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NOTES

Questions and parts of some solutions have been taken from material copyrighted by the Casualty Actuarial Society and the Society of Actuaries. They are reproduced in this study manual with the permission of the CAS and SoA solely to aid students studying for the actuarial exams. Some editing of questions has been done. Students may also request past exams directly from both societies. I am very grateful to these organizations for their cooperation and permission to use this material. They are, of course, in no way responsible for the structure or accuracy of the manual.

Exam questions are identified by numbers in parentheses at the end of each question. CAS questions have four numbers separated by hyphens: the year of the exam, the number of the exam, the number of the question, and the points assigned. SoA or joint exam questions usually lack the number for points assigned. W indicates a written answer question; for questions of this type, the number of points assigned is also given. A indicates a question from the afternoon part of an exam. MC indicates that a multiple choice question has been converted into a true/false question.

Page numbers (p.) with solutions refer to the reading to which the question has been assigned unless otherwise noted.

Although I have made a conscientious effort to eliminate mistakes and incorrect answers, I am certain some remain. I am very grateful to students who discovered errors in the past and encourage those of you who find others to bring them to my attention. Please check our web site for corrections subsequent to publication and possibly for additional problems.

Hanover, NH 8/2/11
PJM

KPW 9: PAST CAS AND SoA EXAMINATION QUESTIONS

A. The Collective and Individual Models

A1. The variance of the compound process is given by ${}_x\sigma^2 = {}_{x_1}E\ {}_k\sigma^2 + {}_{x_1}\sigma^2\ {}_kE^2$, where

x - compound process (or distribution of total losses)
k - discrete process of determining the number of occurrences
x_1 - distribution of the values of a single claim. (81–9–1l–1/2)

A2. Klugman et al. describe a formula for the process variance in terms of the means and variances of the frequency and severity. Which of the following are circumstances under which the use of this formula would not be valid?

1. The third moment of the severity distribution is nonzero.
2. The frequency and severity are correlated.
3. The size of a second claim for a risk depends on the size of the first claim for that risk.

A. 1 B. 1,2 C. 1,3 D. 2,3 E. 1,2,3 (84–4–38–1)

A3. If S is the random variable representing the aggregate loss amount; N, the random variable representing number of claims; and X_i, the random variable representing the amount of the ith claim, then:

$$\text{Var}(S) = E[X_i]\ \text{Var}(N) + \text{Var}(X_i)\ E[N]^2. \qquad (86\text{–}4\text{–}40\text{–MC})$$

A4. Suppose an insurance portfolio has a frequency distribution with mean n and variance w; and a severity distribution with mean m and variance v. What is the variance of the aggregate claim distribution?

A. w + v B. wv C. nv D. nv + mw E. $nv + m^2w$ (88–5–43–1)

A5. According to Klugman et al. , which of the following assumptions are made in the collective risk model?

1. The individual claim amounts are identically distributed random variables.
2. The distribution of the aggregate losses generated by the portfolio is continuous.
3. The number of claims and the individual claim amounts are mutually independent.

A. 1 B. 2 C. 1,3 D. 2,3 E. 1,2,3 (95F–5A–21–1)

A6. According to Klugman et al., Var(S) = E[N] Var(X) + Var(N) E[X] for the collective risk model. (96S–5A–21–MC)

A7. Which of the following are assumptions of the collective risk model as described in Klugman et al.?

1. $X_1, X_2, \dots, X_n$ are identically distributed random variables.
2. X_i represents the loss on insured unit i, and N is the number of risks insured.
3. The random variables $N, X_1, X_2, \dots, X_n$ are mutually independent.

A. 1,2 B. 1,3 C. 2,3 D. 1,2,3 E. None of these answers are correct. (96F–5A–17–1)

Solutions are based on pp. 199–202, 205, 253–56.

A1. F – Substitute "${}_{x1}E^2$" for "${}_{x1}E$" and "${}_kE$" for "${}_kE^2$."

A2. 1. F – This is not mentioned.

2. T

3. T

Answer: D

A3. Substitute "$(E[X_i])^2$" for "$E[X_i]$" and "$E[N]$" for "$(E[N])^2$."

A4. E.

A5. 1. T

2. F – This is not mentioned.

3. T

Answer: C

A6. F – Substitute "$(E[X])^2$" for "$E[X]$."

A7. 1. T

2. F – X_i represents the amount of the ith claim and N is the number of claims.

3. T

Answer: B

A8. Consider the following aggregate claim distribution developed by Klugman et al: $S = X_1 + X_2 + X_3 + \cdots + X_N$, where the X_i's are the random severities and N is the random frequency. Which of the following are true?

1 . The frequency of claims and the severity of claims are always assumed to be mutually independent.
2. The X_i's are assumed to be identically distributed.
3. $Var(S) = E[N]\ Var(X) + (E[X])^2\ Var(N)$

A. 2 B. 1,2 C. 1,3 D. 2,3 E. 1,2,3 (97S–5A–22–1)

A9. For an individual risk model for the insured claims for a particular line of business of an insuring organization, usually the claims for individual insureds are assumed to be independent. (97F–5A–23–MC)

A10. According to Klugman et al., under the collective risk model, it is assumed that individual claim amounts $X_1, X_2 \ldots$ are identically distributed random variables. (97F–5A–24–MC)

A8. E.

A9. T.

A10. T.

B. Probabilities of Aggregate Claims: the General Case

B1. Based on the individual risk model with independent losses, the distribution of aggregate claims for a portfolio of life insurance policies is:

x	0	100	200	300	400	500	600	700	800
F(x)	.648	.720	.720	.792	.962	.980	.980	.998	1.000

One policy with face amount 100 and probability of claim .1 is deleted from the portfolio. What is the probability that the aggregate claims for the remaining policies do not exceed 500?

A. .980 B. .984 C. .988 D. .990 E. .995 (84F–5–4)

B2. X_1 and X_2 are independent random variables whose possible values are nonnegative integers. Given the values below, define $f_2(1)$.

x	$F_1(x)$	$F^{(2)}x$
0	.50	.20
1	.80	.42
2	.90	.78
3	1.00	.92

A. .17 B. .20 C. .22 D. .30 E. .40 (86S–5–6)

B3. Based on the individual risk model with independent claims, the cumulative distribution function of aggregate claims for a portfolio of life insurance policies is as follows:

x	0	100	200	300	400	500	600	700
F(x)	.40	.58	.64	.69	.70	.78	.96	.100

One policy with face amount 100 and probability of claim .20 is increased in face amount to 200. Determine the probability that aggregate claims for the revised portfolio will not exceed 500.

A. .72 B. .74 C. .76 D. .78 E. .80 (87S–151–5)

B4. X_1, X_2, and X_3 are mutually independent random variables with probability functions as follows:

x	$f_1(x)$	$f_2(x)$	$f_3(x)$
0	.90	.50	.25
1	.10	.30	.25
2	.00	.20	.25
3	.00	.00	.25

If $S = X_1 + X_2 + X_3$, determine $F_S(5)$.

A. .975 B. .980 C. .985 D. .990 E. .995 (88S–151–5)

Solutions are based on pp. 203–8.

B1. Let x be the policy in question. Use the formula F(x) = F'(x – 100) P(Claim by x) + F'(x) P(No Claim by x) to derive F'(x) the cumulative distribution minus the particular policy.

$F(0) = .648 = F'(0)\,(.90)$	$F'(0) = .720$
$F(x) = F'(x-100)(.10) + F'(x)\,(.90)$	
$F(100) = .720 = (.720)(.10) + F'(100)\,(.90)$	$F'(100) = .720$
$F(200) = .720 = (.720)(.10) + F'(200)\,(.90)$	$F'(200) = .720$
$F(300) = .792 = (.720)(.10) + F'(300)\,(.90)$	$F'(300) = .800$
$F(400) = .962 = (.800)(.10) + F'(400)\,(.90)$	$F'(400) = .980$
$F(500) = .980 = (.980)(.10) + F'(500)\,(.90)$	$F'(500) = .980$

Answer: A

B2. $f_1(0) = F_1(0) = .50 \qquad f_1(1) = F_1(1) - F_1(0) = .80 - .50 = .30$

$f^{*(2)}(0) = F^{*(2)}(0) = .20 \qquad f_2(0) = f^{*(2)}(0)/f_1(0) = .20/.50 = .40$

$f^{*(2)}(1) = F^{*(2)}(1) - F^{*(2)}(0) = .42 - .20 = .22$

$$f_2(1) = \frac{f^{*(2)}(1) - f_1(1)\,f_2(0)}{f_1(0)} = \frac{.22 - (.30)(.40)}{.50} = .20$$

Answer: B

B3. Let x be the policy in question. Use the formula F(x) = F*(x – 100) P(Claim by x) + F*(x) P(No Claim by x) to derive F*(x) the cumulative distribution minus the particular policy.

$F(0) = .40 = F^*(0)\,(.80)$	$F'(0) = .50$
$F(x) = F'(x-100)\,(.10) + F'(x)\,(.90)$	
$F(100) = .58 = (.50)(.20) + F'(100)\,(.80)$	$F'(100) = .60$
$F(200) = .64 = (.60)(.20) + F'(200)\,(.80)$	$F'(200) = .65$
$F(300) = .69 = (.65)(.20) + F'(300)\,(.80)$	$F'(300) = .70$
$F(400) = .70 = (.70)(.20) + F'(400)\,(.80)$	$F'(400) = .70$
$F(500) = .78 = (.70)(.20) + F'(500)\,(.80)$	$F'(500) = .80$

Since f'(400) = 0, the addition of a policy having a claim of 200 instead of 100 will have no effect on F(500). If other claims total 300 or less, total claims will not exceed 500 in both cases. If other claims total 500 or more, total claims will exceed 500 in both cases. Thus F(500) remains the same.

Answer: D

B4. $F_S(5) = 1 - f(6) = 1 - f_1(1)\,f_2(2)\,f_3(3) = 1 - (.10)(.20)(.25) = .995$

Answer: E

B5. X_1, X_2, and X_3 are mutually independent random variables with probability functions as follows:

x	$f_1(x)$	$f_2(x)$	$f_3(x)$
0	p	.6	.25
1	1 – p	.2	.25
2	0	.1	.25
3	0	.1	.25

If $S = X_1 + X_2 + X_3$ and $f_S(5) = .06$, determine p.

A. .1 B. .2 C. .5 D. .8 E. .9 (89F–151–6)

B6. An insurance portfolio produces N claims, where: P(N = 0) = .5, P(N = 1) = .4, and P(N = 3) = .1. Individual claim amounts have the following distribution: p(1) = .9 and p(10) = .1. Individual claim amounts and N are mutually independent. Calculate the probability that the ratio of aggregate claims to expected claims will exceed 3.

A. .05 B. .07 C. .09 D. .11 E. .13 (89F–151–7)

B7. The random variables X_1, X_2 and X_3 are independent with probability functions $f_1(x)$, $f_2(x)$ and $f_3(x)$, respectively. You are given:

x	$f_1(x)$	$f_2(x)$	$f_3(x)$
0	.4	.5	.1
1	.6	.3	.3
2		.2	.6

Determine $P(X_1 + X_2 + X_3 = 2)$.

A. .12 B. .17 C. .22 D. .27 E. .32 (90F–151–4)

B8. Let $S = X_1 + X_2$, where X_1 and X_2 are independent with distribution functions defined by the table below.

x	$F_1(x)$	$F_2(x)$
0	.4	.5
1	.7	.8
2	.9	1.0
3	1.0	

What is $P(S \leq 2)$?

A. < .6 B. ≥ .6 but < .7 C. ≥ .7 but < .8 D. ≥ .8 but < .9 E. ≥ .9 (91–5–21–1)

B9. Let $S = X_1 + X_2$, where X_1 and X_2 are independent with the following probability functions defined. What is $P(S \leq 2)$?

x	$f_1(x)$	$f_2(x)$
0	.5	.6
1	.2	.2
2	.2	.1
3	.1	.1

A. < .60 B. ≥ .60 but < .65 C. ≥ .65 but < .70 D. ≥ .70 but < .75 E. ≥ .75 (92–5–22–1)

B5. $f_S(5) = .06 = P(X_1 = 0, X_2 = 2, \text{ and } X_3 = 3) + P(X_1 = 0, X_2 = 3, \text{ and } X_3 = 2) +$
$P(X_1 = 1, X_2 = 1, \text{ and } X_3 = 3) + P(X_1 = 1, X_2 = 3, \text{ and } X_3 = 1) +$
$P(X_1 = 1, X_2 = 2, \text{ and } X_3 = 2)$

$.06 = p(.1)(.25) + p(.1)(.25) + (1-p)(.2)(.25) + (1-p)(.1)(.25) + (1-p)(.1)(.25)$

$.06 = [.25][.2p + (1-p)(.4)] \qquad p = .8$

Answer: D

B6. $q = (0)(.5) + (1)(.4) + (3)(.1) = .7 \qquad \mu = (1)(.9) + (10)(.1) = 1.9$

$E[S] = \mu q = (1.9)(.7) = 1.33$

$f(0) = .5 \qquad f(1) = P(N = 1)\, p(1) = (.4)(.9) = .36 \qquad f(2) = 0$

$f(3) = [P(N = 3)][p(1)]^3 = (.1)(.9)^3 = .0729$

$P(S > 3) = 1 - f(0) - f(1) - f(2) - f(3) = 1 - .5 - .36 - .0729 = .07$

Answer: B

B7. $f_S(2) = P(X_1 = 0, X_2 = 1, \text{ and } X_3 = 1) + P(X_1 = 0, X_2 = 2, \text{ and } X_3 = 0) +$
$P(X_1 = 0, X_2 = 0, \text{ and } X_3 = 2) + P(X_1 = 1, X_2 = 1, \text{ and } X_3 = 0) +$
$P(X_1 = 1, X_2 = 0, \text{ and } X_3 = 1)$

$f_S(2) = (.4)(.3)(.3) + (.4)(.2)(.1) + (.4)(.5)(.6) + (.6)(.3)(.1) + (.6)(.5)(.3) = .27$

Answer: D

B8. $f_1(0) = F_1(0) = .4 \qquad f_2(0) = F_2(0) = .5 \qquad f_2(1) = F_2(1) - F_2(0) = .8 - .5 = .3$

$f_2(2) = F_2(2) - F_2(1) = 1.0 - .8 = .2$

$P(S \le 2) = F_1(2)\, f_2(0) + F_1(1)\, f_2(1) + f_1(0)\, f_2(2) = (.9)(.5) + (.7)(.3) + (.4)(.2) = .74$

Answer: C

B9. $F_1(1) = f_1(0) + f_1(1) = .5 + .2 = .7 \qquad F_1(2) = F_1(1) + f_1(2) = .7 + .2 = .9$

$P(S \le 2) = F_1(2)\, f_2(0) + F_1(1)\, f_2(1) + f_1(0)\, f_2(2) = (.9)(.6) + (.7)(.2) + (.5)(.1) = .73$

Answer: D

B10. For three mutually independent integer-valued random variables X_1, X_2, and X_3, you are given:

x	$f_1(x)$	$f_2(x)$	$f_3(x)$	$f_1(x)f_2(x)$	$f_1(x)f_2(x)f_3(x)$
0	.4		y		
1	.6	.3	.3	.42	
2	0		.7 – y	.26	.272
3	0	0	0		

Determine y.

A. .07 B. .08 C. .09 D. .10 E. .11 (93S–151–7–2)

B11. Let $S = X_1 + X_2$, where X_1 and X_2 are independent random variables with probability density functions defined below:

x	$f_1(x)$	$f_2(x)$
0	.2	.8
1	.3	.2
2	.4	
3	.1	

Calculate $P(S \le 2)$.

A. < .6 B. ≥ .6 but < .7 C. ≥ .7 but < .8 D. ≥ .8 but < .9 E. ≥ .9 (93–5A–22–2)

B12. For three mutually independent random variables X_1, X_2, and X_3, you are given:

x	$f_1(x)$	$f_2(x)$	$f_3(x)$
0	.40	.25	p
1	.40	.25	0
2	.10	.25	1 – p
3	.10	.25	0

You are also given that $f_1 * f_2 * f_3(6) = .10$. Determine p.

A. .32 B. .34 C. .36 D. .38 E. .40 (93F–151–7–2)

B13. Let $S = X_1 + X_2$, where X_1 and X_2 are independent random variables with distribution functions defined below:

X	0	1	2	3	4
$F_1(X)$	.3	.4	.6	.7	1.0
$F_2(X)$	.6	.8	1.0		

Calculate $P(S \le 2)$.

A. < .25 B. ≥ .25 but < .35 C. ≥ .35 but < .45 D. ≥ .45 but < .55 E. ≥ .55 (94F–5A–21–1)

B10. $f_2(0) = \dfrac{f_1(x)f_2(x) - f1(0)f_2(1)}{f_1(1)} = \dfrac{.42 - (.4)(.3)}{.6} = .5$

$f_2(2) = 1 - f_2(0) - f_2(1) = 1 - .5 - .3 = .2$

$f_S(2) = .272 = P(X_1 = 0, X_2 = 0, \text{and } X_3 = 2) + P(X_1 = 0, X_2 = 1, \text{and } X_3 = 1) +$
$P(X_1 = 0, X_2 = 2, \text{and } X_3 = 0) + P(X_1 = 1, X_2 = 0, \text{and } X_3 = 1) +$
$P(X1 = 1, X2 = 1, \text{and } X3 = 0)$

$.272 = (.4)(.5)(.7 - y) + (.4)(.3)(.3) + (.4)(.2)y + (.6)(.5)(.3) + (.6)(.3)y = .266 + .06y$

$y = .10$

Answer: D

B11. $F_1(1) = f_1(0) + f_1(1) = .2 + .3 = .5 \qquad F_1(2) = F_1(1) + f_1(2) = .5 + .4 = .9$

$P(S \leq 2) = F_1(2)\, f_2(0) + F_1(1)\, f_2(1) = (.9)(.8) + (.5)(.2) = .82$

Answer: D

B12. $f_S(6) = .10 = P(X_1 = 1, X_2 = 3, \text{and } X_3 = 2) + P(X_1 = 2, X_2 = 2, \text{and } X_3 = 2) +$
$P(X_1 = 3, X_2 = 1, \text{and } X_3 = 2) + P(X_1 = 3, X_2 = 3, \text{and } X_3 = 0)$

$.10 = (.40)(.25)(1 - p) + (.10)(.25)(1 - p) + (.10)(.25)(1 - p) + (.10)(.25)p$

$.10 = .15 - .125p \qquad p = .40$

Answer: E

B13. $f_1(0) = F_1(0) = .3 \qquad f_2(0) = F_2(0) = .6 \qquad f_2(1) = F_2(1) - F_2(0) = .8 - .6 = .2$

$f_2(2) = F_2(2) - F_2(1) = 1.0 - .8 = .2$

$P(S \leq 2) = F_1(2)\, f_2(0) + F_1(1)\, f_2(1) + f_1(0)\, f_2(2) = (.6)(.6) + (.4)(.2) + (.3)(.2) = .50$

Answer: D

B14. Aggregate claims $S = X_1 + X_2 + X_3$, where X_1, X_2, and X_3 are mutually independent random variables with probability functions as follows:

x	0	1	2	3	4
$f_1(x)$	.6	.4	0	0	0
$f_2(x)$	p	.3	.5	0	.7 – p
$f_3(x)$	0	.5	.5	0	0

You are given $F_S(4) = .6$. Determine p.

A. 0 B. .1 C. .2 D. .3 E. .4 (95S–151–8–2)

B15. Let $S = X_1 + X_2$, where X_1 and X_2 are independent random variables with the following distribution functions:

X	$F_1(X)$	$F_2(X)$
0	.5	.3
1	.8	.6
2	1	1

What is $P(S > 2)$?

A. < .20 B. ≥ .20 but < .40 C. ≥ .40 but < .60 D. ≥ .60 but < .80 E. ≥ .80 (95F–5A–19–1)

B16. Mutual Motorists Insurance Company writes only automobile liability insurance. Depending on the initial claim information received by the company, preliminary opening reserves are established by the claims department in the following manner:

Type of Claim	State A	State B	State C
Bodily injury (serious)	\$5,000	\$3,000	\$2,000
Bodily injury (nonserious)	\$1,000	\$3,000	\$2,000
Property damage	\$1,000	\$2,000	\$1,000

According to the company's statistics, 60% of its claims are property damage, 30% are nonserious bodily injury, and 10% are serious bodily injury. Also, the company's claims are distributed 40% in state A, 40% in state B, and 20% in state C. The claims are mutually independent. Each claim file includes the details for only one of the three types of claim. Every month, the claims manager does an audit of each examiner's files (to ensure that the initial reserves are properly set) by pulling three files at random, one for each type of claim. What is the probability that the total dollars of reserves involved in any examiner audit exceeds \$7,500? Show all work. (97S–5A–40–3)

B17. The following information is given regarding three mutually independent random variables:

x	0	1	2	3	4
$f_1(x)$	.5	.4	.1		
$f_2(x)$	.2	.2	.2	.2	.2
$f_3(x)$	.1	.9			

If $S = X_1 + X_2 + X_3$, calculate the probability that $S = 5$.

A. < .10 B. ≥ .10 but < .15 C. ≥ .15 but < .20 D. ≥ .20 but < .25 E. ≥ .25 (97F–5A–22–1)

B14. $f_S(0) = 0$ $f_S(1) = P(X_1 = 0, X_2 = 0, \text{ and } X_3 = 1) = .3p$

$f_S(2) = P(X_1 = 1, X_2 = 0, \text{ and } X_3 = 1) + P(X_1 = 0, X_2 = 1, \text{ and } X_3 = 1) + P(X_1 = 0, X_2 = 0, \text{ and } X_3 = 2) = .2p + .09 + .3p = .5p + .09$

$f_S(3) = P(X_1 = 1, X_2 = 1, \text{ and } X_3 = 1) + P(X_1 = 0, X_2 = 1, \text{ and } X_3 = 2) + P(X_1 = 1, X_2 = 0, \text{ and } X_3 = 2) = .06 + .09 + .2p = .2p + .15$

$f_S(4) = P(X_1 = 1, X_2 = 1, \text{ and } X_3 = 2) = .06$

$F_S(4) = f_S(0) + f_S(1) + f_S(2) + f_S(3) + f_S(4)$

$.6 = 0 + .3p + (.5p + .09) + (.2p + .15) + .06 = p + .3$ $p = .3$

Answer: D

B15. $f_1(1) = F_1(1) - F_1(0) = .8 - .5 = .3$ $f_1(2) = F_1(2) - F_1(1) = 1 - .8 = .2$

$f_2(1) = F_2(1) - F_2(0) = .6 - .3 = .3$ $f_2(2) = F_2(2) - F_2(1) = 1 - .6 = .4$

$P(S > 2) = f_1(1)\, f_2(2) + f_1(2)\, f_2(1) + f_1(2)\, f_2(3) = (.3)(.4) + (.2)(.3) + (.2)(.4) = .26$

Answer: B

B16. $f(8{,}000) = f(BIS_A + BIN_A + PD_B) + f(BIS_A + BIN_C + PD_A) + f(BIS_A + BIN_C + PD_C) + f(BIS_B + BIN_B + PD_B)$

$f(8{,}000) = (.40)(.40)(.40) + (.40)(.20)(.40) + (.40)(.20)(.20) + (.40)(.40)(.40) = .176$

$f(9{,}000) = f(BIS_A + BIN_B + PD_A) + f(BIS_A + BIN_B + PD_C) + f(BIS_A + BIN_C + PD_B)$

$f(9{,}000) = (.40)(.40)(.40) + (.40)(.40)(.20) + (.40)(.20)(.40) = .128$

$f(10{,}000) = f(BIS_A + BIN_B + PD_B) = (.40)(.40)(.40) = .064$

$P(S > 7{,}500) = 1 - F(7{,}500) = f(8{,}000) + f(9{,}000) + f(10{,}000) = .176 + .128 + .064 = .368$

B17. $f_S(5) = P(X_1 = 0, X_2 = 4, \text{ and } X_3 = 1) + P(X_1 = 1, X_2 = 4, \text{ and } X_3 = 0) + P(X_1 = 1, X_2 = 3, \text{ and } X_3 = 1) + P(X_1 = 2, X_2 = 3, \text{ and } X_3 = 0) + P(X_1 = 2, X_2 = 2, \text{ and } X_3 = 1)$

$f_S(5) = (.5)(.2)(.9) + (.4)(.2)(.1) + (.4)(.2)(.9) + (.1)(.2)(.1) + (.1)(.2)(.9) = .19$

Answer: C

B18. You are given the following:

i) The number of claims for a given risk follows a distribution with probability function:

$$p(n) = (e^{\lambda} - 1)^{-1} \frac{\lambda^n}{n!}, \text{ where } n = 1, 2, 3, \ldots \quad \lambda > 0$$

ii) Claim sizes for this risk follow a distribution with density function

$$f(x) = e^{-x}, \text{ where } 0 < x < \infty$$

iii) For this risk, the number of claims and claim sizes are independent.

Determine the probability that the largest claim for this risk is less than k.

A. $e^{\lambda} - 1$ B. $\{e^{\lambda} - 1\}\{\exp[\lambda(1 - e^{-k}) - 1]\}$ C. $\{e^{\lambda} - 1\}\{\exp[\lambda(1 - e^{-k})]\}$
D. $\{e^{\lambda} - 1\}^{-1}\{\exp[\lambda(1 - e^{-k})] - 1\}$ E. $\{e^{\lambda} - 1\}^{-1}\{\exp[\lambda(1 - e^{-k})]\}$ (98S–4B–13–3)

B19. You are in Las Vegas in a dice game. The house has devised a game in which a player rolls two dice, and the sum of 7 or under pays even money (i.e., 1:1 odds). Before playing, you are informed that one die is fair, but the other die is biased according to the number on its face. For example, the biased die rolls a "5" five times more than it rolls a "1," and similarly for the other values, so the probability of a "5" is 5/21.

a. What is the probability of rolling a sum from two dice of eight or over? Show all work.
b. Are you willing to play this game if the odds are revised to 1.2 to 1? Explain your reason. Show all work. (98S–5A–35–2)

B20. Aggregate claims $S = X_1 + X_2 + X_3$, where X_1, X_2, and X_3 are mutually independent random variables with probability functions as follows:

x	0	1	2	3	4
$f_1(x)$	.2	.3	.5	0	0
$f_2(x)$	0	0	p	$1 - p$	0
$f_3(x)$	.5	.5	0	0	0

You are given $F_S(4) = .43$. Determine p.

A. .1 B. .2 C. .3 D. .4 E. .5 (98S–151–11–2)

B21. ABC Insurance Company writes liability coverage with one policy limit of $90,000 offered to all insureds. Only two types of losses to the insurer arise out of this coverage: 1) total limits plus loss expenses of $100,000 or 2) loss expenses only of $50,000. You are given the following distribution of aggregate losses that applies in years when the insurer faces 2 claims.

x	100,000	150,000	200,000
$f(x)$	90.25%	9.5%	.25%

If, next year, the insurer faces 3 claims, what is the likelihood that the aggregate losses will exceed $150,000? Show all work. (99S–5A–37–2)

B18. $P(\text{All } X_i < k) = P(N = 1)\, P(X_1 < k) + P(N = 2)\, P(X_1 < k) \Pr(X_2 < k) + \cdots$
$P(\text{All } X_i < k) = (e^{\lambda} - 1)^{-1}(\lambda/1!)\,(1 - e^{-k}) + (e^{\lambda} - 1)^{-1}(\lambda^2/2!)(1 - e^{-k})^2 + \cdots$
$P(\text{All } X_i < k) = (e^{\lambda} - 1)^{-1}[(\lambda/1!)(1 - e^{-k}) + (\lambda^2/2!)(1 - e^{-k})^2 + \cdots]$

Since the term in brackets is a power series for e minus one, we get:

$P(\text{All } X_i < k) = \{e^{\lambda} - 1\}^{-1}\{\exp[\lambda(1 - e^{-k})] - 1\}$

Answer: D

B19. a. The probability of each outcome less than 8 equals the sum of the products of the probabilities of each lower number on the unfair dice times the probability of the outcome minus the lower number on the fair dice. But since this later probability always equals 1/6 and the denominator of the former probability is always 21, we get:

$f_S(i) = [1/6][(i - 1) + (i - 2) + \cdots]/21$ $\quad f_S(2) = 1/126$ $\quad f_S(3) = 3/126$

$f_S(4) = 6/126$ $\quad f_S(5) = 10/126$ $\quad f_S(6) = 15/126$ $\quad f_S(7) = 21/126$

$F_S(7) = \sum_{i=1}^{7} = (1 + 3 + 6 + 10 + 15 + 21)/126 = 56/126$

$P(X > 7) = 1 - F_S(7) = 1 - 56/126 = 70/126$

b. Correct Odds $= [1 - F_S(7)]/F_S(7) = (70/126)/(56/126) = 1.25$

Since the offered odds are less than the correct odds, I am not willing to play the game.

B20. $f_S(0) = f_S(1) = 0$ $\quad f_S(2) = P(X_1 = 0, X_2 = 2, \text{and } X_3 = 0) = .1p$

$f_S(3) = P(X_1 = 0, X_2 = 3, \text{and } X_3 = 0) + P(X_1 = 1, X_2 = 2, \text{and } X_3 = 0) +$
$P(X_1 = 0, X_2 = 2, \text{and } X_3 = 1) = (.1 - .1p) + .15p + .1p = .1 + .15p$

$f_S(4) = P(X_1 = 1, X_2 = 2, \text{and } X_3 = 1) + P(X_1 = 2, X_2 = 2, \text{and } X_3 = 0) +$
$P(X_1 = 1, X_2 = 3, \text{and } X_3 = 0) + P(X_1 = 0, X_2 = 3, \text{and } X_3 = 1)$
$f_S(4) = .15p + .25p + (.15 - .15p) + (.1 - .1p) = .25 + .15p$

$F_S(4) = f_S(0) + f_S(1) + f_S(2) + f_S(3) + f_S(4)$

$.43 = 0 + 0 + .1p + (.1 + .15p) + (.25 + .15p) = .4p + .35$ $\quad p = .2$

Answer: B

B21. 1) Calculate the conditional probabilities of each claim type:

$.9025 = P(2 \text{ Type 2 Claims} \mid 2 \text{ Claims}) = [P(\text{Type 2 Claim} \mid 1 \text{ Claim})]^2 = (.95)^2$

$P(\text{Type 2 Claim} \mid 1 \text{ Claim}) = .95$ $\quad P(\text{Type 1 Claim} \mid 1 \text{ Claim}) = .05$

2) Calculate the probability of 150,000 aggregate losses given 3 claims. (Note that this is the lowest possible amount of aggregate losses, given three claims.)

$150{,}000 = P(3 \text{ Type 2 Claims} \mid 3 \text{ Claims}) = (.95)^3 = .857375$

3) Calculate the probability that aggregate losses exceed 150,000, given three claims:

$P(> 150{,}000 \mid 3 \text{ Claims}) = 1 - P(3 \text{ Type 2 Claims} \mid 3 \text{ Claims}) = 1 - .857375$
$P(> 150{,}000 \mid 3 \text{ Claims}) = .142625$

