Practice Test Questions

Exam P/1: Probability
Society of Actuaries

(Sample Only – Purchase the Full Version)

Created By: Digital Actuarial Resources

Introduction:

This guide from Digital Actuarial Resources (DAR) contains sample test problems for Exam P offered through the Society of Actuaries. The book has 250 practice questions to test your knowledge of the principles of probability and statistics. The problems encompass applications of chance in discrete distributions, continuous distributions, Venn diagrams, multivariate distributions, and moment-generating functions. The set of questions is very comprehensive and attempts to cover all major topics featured on the actual test. Nearly all of these questions are math-based. Some of the examples require calculus, while others entail advanced algebra.

You should expect to spend several days taking this test. There is no time limit. You can use your notes, textbooks, other actuaries, and whatever will help you answer the questions. These problems test your introductory actuary skills and also attempt to teach you something. If you can correctly answer 75% of the questions (about 188 problems), you are prepared for the actual test.

The first half of this guide contains the practice test questions. You should use your own scratch paper when taking the test so that you can retake it several times. The detailed solutions start on page 44. If you find any errors in the solutions or would like to debate an answer, please contact the Digital Actuaries.

(18.) A computer chip has a 9% chance of a manufacturing defect. Drur tests a set of 20 chips. What is the probability that at most 2 of the chips are defective?

(29.) Dredge is a terrible driver but is taking a test for his license. Dredge will keep taking the test until he passes. The probability that Dredge passes any given test is 35%. Use this information for the next few questions.

What is the probability that Dredge passes the test on the third attempt?

(39.) A football kicker is making practice field goal attempts. The probability of making a field goal is 85%. The kicker will continue making field goal attempts until he makes 3 goals.

What is the probability that the player makes the third goal on the fourth kick?

(43.) A carpenter with a nail gun must put 12 nails through a board. The probability that the carpenter correctly drives in a nail with a given shot is 90%. What is the probability that the carpenter completes the job with exactly 15 nails?

(67.) Suppose the continuous distribution of an event is uniform with an expected value of $700,000 and a standard deviation of $100,000. Find the endpoints of the distribution.

(94.) A farmer uses 2 types of fertilizer and 2 levels of irrigation – light or heavy. The probability of light irrigation given that the farmer used fertilizer number 1 is 70%. The probability of heavy irrigation given that the farmer used fertilizer number 2 is 45%. The probability of using fertilizer number 1 is 62%, and the probability of using fertilizer number 2 is 38%. Given that the farmer used heavy irrigation, what is the probability he used fertilizer number 2?

(112.) Suppose X equals the number of service calls under a warranty during a given year. The company sold 4,000 warranties, and the probability of a service call is 0.03. Find the probability that the company receives at least 135 calls this year.

(116.) Consider 2 classes of 45-year-olds under a health insurance plan. The nonsmoker class has 750 insureds with annual clams following a uniform distribution between $200 and $2,600. The smoker class has 125 insureds with annual claims following a uniform distribution between $500 and $4,500. What is the probability that the difference between the average smoker claims and the average nonsmoker claims is under $1,000?

(173.) Consider the discrete bivariate distribution below:

Y \ X	100	150	180
30	0.22	0.15	0.08
42	0.18	0.04	0.09
55	0	0.13	0.11

Find the covariance between X and Y.

(189.) Tamir is attempting to estimate the yield on his crops this season (in tons). With 24% chance the yield follows a gamma distribution with parameters $\alpha = 30$ and $\beta = 17$. With 9% chance the yield follows a chi-square distribution with 400 degrees of freedom. The third possible distribution is an exp. distribution with mean 520 tons. Find the variance of the crop yield.

Solutions

(18.)

Bimodal distribution…
Let p = prob. of success = prob. a chip is defective = 0.09
Let q = prob. of failure = prob. a chip is not defective = 0.91
n = number of trials = 20

$$P = p(0) + p(1) + p(2)$$
$$= {}_{20}C_0 * p^0 * q^{20} + {}_{20}C_1 * p^1 * q^{19} + {}_{20}C_2 * p^2 * q^{18}$$
$$= 1 * 0.09^0 * 0.91^{20} + 20 * 0.09^1 * 0.91^{19} + 190 * 0.09^2 * 0.91^{18}$$
$$= 0.1516 + 0.3 + 0.2818$$
$$= \mathbf{0.7334}$$

(29.)

Geometric Distribution

$$p = 0.35, q = 0.65, n = \infty$$
$$p(3) = q^{3-1} * p = 0.65^2 * 0.35 = \mathbf{0.1479}$$

(39.)

Negative Binomial Distribution

$$n = \infty, p = 0.85, q = 0.15, r = 3$$
$$p(4) = \begin{pmatrix} x-1 \\ r-1 \end{pmatrix} * q^{x-r} * p^r$$
$$\downarrow$$
$$= \begin{pmatrix} 4-1 \\ 3-1 \end{pmatrix} * q^{4-3} * p^3$$
$$\downarrow$$
$$= \begin{pmatrix} 3 \\ 2 \end{pmatrix} * q^1 * p^3 =$$
$$\mathbf{0.2764}$$

(43.)

A negative binomial distribution with X = index for the trial with the 12th accurate nail-in.

$$n = \infty \,, p = 0.90, q = 0.10$$

$$p(15) = \binom{15-1}{12-1} * q^{15-12} * p^{12} = \binom{14}{11} * q^{3} * p^{12}$$

= **0.1028**

(67.)

$$E(X) = 700{,}000 = \frac{a+b}{2} \rightarrow 1{,}400{,}000 = a+b \rightarrow a = 1{,}400{,}000 - b$$

$$\sqrt{Var(X)} = 100{,}000 = \sqrt{\frac{(b-a)^2}{12}}$$

$$100{,}000^2 = \frac{(b-a)^2}{12}$$

$$\downarrow$$

$$12 * 100{,}000^2 = (b-a)^2$$

$$\sqrt{12 * 100{,}000^2} = b - a$$

$$\sqrt{12 * 100{,}000^2} = b - (1{,}400{,}000 - b)$$

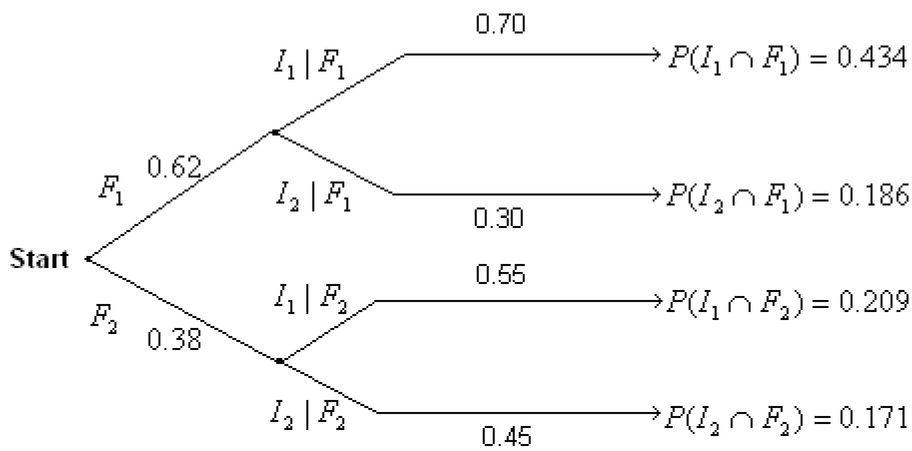
$$346{,}410.16 = b - 1{,}400{,}000 + b$$

$$1{,}746{,}410.16 = 2b$$

b = 873,205

$a = 1{,}400{,}000 - b$ **= a = 526,795**

(94.)



Let F_1 = fertilizer # 1

F_2 = fertilizer #2

I_1 = light irrigation

I_2 = heavy irrigation

$P(I_1 \cap F_2) = 0.38 - 0.171 = 0.209$

$P(I_2 \cap F_1) = 0.62 - 0.434 = 0.186$

$P(I_1 \mid F_1) + P(I_2 \mid F_1) = 1$

$\downarrow$

$0.7 + P(I_2 \mid F_1) = 1 \rightarrow P(I_2 \mid F_1) = 0.3$

$P(I_1 \mid F_2) + P(I_2 \mid F_2) = 1$

$P(I_1 \mid F_2) + 0.45 = 1 \rightarrow P(I_1 \mid F_2) = 0.55$

$$P(F_2 \mid I_2) = \frac{P(F_2) * P(I_2 \mid F_2)}{P(F_1) * P(I_2 \mid F_1) + P(F_2) * P(I_2 \mid F_2)}$$

$\downarrow$

$$= \frac{0.38 * 0.45}{0.62 * 0.3 + 0.38 * 0.45}$$

= 0.479

(112.)

X ~ Binomail(n = 4,000, p = 0.03)

Since n is large, a Normal distribution can approximate the time Binomial distribution.
↓

$X \sim Normal(\mu, \sigma^2)$

$\mu = E(X) = np = 4{,}000 * 0.03 = 120$

$\sigma^2 = Var(X) = npq = 4{,}000 * 0.03 * 0.97 = 116.4$

$\sigma = \sqrt{116.4} = 10.7889$

$$z = \frac{X - \mu}{\sigma} = \frac{135 - 120}{10.7889} \approx 1.39$$

The z-value above is incorrect. The variable X is discrete, but we are using a continuous distribution to estimate a probability. We must apply the continuity correction because we are switching between the discrete and continuous realms.

$P(X \geq 135$ (in Binomial distribution))=
$= P(X \geq 134.5$ (in Normal distribution))

$$z = \frac{134.5 - 120}{10.7889} \approx 1.344$$

$P(X \geq 135) = P(z > 1.344)$ **= 0.0901**

(116.)

Let A = nonsmoker insured, $\overline{X_A}$ = average claims for a nonsmoker.
B = smoker insured, $\overline{X_B}$ = average claims for a smoker.

Note that $\overline{X_A}$ and $\overline{X_B}$ are ~N.

Let Y = $(\overline{X_B} - \overline{X_A}) \sim Normal(\mu = \mu_{\overline{X_B}} - \mu_{\overline{X_A}}, \sigma^2 = \sigma_{\overline{X_B}}^2 + \sigma_{\overline{X_A}}^2)$

$$\mu_{\overline{X_A}} = \mu_{X_A} = \frac{200 + 2600}{2} = 1{,}400$$

$$\mu_{\overline{X_B}} = \mu_{X_B} = \frac{500 + 4500}{2} = 2{,}500$$

$$\sigma_{\overline{X_A}}^2 = \sigma_{X_A}^2 / n = \left(\frac{(2{,}600 - 200)^2}{12} \right) / 750 = 640$$

$$\sigma_{\overline{X_B}}^2 = \sigma_{X_B}^2 / n = \left(\frac{(4{,}500 - 500)^2}{12} \right) / 125 = 10{,}667$$

$$Y \sim Normal(\mu = 2{,}500 - 1{,}400, \sigma^2 = 10{,}667 + 640)$$
$$\downarrow$$
$$Y \sim Normal(\mu = 1{,}100, \sigma^2 = 11{,}307)$$

$$\Pr(Y < 1{,}000) = \Pr(z < -0.94) \mathbf{= 0.1736}$$

$$z = \frac{1{,}000 - 1{,}100}{\sqrt{11{,}307}} = -0.94$$

(173.)

$$Cov(X,Y) = E(XY) - E(X) * E(Y)$$
$$E(X) = \sum_X \sum_Y X * P(X,Y) = 100*(0.22 + 0.18) + 150*(0.15 + 0.04 + 0.13) + 180*(0.08 + 0.09 + 0.11)$$
$$= 138.4$$
$$E(Y) = \sum_Y \sum_X Y * P(X,Y) = 30*(0.22 + 0.15 + 0.08) + 42*(0.18 + 0.04 + 0.09) + 55*(0.13 + 0.11)$$
$$= 39.72$$
$$E(XY) = \sum_X \sum_Y XY * P(X,Y) = 5{,}616.9$$
$$Cov(X,Y) = 5{,}616.9 - 138.4 * 39.72 \mathbf{= 119.652}$$

(189.)

$$w_1 = 0.24, X_1 \sim gamma(\alpha = 30, \beta = 17), Var(X_1) = \alpha\beta^2 = 8{,}670$$
$$w_2 = 0.09, X_2 \sim \chi^2(400), Var(X_2) = 2m = 800$$
$$w_3 = 0.67, X_3 \sim Exp(520), Var(X_3) = \theta^2 = 270{,}400$$

Let Y = mixture of X_1, X_2, X_3

$Var(Y) = 0.24^2 * Var(X_1) + 0.09^2 * Var(X_2) + 0.67^2 * Var(X_3)$

$Var(Y)$ **= 121,888**