumptions:
- liabilities are benefit plus expense reserves
- gross premiums, interest rate, mortality rate are based on the original policy assumption
- policy lapse rate is 20% with cash value ${}_{10}CV$ paid at end of 10th year for lapses that occur
Calculate the cash value that would result in an expected profit of \$5000 at the end of the 10th policy year.

4. (9 points)

An insurance company issues a special 3-year insurance to a high-risk individual. You are given the following homogeneous Markov chain model:

(i) Transition probability matrix:

		1	2	3	4
State 1: active	1	0.6	0.2	0.1	0.1
State 2: disabled	2	0.2	0.6	0	0.2
State 3: withdrawn	3	0	0	1	0
State 4: dead	4	0	0	0	1

(ii) Changes in state occur in the instant before the end of a year.

(iii) The death benefit is 100,000, payable at the end of the year of death.

(iv) The disability benefit is a payment of 25,000 at the start of the year if the individual is disabled at that time.

(iv) $i = 0.05$

(v) The insured is active at the start of year 1.

(a) (3 points) Calculate the probability that the individual will die while the policy is still in force before the three year term elapses.

(b) (2 points) Calculate the actuarial present value of the death benefit at the time the policy is issued.

(c) (2 points) Calculate the actuarial present value of the disability benefit.

(d) (2 points) A net annual premium is payable at the start of each year the individual is active. Calculate the net annual premium.

5. (7 points)

Two individuals aged (x) and (y) each have constant, but different, forces of mortality:

$\mu_{x+t} = .25$, $\mu_{y+t} = .15$ for $t \geq 0$

(a) (3 points) Assuming T_x and T_x are independent, calculate $\mathring{e}_{xy}$, $P[T_x < T_y]$, and $A_{\overset{1}{x}y}$ based on a force of interest $\delta = .1$.

(b) (3 points) The common shock model is assumed, with common shock factor $\lambda = .05$. The forces of mortality for T_x and T_y are still the originally assumed values. Calculate $\mathring{e}_{xy}$, $P[T_x < T_y]$, and $A_{\overset{1}{x}y}$ based on a force of interest $\delta = .05$.

(c) (1 point) Provide a brief explanation as to why ${}_tp_{xy}$ is smaller in (a) than it is in (b) even though ${}_tp_x$ and ${}_tp_y$ are the same in both cases.

6. (11 points)
Max is a member of a defined benefit pension plan. Max is 60 on January 1, 2010 and has been a plan member for 30 years. Max gets a salary increase each January 1, and his salary as of January 1, 2010, after the increase, is 75,000. His annual retirement benefit is 2% of his two-year final average salary for each year of service. Max's normal retirement date is December 31, 2014, at age 65. Retirements occur on December 31 and there is a reduction in benefit for early retirement of 4% for each year prior to normal retirement. He expects a 3% salary increase each January 1 and his benefit will be payable as a single annuity.
(a) (3 points) Calculate the replacement ratio in Max's pension if he retires at age 64 (on December 31, 2013).
(b) (5 points) Assume that the rates in the Illustrative Table are used as the mortality rates in a two-decrement model for death and retirement. q_x from the Illustrative Table is $q_x^{(d)}$ in the two-decrement table. Assume also that $q_x^{(r)} = 0$ for $x < 63$, $q_{63}^{(r)} = .1$, $q_{64}^{(r)} = .25$ and $q_{65}^{(r)} = 1$. Assume further that post-retirement mortality follow the Illustrative Table as a one-decrement table, with $i = .06$. Max's retirement annuity is an annuity-due that commences on his retirement date Calculate the APV of Max's retirement benefit on January 1, 2010.
(c) (3 points) Suppose that Max retires at age 65. Max is entitled to an annual pension benefit of 58,232 commencing at the time of retirement. Max has a spouse who is also exactly 65 at the time of Max's retirement and whose mortality is independent of that of Max. Max is given an optional alternative benefit annuity-due that begins at the same time the original benefit would have begun and that is actuarially equivalent to his original benefit at age 65. Under this option, Max and his spouse will receive a 10-year certain last-survivor annuity of K per year.
Calculate K.

7. (6 points)
A Type B Universal Life policy has death benefit of 25,000 plus account value. The policy is issued at age (50). You are given:
(i) Premium paid at the start of the first year is 6,000.
(ii) Expense charges each year are 5% of premium plus 100, charged at the start of the year.
(iii) The cost of insurance rate is 120% of the mortality rate at the attained age in the Illustrative Life Table.
(iv) Credited interest rate applied to investments is $i^c = .05$ each year.
(v) COI interest rate is $i^q = .04$ each year.

(a) (3 points) The premium at the start of the second year is 6,000. Calculate the account value at the end of the second year.
(b) (3 points) Suppose that policy is Type A, with a total death benefit of 25,000. All other assumptions regarding premium, interest rates, etc., are unchanged.. Calculate the account value at the end of the second year.

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PRACTICE EXAM 1 SOLUTIONS

SECTION A SOLUTIONS - MULTIPLE CHOICE

1. We use the recursive reserve formula, $({}_kV+P)(1+i)-b_{k+1}\cdot q_{x+k}=p_{x+k}\cdot {}_{k+1}V$.
Also, ${}_0V=0$ for net reserves. For the first year, we have
$(0+P)(1.06)-10,000(.02)=.98\cdot {}_1V \rightarrow {}_1V=1.0816P-204.08$.
For the second year, we have
$(1.0816P-204.08+P)(1.06)-9,000(.021)=.979\cdot {}_2V=(.979)(77.66)=76.03$
$\rightarrow P=218$. Answer: E

2. $\mu_{40+t}^{(\tau)}=\frac{-\frac{d}{dt}\,{}_tp_{40}^{(\tau)}}{{}_tp_{40}^{(\tau)}}$. ${}_tp_{40}^{(\tau)}={}_tp_{40}^{\prime(1)}\cdot{}_tp_{40}^{\prime(2)}=(1-\frac{t^2}{1600})(1-\frac{t^2}{900})=1-\frac{t^2}{576}+\frac{t^4}{1,440,000}$
$\frac{-\frac{d}{dt}\,{}_tp_{40}^{(\tau)}}{{}_tp_{40}^{(\tau)}}=\frac{\frac{t}{288}-\frac{4t^3}{1,440,000}}{1-\frac{t^2}{576}+\frac{t^4}{1,440,000}} \rightarrow \mu_{40+20}^{(\tau)}=\frac{\frac{20}{288}-\frac{32,000}{1,440,000}}{1-\frac{400}{576}+\frac{160,000}{1,440,000}}=.113$. Answer: C

3. The probability that the first death occurs in the 6th year is ${}_{5|}q_{35:45}$.
This can be formulated as
${}_{5|}q_{35:45}={}_5p_{35:45}-{}_6p_{35:45}={}_5p_{35}\cdot{}_5p_{45}-{}_6p_{35}\cdot{}_6p_{45}={}_5p_{35}\cdot{}_5p_{45}\cdot(1-p_{40}\cdot p_{50})$
From the given information we have $p_{40}=0.97$ and $p_{50}=0.95$.
Then, ${}_{5|}q_{35:45}=(.9)(.8)[1-(.97)(.95)]=.05652$. Answer: B

4. $PAB_{65.5}=(.02)(25{,}000)(25)+(.025)\left[35{,}000\frac{S_{65}}{S_{45}}-25{,}000\right](25)$
$+(.03)(25{,}000)(5)+(.035)\left[35{,}000\frac{S_{65}}{S_{45}}-25{,}000\right](5)=-3750+28{,}000\frac{S_{65}}{S_{45}}$
$\rightarrow A+B=24,250$. Answer: B.

5. ${}_{4|14}q_{50}={}_4p_{50}-{}_{18}p_{50}$.
${}_4p_{50}=(.95)^4=.8145$, ${}_{18}p_{50}={}_{10}p_{50}\cdot{}_8p_{60}=(.95)^{10}(.96)^8=.4319$.
Then ${}_{4|14}q_{50}=.8145-.4319=.3826$. Answer: C

6. We are given $100,000(IA)^{1}_{40:\overline{10}|} = 15,551$. We wish to find $100,000(IA)^{1}_{41:\overline{10}|}$.
We use the relationship $(IA)^{1}_{x:\overline{n}|} = A^{1}_{x:\overline{n}|} + vp_x\,(IA)^{1}_{x+1:\overline{n}|} - nv^{n+1}\,{}_{n|}q_x$. This can be seen by looking at the time line of possible death benefit payments; the first row is the sum of the second and third rows.

	x	$x+1$	$x+2$	$x+3$	$\dots$	$x+n-1$	$x+n$	$x+n+1$
$(IA)^{1}_{x:\overline{n}|}$	$\leftarrow$	1	2	3		$n-1$	n	
$A^{1}_{x:\overline{n}|}$	$\leftarrow$	1	1	1		1	1	
$vp_x\,(IA)^{1}_{x+1:\overline{n}|}$		$\leftarrow$	1	2		$n-2$	$n-1$	n
$-\,nv^{n+1}\,{}_{n|}q_x$								$-\,n$

Then, $(IA)^{1}_{40:\overline{10}|} = A^{1}_{40:\overline{10}|} + vp_{40}\,(IA)^{1}_{41:\overline{10}|} - 10v^{11}\,{}_{10|}q_{40}$
$A^{1}_{40:\overline{10}|} = A_{40} - v^{10}\,{}_{10}p_{40}A_{50} = (.16132) - (.53667)(.24905) = .027662$
(we use values from the Illustrative Table, and note that $v^{10}\,{}_{10}p_{40} = {}_{10}E_{40}$).
Also $v^{11}\,{}_{10|}q_{40} = v \cdot v^{10}\,{}_{10}p_{40} \cdot q_{50} = \frac{1}{1.06} \cdot (.53667)(.00592) = .002997$.
Then $.15551 = .027662 + \frac{1}{1.06} \cdot (1 - .00278)\,(IA)^{1}_{41:\overline{10}|} - 10(.002997)$.
Solving for $(IA)^{1}_{41:\overline{10}|}$ results in $(IA)^{1}_{41:\overline{10}|} = .1678$
We then multiply by 100,000 to get $16,780$. Answer: D

7. With constant force of interest δ and constant force of mortality μ, we have
$A = \overline{a}_x = \frac{1}{\mu+\delta} = \frac{1}{.01+.05} = 16.6667$, and $\ddot{a}_x = \frac{1}{1-vp_x} = \frac{1}{1-e^{-.05}\cdot e^{-.01}} = 17.1717$.
With $m = \infty$, $B = 17.1717 - \frac{1}{2} - \frac{1}{12}(.06) = 16.6667$. $A/B = 1.00$. Answer: C

8. The equivalence principle premium is
$1000P_{x:\overline{3}|} = 1000(\frac{1}{\ddot{a}_{x:\overline{3}|}} - d) = 1000(\frac{1}{2.70182} - \frac{.1}{1.1}) = 279.21$.
We are given that (x) survives to the end of the third year from issue. Using the end of the second year as a reference point, there will be the endowment benefit of 1000 paid one year later (end of the third year) and there will be one premium received just at the start of the third year. ${}_2L$ is the present value, value at the end of the second year, of the insurance payment minus the present value of the future premiums. This will be ${}_2L = 1000v - 279.21 = 629.88$.
Answer: B

9. We assume that we are to find premiums based on the equivalence principle.
We will denote each of the two premiums as Q (assume to be paid the start of each half-year during the first year). The APV of the premiums is $Q[1+v^{.5}\cdot {}_{.5}p_{30}]$.
The APV of the benefit is $1000vq_{30}+500v^2\ {}_{1|}q_{30}+250v^3\ {}_{2|}q_{30}$.
From the Illustrative Table, we have $q_{30}=.00153$, $q_{31}=.00161$ and $q_{32}=.00170$.
Using UDD, the APV of premiums is $Q[1+v^{.5}(1-.5(.00153))]=1.961514Q$.
The APV of the benefit is
$1000v(.00153)+500v^2(.99847)(.00161)+250v^3(.99847)(.99839)(.00170)=2.442089$.
Then $Q=\frac{2.442089}{1.961514}=1.245$. Answer: E

10. Let q denote the probability of dying during the second year. Then the number of deaths N in the second year has a binomial distribution based on $m=100$ trials and success (dying) probability q. The variance of the binomial is $Var[N]=mq(1-q)=100q(1-q)$.
q can be formulated as $q=P[\text{survive 1st year and die in 2nd year}]$, which is equal to
$=Q^{(0,0)}\cdot Q^{(0,2)}+Q^{(0,1)}\cdot Q^{(1,2)}=(.7)(.1)+(.2)(.25)=.12$
(this is the combination of staying healthy for the 1st year and dying in the 2nd year, or becoming disabled in the 1st year and dying in the 2nd year).
Therefore $Var[N]=100(.12)(.88)=10.56$. Answer: C

11. The annual effective rate for the 1 year certificate is $(1.01)^4-1=.0406$, for the 3-year certificate it is $(1.0125)^4-1=.0509$, and for the 5-year certificate it is $(1.014125)^4-1=.0577$. In order to withdraw the investment at the end of 6 years, the investor must choose one of the following patterns of investment:
(i) 6 successive one-year certificates , annual effective rate is .0406 .
(ii) a 3 year certificate combined with 3 one-year certificates (in any order), annual effective rate is $[(1.0125)^{12}(1.01)^{12}]^{1/6}-1=.0458$ (we have found the 6-year accumulation and then the equivalent annual effective rate that would compound to the same amount in 6 years).
(iii) Two 3-year certificates, annual effective rate .0509 .
(iv) A one-year certificate and a 5-year certificate (either order), annual effective rate is $[(1.014125)^{20}(1.01)^4]^{1/6}-1=.0548$.
The maximum annual effective return is .0548 and is obtained with a 5-year and a 1-year certificate, in either order. Answer: D

12. With a one-year select period, $\overset{\circ}{e}_{[81]+1} = \overset{\circ}{e}_{82}$, so that
$\overset{\circ}{e}_{[81]} = \overset{\circ}{e}_{[81]:\overline{1}|} + p_{[81]} \cdot \overset{\circ}{e}_{82} = \int_0^1 {}_tp_{[81]}\, dt + p_{[81]} \cdot \overset{\circ}{e}_{82} = \int_0^1 (1 - {}_tq_{[81]})\, dt + p_{[81]} \cdot \overset{\circ}{e}_{82}$
$= \int_0^1 (1 - t \cdot q_{[81]})\, dt + p_{[81]} \cdot \overset{\circ}{e}_{82} = \int_0^1 (1 - \frac{90}{920}t)\, dt + (\frac{830}{920}) \cdot \overset{\circ}{e}_{82} = .9511 + .9022 \cdot \overset{\circ}{e}_{82}$
(using UDD and $q_{[81]} = \frac{90}{920} = .0978$). Therefore $8.2 = .9511 + .9022 \cdot \overset{\circ}{e}_{82}$, so that
$\overset{\circ}{e}_{82} = 8.035$. From the table we have $\ell_{[80]+1} = \ell_{81} = 910$ and $\ell_{[81]+1} = \ell_{82} = 830$, so that
$p_{81} = \frac{830}{910}$ and $q_{81} = \frac{8}{91}$. Then, we use the relationship
$\overset{\circ}{e}_{[80]} = \overset{\circ}{e}_{[80]:\overline{1}|} + p_{[80]} \cdot \overset{\circ}{e}_{81:\overline{1}|} + {}_2p_{[80]} \cdot \overset{\circ}{e}_{82}$ to solve for $\overset{\circ}{e}_{[80]}$.
From UDD we have $\overset{\circ}{e}_{[80]:\overline{1}|} = \int_0^1 (1 - .09t)\, dt = .955$ and $\overset{\circ}{e}_{81:\overline{1}|} = \int_0^1 (1 - \frac{8}{91}t)\, dt = .956$,
$p_{[80]} = .91$, ${}_2p_{[80} = p_{[80]} \cdot p_{81} = (.91)(\frac{83}{91}) = .83$. Then
$\overset{\circ}{e}_{[80]} = .955 + (.91)(.956) + (.83)(8.035)$, so that $\overset{\circ}{e}_{[80]} = 8.494$. Answer: E

13. The loss on one policy is $L = Z - QY = Z - .016Y$,
where Z is the present value random variable for a continuous whole life insurance of 1 and Y is the PVRV of a continuous life annuity of 1 per year.
$E[L] = \overline{A}_x - .016\overline{a}_x = \frac{.015}{.06+.015} - (.016)(\frac{1}{.06+.015}) = -\frac{1}{75}$,
Since $Y = \frac{1-Z}{\delta} = \frac{1-Z}{.06}$, L can be written as $L = (1 + \frac{.016}{.06})Z - \frac{.016}{.06}$ and then
$Var[L] = (1 + \frac{.016}{.06})^2 Var[Z] = (1 + \frac{.016}{.06})^2 ({}^2\overline{A}_x - \overline{A}_x^2)$
$= (1 + \frac{.016}{.06})^2 [\frac{.015}{.12+.015} - (\frac{.015}{.06+.015})^2] = .114094$.

If the total loss is denoted S, then $S = L_1 + L_2 + \cdots + L_n$ (sum of losses on each of the n policies), and $E[S] = nE[L] = -\frac{n}{75}$, and $Var[S] = nVar[L] = .114094n$.

Using the normal approximation, $P(S > 0) = P(\frac{S-E[S]}{\sqrt{Var[S]}} > \frac{-E[S]}{\sqrt{Var[S]}})$.
In order for this probability to be .05, we must have $\frac{-E[S]}{\sqrt{Var[S]}} = 1.645$,
or equivalently, $\frac{-(-\frac{n}{75})}{\sqrt{.114094n}} = 1.645$. Solving for n results in $n = 1737$.
The number of independent policies needed to have a positive loss probability of .025, say m, must satisfy the equation $\frac{-(-\frac{m}{75})}{\sqrt{.114094m}} = 1.96$, and solving for m results in $m = 2466$.
The additional number of policies needed is $2466 - 1737 = 729$. Answer: D

14. The initial net reserve for policy year 10 is ${}_9V + P = 296.1 + P$ (where P is the net premium). $P = bP_x = b(\frac{1}{\ddot{a}_x} - d) = .039088b$.
The net amount at risk for policy year 10 is $b - {}_{10}V = 872$.
Using the net amount at risk form of the recursive relationship for net reserve, for year 10, we have $({}_9V + P)(1+i) - (b - {}_{10}V)q_{x+9} = {}_{10}V$, which becomes
$(296.1 + .039088b)(1.03) - (872)(.02904) = b - 872 \to b = 1200$.
Then, $P = bP_x = 1200(.039088) = 46.91.$
Then the initial net reserve for policy year 10 is ${}_9V + P = 296.1 + 46.91 = 343$.
Answer: C

15. The APV of the death benefit is $10,000[vq_{40}^{(1)} + v^2\,{}_{1|}q_{40}^{(1)} + v^3\,{}_{2|}q_{40}^{(1)}]$.
Since decrement 2 occurs at the end of the year, for each year, $q_x^{(1)} = q_x'^{(1)}$, and
$q_x^{(2)} = (1 - q_x'^{(1)}) \cdot q_x'^{(2)}$.
For decrement 1, we have $q_x^{(1)} = q_x'^{(1)} = .02$ for $x = 40,\ 41,\ 42$, since the force of decrement is constant. Also $p_x^{(\tau)} = p_x'^{(1)} \cdot p_x'^{(2)} = (.98) \cdot (.96) = .9408$ for $x = 40,\ 41.$
Then, ${}_{1|}q_{40}^{(1)} = p_{40}^{(\tau)} \cdot q_{41}^{(1)} = (.9408)(.02) = .018816$ and
${}_{2|}q_{40}^{(1)} = {}_2p_{40}^{(\tau)} \cdot q_{42}^{(1)} = p_{40}^{(\tau)} \cdot p_{41}^{(\tau)} \cdot q_{42}^{(1)} = (.9408)^2(.02) = .01770$.
The APV of the death benefit is
$10,000[v(.02) + v^2(.018816) + v^3(.01770)] = 512$. Answer: C

16. The benefit plus expense reserve at time 4 is
${}_4V_e = 100,000\,A_{54:\overline{1}|} - 1.1G\,\ddot{a}_{54:\overline{1}|} - 50$.
The full preliminary term reserve is ${}_4V^{FPT} = 100,000\,A_{54:\overline{1}|} - \beta^{FPT}$.
Since the experience in the 4th year is the same as the assumptions, the asset value at the end of the 4th year is ${}_4V_e$, so the gain is ${}_4V_e - {}_4V^{FPT} = \beta^{FPT} - 1.1G\,\ddot{a}_{54:\overline{1}|} - 50$.
We find G from $G\,\ddot{a}_{50:\overline{5}|} = 100,000\,A_{50:\overline{5}|} + .3G + .1G\,\ddot{a}_{50:\overline{5}|} + 200 + 50\,\ddot{a}_{50:\overline{5}|}$,
so that $G = \frac{100,000\,A_{50:\overline{5}|} + 200 + 50\,\ddot{a}_{50:\overline{5}|}}{.9\,\ddot{a}_{50:\overline{5}|} - .3}$, where $\ddot{a}_{50:\overline{5}|} = \ddot{a}_{50} - {}_5E_{50} \cdot \ddot{a}_{55} = 4.4114$
from the Illustrative Table. Then, $A_{50:\overline{5}|} = 1 - d \cdot \ddot{a}_{50:\overline{5}|} = .7503$.
Solving for G results in $G = 20,557$. Then,
${}_4V_e = 100,000\,A_{54:\overline{1}|} - 1.1G\,\ddot{a}_{54:\overline{1}|} - 50 = 100,000v - 1.1G - 50 = 71,677$.
The FPT premium β^{FPT} is the same as the level net premium for a similar policy issued at one age higher for one year shorter policy, so that

$\beta^{FPT} = 100,000\,P_{51:\overline{4}|} = \frac{100,000\,A_{51:\overline{4}|}}{\ddot{a}_{51:\overline{4}|}} = 100,000\left[\frac{1}{\ddot{a}_{51:\overline{4}|}} - d\right]$.

From $\ddot{a}_{51:\overline{4}|} = \ddot{a}_{51} - {}_4E_{51}\cdot\ddot{a}_{55} = 3.6376$, and then $\beta^{FPT} = 21,830$.

The gain at time 4 due to the liability basis change is

$\beta^{FPT} - 1.1G\,\ddot{a}_{54:\overline{1}|} - 50 = 21,830 - 1.1\times 20,577 - 50 = -855$. Answer: B

17. The APV of the annuity is

$30,000\ddot{a}_{xy:\overline{3}|} + 20,000(\ddot{a}_{y:\overline{3}|} - \ddot{a}_{xy:\overline{3}|}) + 10,000(\ddot{a}_{x:\overline{3}|} - \ddot{a}_{xy:\overline{3}|})$.

$= 20,000\ddot{a}_{y:\overline{3}|} + 10,000\ddot{a}_{x:\overline{3}|} = 30,000\ddot{a}_{x:\overline{3}|}$.

$\ddot{a}_{x:\overline{3}|} = 1 + vp_x + v^2\,{}_2p_x = 2.61043$

so the APV is 78,312.9 . Answer: A

18. With constant force of mortality $\mu = .03$ and force of interest $\delta = .05$, $\overline{a}_x = \frac{1}{\delta+\mu} = 12.5$.

The variance of $\overline{a}_{\overline{T(x)}|}$ is

$\frac{1}{\delta^2}[{}^2\overline{A}_x - (\overline{A}_x)^2] = \frac{1}{\delta^2}[\frac{\mu}{2\delta+\mu} - (\frac{\mu}{\delta+\mu})^2] = \frac{1}{(.05)^2}[\frac{.03}{.1+.03} - (\frac{.03}{.05+.03})^2] = 36.06$.

The standard deviation is $\sqrt{36.06} = 6.00$.

We wish to find $P[12.5 - 6.00 < \overline{a}_{\overline{T_x}|} < 12.5 + 6.00] = P[\,6.5 < \overline{a}_{\overline{T_x}|} < 18.5]$.

We solve for n, from the equation $\overline{a}_{\overline{n}|} = 6.5$, so that $\frac{1-e^{-.05n}}{.05} = 6.5$, so that $e^{-.05n} = .675$

(which is equivalent to $n = -\frac{ln(.675)}{.05} = 7.86$ years), and we solve for m from the equation

$\overline{a}_{\overline{m}|} = 18.5$, so that $\frac{1-e^{-.05m}}{.05} = 18.5$, so that $e^{-.05m} = .075$

(which is equivalent to $m = -\frac{ln(.075)}{.05} = 51.81$ years)

$P[\,6.5 < \overline{a}_{\overline{T(x)}|} < 18.5] = P[7.86 < T(x) < 51.81] = {}_{7.86}p_x - {}_{51.81}p_x$

$= e^{-.03(7.86)} - e^{-.03(51.81)} = .790 - .211 = .579$. Answer: C

19. We use the equivalence principle relationship

APV expense-loaded premium $=$ APV benefit plus expenses.

$G\ddot{a}_{35} = 100,000A_{35} + .1G\ddot{a}_{35} + 25\ddot{a}_{35} + 250\ddot{a}_{35}$.

Then $1234 = 100,000P_{35} + .1(1234) + 25 + 250$,

so that $100,000P_{35} = 835.60$. Answer: D

20. The 2nd year terminal reserve for a 3-year endowment insurance can be formulated as

$${}_2V_{x:\overline{3}|} = 1 - \frac{\ddot{a}_{x+2:\overline{1}|}}{\ddot{a}_{x:\overline{3}|}} \ ,$$

where $\ddot{a}_{x+2:\overline{1}|} = 1$ and $\ddot{a}_{x:\overline{3}|} = 1 + vp_x + v^2\,{}_2p_x = 1 + \frac{p}{1.05} + \frac{p^2}{(1.05)^2}$.

Then ${}_2V_{x:\overline{3}|} = 1 - \frac{1}{1+\frac{p}{1.05}+\frac{p^2}{(1.05)^2}} = .526$, so that $\frac{p^2}{(1.05)^2} + \frac{p}{1.05} - 1.1097 = 0$.

Solving the quadratic equation, or substituting the possible answers results in $p = .7$.

Answer: E

SECTION B SOLUTIONS - WRITTEN ANSWER

1.(a) $Z_1 = \begin{cases} 0 \text{ if } T<20 \\ v^{20} \text{ if } T\geq 20 \end{cases}$, $Z_2 = \begin{cases} 0 \text{ if } T<30 \\ v^{30} \text{ if } T\geq 30 \end{cases}$

(b) $Var[Z_1] = E[Z_1^2] - (E[Z_1])^2 = v^{40}\,{}_{20}p_x - (v^{20}\,{}_{20}p_x)^2 = e^{-40\delta}\,{}_{20}p_x \times (1 - {}_{20}p_x)$
$= e^{-1.6} \times e^{-.2} \times (1 - e^{-.2}) = .02996$

$Var[Z_2] = E[Z_2^2] - (E[Z_2])^2 = v^{60}\,{}_{30}p_x - (v^{30}\,{}_{30}p_x)^2 = e^{-60\delta}\,{}_{30}p_x \times (1 - {}_{30}p_x)$
$= e^{-2.4} \times e^{-.3} \times (1 - e^{-.3}) = .01742$

$Cov[Z_1, Z_2] = E[Z_1 \times Z_2] - E[Z_1] \times E[Z_2]$.
$E[Z_1 \times Z_2] = v^{50}\,{}_{30}p_x = e^{-2} \times e^{-.3} = .10026$, $E[Z_1] = v^{20}\,{}_{20}p_x = .36788$
$E[Z_2] = v^{30}\,{}_{30}p_x = .22313 \rightarrow Cov[Z_1, Z_2] = .10026 - .36788 \times .22313 = .01817$.

(c) $Z_3 = \begin{cases} v^{K+1} \text{ if } K<20 \\ v^{20} \text{ if } K\geq 20 \end{cases}$. $E[Z_3] = A_{x:\overline{20}|}$.

$Z_1 \times Z_3 = \begin{cases} 0 \text{ if } K<20 \\ v^{40} \text{ if } K\geq 20 \end{cases}$. $E[Z_1 \times Z_3] = v^{40}\,{}_{20}p_x = v^{20} \times A_{x:\overset{1}{\overline{20}|}}$.

$Cov[Z_1, Z_3] = E[Z_1 \times Z_3] - E[Z_1] \times E[Z_3] = v^{20} \times A_{x:\overset{1}{\overline{20}|}} - A_{x:\overset{1}{\overline{20}|}} \times A_{x:\overline{20}|}$
$= A_{x:\overset{1}{\overline{20}|}} \times (v^{20} - A_{x:\overline{20}|})$. Since $v^{20} < A_{x:\overline{20}|}$, this covariance is negative.

The later death occurs, the smaller is Z_3 and eventually Z_1 goes from 0 to positive as time of death increases. Z_1 and Z_3 move in opposite directions as time of death increases. This is characteristic of negative covariance.

2.(a)(i) Single premium $=$ APV of benefit $= {}_{5|}\overline{a}_x = e^{-5\delta} \times {}_5p_x \times \overline{a}_{x+5}$
$= e^{-.15} \times e^{-.1} \times \overline{a}_{x+5}$;
$\overline{a}_{x+5} = \overline{a}_{x+5:\overline{5}|} + e^{-5\delta} \times {}_5p_{x+5} \times \overline{a}_{x+10}$
$= \int_0^5 e^{-.03t}e^{-.02t}\,dt + e^{-.25} \times \int_0^\infty e^{-.03t}e^{-.01t}\,dt = \frac{1-e^{-.05(5)}}{.03+.02} + e^{-.25} \times \frac{1}{.03+.01}$
$= 23.8940$. Then, the single premium is $e^{-.25} \times 23.8940 = 18.6087$

(ii) Annual premium $= \frac{\text{APV of benefit}}{\overline{a}_{x:\overline{5}|}}$. Because the force of interest is constant for the first 10 years, we have $\overline{a}_{x:\overline{5}|} = \overline{a}_{x+5:\overline{5}|} = 4.4240$, so that the annual premium is $\frac{18.6087}{4.4240} = 4.206$.

(b)(i) The loss at issue is $L = Y - 18.6087$, where Y is the PVRV for the annuity. The expected loss at issue is $E[Y] - 18.6087$, where $E[Y]$ is based on the revised mortality assumption. $E[Y] = {}_{5|}\overline{a}_x^{rev} = e^{-5\delta} \times {}_5p_x \times \overline{a}^r_{x+5} = e^{-.25} \times \frac{1}{.03+.02} = 15.5760$.
The expected loss at issue will be $15.5760 - 18.6087 = -3.03$, or an expected gain of 3.03.

(ii) The loss at issue is $L = Y_1 - Y_2$, where Y_1 is the PVRV for the deferred annuity and Y_2 is the PVRV for the premium annuity. Under the revised mortality assumption, $E[Y_1]$ is the same as $E[Y]$ in (i), which is 15.5760. Under the revised mortality assumption,
$E[Y_2] = 4.206 \times \overline{a}^r_{x:\overline{5}|} = 4.206 \times \int_0^5 e^{-.03t}e^{-.02t}\,dt = 18.6087$, so that
$E[L] = 15.5760 - 18.6087 = -3.03$, or an expected gain of 3.03, as in part (i)

(iii) The mortality change occurs after time 10, so the APV of the premium annuity for the first five years is unaffected by that change and so has the same APV as the single premium in (i).

3.(a) Net Premium: $P\ddot{a}_{50} = 1,000,000A_{50} \to P = 18,772$.
$G\ddot{a}_{50} = 1,000,000A_{50} + .2G + .05Ga_{50} + 5,000 + 1000 + (1000 + 200)a_{50} + 1000A_{50}$
$G = \frac{1,001,000A_{50}+6,000+1200a_{50}}{.95\ddot{a}_{50}-.15} = 21,682.$

(b) The 9th year terminal expense augmented reserve is
${}_9V^g =$ APV$_9$ benefit + expenses $-$ APV$_9$ gross premium
$= 1,000,000A_{59} + (.05 \times 21,682 + 1200) \times \ddot{a}_{59} + 1000A_{59} - 21,682\,\ddot{a}_{59}$
$= 135,323.$

The accumulation relationship from time 9 to 10 is
$({}_9V^g + G - E)(1+i) - b \times q_{59} - SE \times q_{59} = p_{59} \times {}_{10}V^g$,
where SE is settlement expense.

This becomes
$(135,323 + 21,682 - .05 \times 21,682 - 1200) \times 1.06 - 1,001,000 \times .01262$
$= .98738 \times {}_{10}V^q \rightarrow {}_{10}V^q = 153,306.$

(The 10th year terminal expense augmented reserve formulated prospectively is
${}_{10}V^g = \text{APV}_{10}$ benefit + expenses $-$ APV_{10} gross premium
$= 1,000,000A_{60} + (.05 \times 21,682 + 1200) \times \ddot{a}_{60} + 1000A_{60} - 21,682\,\ddot{a}_{60}$
$= 153,302$ - difference is do to round off error.)

(c) The 10th year terminal expense reserve is equal to
APV_{10} future expenses $-$ APV_{10} future expense loading.
The expense loading is gross premium $-$ net premium $= 21,682 - 18,772 = 2910$.
Then 1
0th year terminal expense reserve
$= (.05 \times 21,682 + 1200) \times \ddot{a}_{60} + 1000A_{60} - 2910 \times \ddot{a}_{60} = -6607.$

The expense loading is a lifetime "average" of all expenses and so is less than the large first year expense and larger than the renewal expenses. When the t-th year terminal expense reserve is formulated as APV future expenses as of time $t -$ APV future expense loading as of time t, this will be negative for $t > 0$ since expense loading is larger then annual expenses after time 0.

(Note also that 10th year terminal expense reserve is equal to
10th year terminal expense augmented reserve $-$ 10th year terminal net reserve
$= {}_{10}V^g - {}_{10}V$.
${}_{10}V = \text{APV}_{10}$ benefit $-$ APV_{10} net premium $= 1,000,000A_{60} - 18,772\ddot{a}_{60} = 159,909$
$\rightarrow$ 10th year terminal expense reserve $= 153,302 - 159,909 = -6607.$

(d) The asset accumulation relationship for the 10th year is
$({}_9V^g + G - E)(1+i) - (b + SE) \times q_{59}^{(d)} - {}_{10}CV \times q_{59}^{(w)} = p_{59}^{(\tau)} \times {}_{10}AS$
The expected asset share at the end of the 10th year is
${}_{10}AS = \frac{(135{,}323 + .95 \times 21{,}682 - 1200)(1.06) - 1{,}001{,}000 \times .01262 - .2 \times {}_{10}CV}{.78738} = 192,247 - .254 \times {}_{10}CV$
The liability at the end of the 10th year is ${}_{10}V^g = 153,302$, so the expected profit for the year is
${}_{10}AS - 153,302 = 192,247 - .254 \times {}_{10}CV - 153,202 = 39,045 - .254 \times {}_{10}CV.$
In order for this to be 5000, we must have ${}_{10}CV =$ 134,035.

4.(a) In order for the individual to die within the three year term, one of the following state paths must be taken: 1-4 (death in first year) , 1-1-4 or 1-2-4 (death in second year) ,
1-1-1-4 or 1-2-1-4 or 1-1-2-4 or 1-2-2-4 . The probabilities of these paths are:
1-4: $.1$, 1-1-4: $.6 \times .1 = .06$, 1-2-4: $.2 \times .2 = .04$, 1-1-1-4: $.6 \times .6 \times .1 = .036$,
1-2-1-4: $.2 \times .2 \times .1 = .004$, 1-1-2-4: $.6 \times .2 \times .2 = .024$, 1-2-2-4: $.2 \times .6 \times .2 = .024$.
Total probability is $.1 + .06 + .04 + .036 + .004 + .024 + .024 = .288.$
Alternatively, the three step transition probability matrix is

$$\boldsymbol{P^3} = \begin{bmatrix} .6 & .2 & .1 & .1 \\ .2 & .6 & 0 & .2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^3 = \begin{bmatrix} .288 & .224 & .2 & .288 \\ .224 & .288 & .044 & .444 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

so the probability that someone in state 1 (active) at time 0 is in state 4 (dead) at time 3 is
${}_3p^{14} = .288.$

(b) The probability that death occurs in the first year is .1, the probability that death occurs in the second year is .1 (paths 1-1-4 or 1-2-4) and the probability that death occurs in the third year is .088 (paths 1-1-1-4 , 1-2-1-4 , 1-1-2-4 or 1-2-2-4). The actuarial present value of the death benefit at the time the policy is issued is $100,000 \times \left(\frac{.1}{1.05} + \frac{.1}{(1.05)^2} + \frac{.088}{(1.05)^3}\right) = 26,196.$

(c) The individual is disabled at the start of the second year with probability .2 (path 1-2), and at the start of the third year with probability .24 (paths 1-1-2 or 1-2-2). The actuarial present value of the disability benefit is $25,000 \times \left(\frac{.2}{1.05} + \frac{.24}{(1.05)^2}\right) = 10,204$.

(d) With net annual premium P, the APV of premium is
$P \times \left(1 + \frac{p^{11}}{1.05} + \frac{{}_2p^{11}}{(1.05)^2}\right) = P \times \left(1 + \frac{.6}{1.05} + \frac{.4}{(1.05)^2}\right) = 1.93424P$.
We set this equal to the total APV of benefit and solve for P:
$1.93424P = 26,196 + 10,204 = 36,400 \rightarrow P = 18,819$.

5.(a) From independence, we get ${}_tp_{xy} = {}_tp_x \times {}_tp_y = e^{-t\mu_x} \times e^{-t\mu_y} = e^{-.4t}$.
$\overset{\circ}{e}_{xy} = \int_0^\infty {}_tp_{xy}\, dt = \int_0^\infty e^{-.4t}\, dt = \frac{1}{.4} = 2.5$.
$P[T_x < T_y] = {}_\infty q_{\overset{1}{x}y} = \int_0^\infty {}_tp_{xy} \times \mu_{x+t}\, dt = \int_0^\infty e^{-.4t} \times .25 = \frac{.25}{.4} = .625.$
$A_{\overset{1}{x}y} = \int_0^\infty e^{-\delta t}\, {}_tp_{xy} \times \mu_{x+t}\, dt = \int_0^\infty e^{-.1t} e^{-.4t} \times .25\, dt = \frac{.25}{.5} = .5.$

(b) The force of mortality for (x) at age $x+t$ is the sum of μ^*_{x+t} and λ, where μ^*_{x+t} is the force of mortality in the absence of a common shock, and similarly for (y). Since the forces are constant and we are given $\lambda = .05$, it follows that $\mu^*_{x+t} = .20$ and $\mu^*_{y+t} = .10$.

$\mathring{e}_{xy} = \int_0^\infty {}_tp_{xy}\,dt = \int_0^\infty {}_tp^*_x \times {}_tp^*_y \times e^{-\lambda t}\,dt = \int_0^\infty e^{-.2t} \times\ e^{-.1t} \times e^{-.05t}\,dt = \frac{1}{.35} = 2.857.$

$P[T_x < T_y] = {}_\infty q^{\,1}_{xy} = \int_0^\infty {}_tp_{xy} \times \mu^*_{x+t}\,dt = \int_0^\infty e^{-.35t} \times .2 = \frac{.2}{.35} = .571.$

$A^{\,1}_{xy} = \int_0^\infty e^{-\delta t}\,{}_tp_{xy} \times \mu^*_{x+t}\,dt = \int_0^\infty e^{-.1t}e^{-.35t} \times .2\,dt = \frac{.2}{.45} = .444.$

(c) The combined force of failure in (b) is smaller than that in (a) because the common shock is a component of both individuals' mortality in (b), but that numerical factor is in both forces of mortality in (a) and so appears twice as large in the joint force of failure.

6.(a) Max's salary in 2012 and 2013 will be $75,000(1.03)^2 = 79,578$ and $75,000(1.03)^{23} = 81,954$, respectively, so for retirement at age 64, the two-year final average salary will be 80,766. Max will have had 34 years of service as of that retirement date, so Max's benefit, before reduction due to early retirement, will be $34 \times .02 \times 80,766 = 54,921.$
There will be a 4% reduction in benefit as a result of retirement occurring one year before normal retirement date, so Max's benefit will be $54,921 \times .96 = 52,724.$ Max's salary in the year of retirement is 81,954, so the replacement ratio is $\frac{52,724}{81,954} = .643$, or 64.3%.

(b) For retirement at age 63, the retirement benefit is $33 \times .02 \times 78,414 \times .92 = 47,613$.
For retirement at age 64, the retirement benefit is $52,724$ (from (a) .
For retirement at age 65, the retirement benefit is $35 \times .02 \times 83{,}189 = 58,232$.
The APV of the retirement benefit as of January 1, 2010 is
$v^3\,{}_3p^{(\tau)}_{60} \times q^{(r)}_{63} \times 47,613 \times \ddot{a}_{63} + v^4\,{}_4p^{(\tau)}_{60} \times q^{(r)}_{64} \times 52,724 \times \ddot{a}_{64}$
$+\,v^5\,{}_5p^{(\tau)}_{60} \times q^{(r)}_{65} \times 58,232 \times \ddot{a}_{65}$, where
${}_3p^{(\tau)}_{60} = {}_3p_{60} = .955521$ from the Illustrative Table,
${}_4p^{(\tau)}_{60} = {}_3p^{(\tau)}_{60} \times (1 - q^{(d)}_{63} - q^{(r)}_{63}) = .842884$, and
${}_5p^{(\tau)}_{60} = {}_4p^{(\tau)}_{60} \times (1 - q^{(d)}_{64} - q^{(r)}_{64}) = .615710$.
The APV is
$\frac{1}{(1.06)^3} \times .955521 \times .1 \times 47,613 \times 10.4084 + \frac{1}{(1.06)^4} \times .842884 \times .25 \times 52,724 \times 10.1544$
$+\ \frac{1}{(1.06)^5} \times .615710 \times 1 \times 58,232 \times 9.8969 = 394,279$.

(c) The APV of Max's original benefit as of age 65 is $58,232 \times \ddot{a}_{65} = 576,316.$

The APV of the optional benefit is $K \times (\ddot{a}_{\overline{10}|} + {}_{10|}\ddot{a}_{\overline{65:65}})$.

With $i = .06$, $\ddot{a}_{\overline{10}|} = 7.80169$. From the Illustrative Table, we have

${}_{10|}\ddot{a}_{\overline{65:65}} = {}_{10|}\ddot{a}_{65} + {}_{10|}\ddot{a}_{65} - {}_{10|}\ddot{a}_{65:65} = {}_{10}E_{65} \times \ddot{a}_{75} + {}_{10}E_{65} \times \ddot{a}_{75} - {}_{10}E_{65:65} \times \ddot{a}_{75:75}$

$= 2 \times .39994 \times 7.217 - {}_{10}p_{65} \times {}_{10}E_{65} \times \ddot{a}_{75:75} = 4.2644$.

Then $4.2644K = 576,316 \rightarrow K = 135,145.$

7.(a) The account value at the end of year 1 is

$AV_1 = [6000(1-.05) - 100 - \frac{1.2q_{50}}{1.04}(25,000)](1.05) = 5701$.

The account value at the end of the second year is

$AV_2 = [5700.69 + 6000(1-.05) - 100 - \frac{1.2q_{51}}{1.04}(25,000)](1.05) = 11,671$.

(b) For A Type A policy, we use the accumulation relationship

$[AV_k + G_k - E_k - (B - AV_{k+1})v^q_{k+1}\, q^*_{x+k}](1+i^c) = AV_{k+1}$, which becomes

$AV_{k+1} = \frac{[AV_k + G_k - E_k - B\cdot v^q \cdot q^*_{x+k}](1+i^c)}{(1 - v^q_{k+1}\, q^*_{x+k})(1+i^c)}$.

In this expression, $B = 25,000$ so that $AV_1 = \frac{\left(6000(1-.05) - 100 - \frac{25,000 \times 1.2q_{50}}{1.04}\right)(1.05)}{1 - \frac{1.2q_{50}}{1.04} \times 1.05} = 5,921$.

For the second year, we have $AV_1 = \frac{\left(5921 + 6000(1-.05) - 100 - \frac{25,000 \times 1.2q_{51}}{1.04}\right)(1.05)}{1 - \frac{1.2q_{51}}{1.04} \times 1.05} = 11,996$.