Practice Exam 1

Questions

1-1. The total present value of the cash flows:

- $1,800 in 4 years
- $400 in 5 years
- $1,500 in 7 years

at an interest rate of 7% per year compounded weekly is:

A) $2,365.51 B) $2,561.95 C) $2,592.53 D) $3,700.00 E) $5,396.35

1-2. Joe has an amount of money that he would like to invest in some way. He has two options:

- He can invest the money into an account that pays 9% per year effective, while reinvesting the interest payments into another account that pays 6% per year effective. This will continue for 15 years.

- He can pay his friend Ted the money, who in return will pass on to Joe a series of 15 yearly payments that he is owed. In particular, these payments are $100, they start in one year, and they are reinvested at 6% per year effective.

Joe notices that the amount of money that he ends up with is the same either way. To the nearest dollar, the amount of money that Joe has to invest is:

A) $406.00 B) $417.00 C) $752.00 D) $1,063.00 E) $1,111.00

2-7. An investor sells short 500 shares of stock at 10 per share and covers the short position one year later when the price of the stock has declined to 7.50. The margin requirement is 50%. Interest on the margin deposit is 8% effective. Four quarterly dividends of .15 per share are paid.

Calculate the yield rate.

A) 19%

B) 23%

C) 38%

D) 46%

E) 58%

2-8. In the list at the left are two items, lettered X and Y. In the list at the right are three items, numbered I, II, and III. ONE of the lettered items is related in some way to EXACTLY TWO of the numbered items. Choose the related items.

X.	Putable Bond	I.	Has unlimited gain potential for the buyer
Y.	Callable Bond	II.	As interest rates decrease, the bond duration decreases
		III.	Has unlimited loss potential for the buyer

A) X is related to I and II only

B) X is related to II and III only

C) Y is related to I and II only

D) Y is related to II and III only

E) None of the above

3-5. The internal rate of return of an investment is the interest rate that equates the present value of all related cash flows to:

A) the opportunity cost of the cash flows

B) the initial investment amount

C) the investment value of the cash flows

D) zero

E) the sum of all positive cash flows

3-6. Select the condition that is **not** a requirement of Redington immunization:

A) The term structure of interest rates is flat.

B) The convexity of liabilities is greater than the convexity of assets.

C) The duration of the liabilities is equal to the duration of the assets.

D) The present value of liabilities is equal to the present value of the assets.

E) All of the above are required.

4-16. Terms of a bond:

Face amount	\$1,000
Redemption amount	\$1,000
Term	10 years
Coupons	6%, payable semi-annually
Yield rate	5%, convertible semi-annually

In what range is the Macaulay duration of the bond?

A) Less than 7.57

B) 7.57 but less than 7.64

C) 7.64 but less than 7.71

D) 7.71 but less than 7.78

E) 7.78 or more

4-17. In the list at the left are two items, lettered X and Y. In the list at the right are three items, numbered I, II, and III. ONE of the lettered items is related in some way to EXACTLY TWO of the numbered items. Choose the related items.

X.	Effective duration matching	I.	Very expensive to implement
Y.	Cash flow matching	II.	Only works for small changes in interest rates
		III.	Accounts for options embedded in the assets and liabilities

A) X is related to I and II only

B) X is related to II and III only

C) Y is related to I and II only

D) Y is related to II and III only

E) None of the above

5-12. You are given $\delta_t = \dfrac{2}{t-1}$ for $2 \le t \le 10$. For any one year interval between n and $n+1$, with $2 \le n \le 9$, calculate the equivalent $d^{(2)}$.

A) $\dfrac{1}{n}$ B) $\dfrac{2}{n}$ C) $\dfrac{n-1}{n}$ D) $\dfrac{n}{n-1}$ E) $\left(\dfrac{n}{n-1}\right)^2$

5-13. The force of interest at time t is kt^3. R is the present value of a four-year continuously increasing annuity which has a rate of payment at time t *of* mt^3.

Calculate R.

A) $\dfrac{k - me^{-4k}}{k}$

B) $\dfrac{k - me^{-64k}}{k}$

C) $\dfrac{1 - me^{-4k}}{k}$

D) $\dfrac{1 - me^{-64k}}{k}$

E) $\dfrac{m(1 - e^{-64k})}{k}$

5-14. Date of loan: 1/1/2005

Terms of loan repayment: 20 annual payments beginning 12/31/2005. The payment is X in the first 10 years, and 50% of X in the second 10 years.

Interest rate: 5%, compounded annually

Y is the ratio of principal repaid in the 10^{th} payment to principal repaid in the 11^{th} payment.

In what range is Y?

A) Less than 2.40

B) 2.40 but less than 2.46

C) 2.46 but less than 2.52

D) 2.52 but less than 2.58

E) 2.58 or more

5-15. A common stock is purchased at a price equal to ten times current earnings. During the next eight years the stock pays no dividends, but earnings increase 50%. At the end of eight years the stock is sold at a price equal to 16.5 times earnings.

Calculate the effective annual yield rate.

A) .052

B) .065

C) .083

D) .102

E) .120

Solutions to Practice Exam 1

Solutions

1-1.

The effective weekly rate is $j = \dfrac{.07}{52} = .001346.$

(Note that at least six decimal places should be retained in order to make an accurate calculation.) Then the present value is

$$PV = 1000\,(1+j)^{-208} + 400\,(1+j)^{-260} + 1500\,(1+j)^{-364}$$
$$= 1360.71 + 281.95 + 919.29 = 2561.95, \text{ Answer B.}$$

1-2. Joe has X to invest. Under the first option, he earns $.09X$ at the end of each year which he invests in the second account at 6%. Then at time 15 he has

$$X + .09X \cdot s_{\overline{15}|.06} = 3.09484X.$$

Under the second option, Joe invests \$100 each year at 6%, so he has

$$100\ s_{\overline{15}|.06} = 2327.60$$

at time 15. Then it follows that

$$X = \frac{2327.60}{3.09484} = 752.09, \text{ Answer C.}$$

2-4. The coupon amount is $Fr = (10{,}000)(.08/2) = 400.$ Since 2 months is 1/3 of a coupon period, we have the following values, recalling that the market price is the flat price minus the accrued coupon:

Theoretical: $P_{1/3} = P_0(1.03)^{1/3} - 400s_{\overline{1/3}|.03}$

Semi-theoretical: $P_{1/3} = P_0(1.03)^{1/3} - (400)\left(1/3\right)$

Then the difference in market price is

$$\left|400s_{\overline{1/3}|.03} - 400\left(1/3\right)\right| = |132.01 - 133.33| = 1.32, \text{ Answer E.}$$

2-5. The amortization payment is $P = \dfrac{100{,}000}{a_{\overline{30}|}}$, and the outstanding balance immediately after the second payment is $OB = \dfrac{100{,}000a_{\overline{28}|}}{a_{\overline{30}|}}.$ The origination fee is $.02(100{,}000) = 2000$ at time $t = 0.$ In return for his 100,000 loan, the lender receives 2000 at $t = 0, P$ at $t = 1,$ and $(P + OB)$ at $t = 2.$ The lender's yield is the value of i which satisfies the equation of value

$100{,}000 = 2000 + Pv + (P + OB)v^2.$

Then $1 = .02 + \dfrac{v}{a_{\overline{30}|}} + \left(\dfrac{1 + a_{\overline{28}|}}{a_{\overline{30}|}}\right)v^2$ or $(1 + a_{\overline{28}|})v^2 + v - .98a_{\overline{30}|} = 0.$

Evaluating $a_{\overline{28}|}$ and $a_{\overline{30}|}$ at $4\%,$ we have $17.6631v^2 + v - 16.94616 = 0.$

The positive root of v in this quadratic is $v = .95159.$

Then $i = v^{-1} - 1 = .050865,$ Answer C.

2-6. Correct answer is B

2-7. Per share we have a profit of $10-7.50=2.50,$ a margin deposit of $(10)(.5)=5,$ interest on the margin of $(5)(.08)=.40,$ and an annual dividend of $4(.15)=.60.$ Then the yield is

$$\frac{\text{Profit + Interest on Margin - Dividends}}{\text{Margin}}=\frac{2.50+.40-.60}{5}=.46, \text{ Answer D.}$$

2-8. A callable bond's duration will shorten if rates fall because the bond is more likely to be called (Y is related to II). Neither bond has an unlimited gain or loss because each is limited by the purchase price and the redemption value, Answer E.

2-9. Correct answer is C (Must be in between 4% and 7% or else arbitrage is possible.)

2-10. Answer C

2-11. After the swap, Lachlin Bank is exposed to risk from the prime due to its obligation to DCB. DCB, on the other hand, has an offsetting asset and liability tied to prime. Answer A.

2-12. The equivalency relationship is

$$1+i=\left(1+\frac{.08}{365}\right)^{365},$$

so

$$i=\left(1+\frac{.08}{365}\right)^{365}-1=.0832776.$$

Note: Often "daily compounding" is considered to be continuous compounding, so the conversion to an effective annual rate would be

$$i=e^{.08}-1=.0832871,$$

which shows how good is the approximation of daily compounding by continuous compounding, Answer E.

2-13. The effective semi-annual rate is $\frac{.125}{2}=.0625$. The first payment is to be on 12/17/13, which is time t=8 (in half years). Then the present value of the perpetuity is $\frac{2500}{.0625}$ at time t = 7, and $\left(\frac{2500}{.0625}\right)(1.0625)^{-7}$ at time t = 0, which is the amount of the required deposit. We have

$$PV=\left(\frac{2500}{.0625}\right)(1.0625)^{-7}$$
$$=(40,000)\,(.65418)=26,167.21, \text{ Answer C.}$$

2-14. Correct answer C

2-15. Statement 1 is false because the price also includes the present value of the redemption amount.

Statement 2 is false; it would be true only if the bond is redeemed at par value.

Statement 3 is false; even if $C = F$ the price could still be either greater than or less than the face value.

Statement 4 is true; this is the correct definition of the modified coupon rate.

Statement 5 is false; purchased at a premium means the price exceeds the redemption value, not the face value.

Answer D

2-16. The equation of value is

$$11,000 = 1300\, a_{\overline{10}|i}.$$

Because the amount invested is always positive, a unique positive solution for the IRR exists, so the first statement is correct, Answer A.

2-17. First, let's refer to the requirements for immunization. Let x and y, respectively, refer to the amounts to be invested in the 2-year and 8-year bonds. This means that $x(1.05)^2$ is the face value of the 2-year bond and that $y(1.05)^8$ is the face value of the 8-year bond.

We know that $P(i) = 0$ and therefore that $x + y = 100$ (i.e. we must invest the whole \$100 to back a liability with a present value of \$100).

We also require that the duration of assets and liabilities are equal, or equivalently that the surplus has a duration of 0.

We write

$$P(i) = x(1.05)^2(1+i)^{-2} - 115.76(1+i)^{-3} + y(1.05)^8(1+i)^{-8} = 0$$

Taking derivatives we get

$$P^{'}(i) = x(1.05)^2(-2)(1+i)^{-3} - 115.76(-3)(1+i)^{-4} + y(1.05)^8(-8)(1+i)^{-9} = 0$$

Since $115.76 = 100(1.05)^3$ and we are evaluating the derivative at $i = 5\%$, this equals:

$$P^{'}(.05) = -2x(1.05)^{-1} + 300(1.05)^{-1} - 8y(1.05)^{-1} = 0$$

or finally

$$x + 4y = 150\,.$$

Since we also know that $x + y = 100$, we can solve these two equations simultaneously to get $x = 250/3$ and $y = 50/3$.

We need to check only that $P^{''}(i) > 0$ at $i = 5\%$ and we will have satisfied the conditions for immunization.

We get

$$P^{''}(i) = x(1.05)^2(-2)(-3)(1+i)^{-4} - 115.76(-3)(-4)(1+i)^{-5} + y(1.05)^8(-8)(-9)(1+i)^{-10}$$
$$P^{''}(.05) = 6(250/3)(1.05)^{-2} - 1200(1+i)^{-2} + 72(50/3)(1+i)^{-2}$$
$$P^{''}(.05) = (1.05)^{-2}\left[500 - 1200 + 1200\right]$$
$$P^{''}(.05) = 500(1.05)^{-2} > 0$$

Answer E

2-18. I) is false. Final prices can differ since margin is required for futures but not necessarily for forwards.

II) is true, as stated on page 147 of *Derivatives Markets*.

III) is false, as we see from I) above.

Answer B

2-19. Here, the option pays off $4 in the "up" state and 0 in the "down" state. The expected payoff is therefore .60(4) = 2.40. The expected gain is therefore 2.40 – 2.00 = .40. Investing the risk-free security would give an ending value of 2.00(1.04) = 2.08. Therefore the risk premium is 2.40 – 2.08 = .32, Answer C.

2-20. Correct answer is D

2-21. Correct answer is E (II only)

2-22. If the one US dollar is invested at home, it grows to 1.05 at year-end. If it is invested in Canada, as 1.4285 CDN, it grows to (1.4285) (1.07) = 1.52850 at year-end. If there is no arbitrage, then 1.05 USD is equivalent to 1.5285 CDN, so the exchange rate one year hence would be

$\frac{1.5285}{1.05} = 1.4557$, Answer E.

2-23. The equivalency relationship is

$$(1-d) = \left(1 - \frac{.17}{2}\right)^2,$$

so

$$d = 1 - (1 - .085)^2 = .162775 \text{ or } 16.2775\%, \text{ Answer B.}$$

2 - 24. Items (i) and (ii) tell us that $200v^5 + 500v^{10} = 400.94v^5$,

or that $200 + 500v^5 = 400.94$, which

implies that $v^5 = \frac{200.94}{500} = .40188.$ We are also given

that $100(1+i)^{10} + 120(1+i)^5 = P$, so

that $P = 120(.40188)^{-1} + 100(.40188)^{-2} = 917.76$, Answer B.

2-25. The bond produces \$40 coupons each half-year for 40 half-years. Since they are then invested at 3% effective per half-year, they grow to $40s_{\overline{40}|.03} = 3016.05$ at the bond's redemption date. Along with the 1000 redemption value of the bond, this original 1014 investment has grown to

$4016.05(1+i)^{20} = 3.96060$, which solves for $i = 7.12\%$, Answer C.

2 - 26. For the first bond, $101.92 = 100(1+i)^{-1} + 6a_{\overline{1}|i} = 106(1+i)^{-1}$. This tell us that

$1+i = \frac{106.00}{101.92} = 1.04$. For the second bond,

$102.84 = 100(1+i)^{-1}(1+j)^{-1} + 6(1+i)^{-1} + 6(1+i)^{-1}(1+j)^{-1}$

$= 106(1.04)^{-1}(1+j)^{-1} + 6(1.04)^{-1} = 101.92(1+j)^{-1} + 5.77$, so that

$(1+j) = \frac{101.92}{102.84 - 5.77} = 1.04996$, and $j = .05$, Answer D.

2-27. Correct answer B

2-28. Correct answer is B

2-29. Correct answer B

2-30. Correct answer is A

A callable bond is more likely to be called if interest rates fall, so the borrower can refinance at a lower rate, so the reason is a correct statement. As rates increase, the bond is less likely to be called, and rather continues on to maturity, so the two types of duration converge.

2-31. The balance in Fund A at time T is $(1.01)^{12T}$, and the balance in Fund B at that time is

$\exp\left[\int_0^T \frac{t}{6}\,dt\right] = e^{T^2/12}$. Since the two fund balances are equal, we have $(1.01)^{12T} = e^{T^2/12}$, so that

$12T \cdot \ln(1.01) = \dfrac{T^2}{12}$, or $T = 144 \cdot \ln(1.01)$, Answer D.

2-32. The present value of the annual payment stream is $\frac{1}{i}$. The present value of the bi-annual payment stream is $\frac{1}{j}$, where j is the effective bi-annual rate. Since $(1+j)=(1+i)^2$, then we have $K=\frac{1}{i}+\frac{1}{(1+i)^2-1}=\frac{1}{i}+\frac{1}{2i+i^2}=\frac{2+i+1}{i(2+i)}=\frac{i+3}{i(i+2)}$, Answer A.

2-33. The theoretical price is the present value of future dividends. The future earnings stream is $J(1.10), J(1.10)^2, J(1.10)^3$, and so on. The earnings in the sixth year will be $J(1.10)^6$, of which 50% is paid as a dividend. Thus $P=.50J\left[(1.10)^6v^6+(1.10)^7v^7+\cdots\right]$, where $v=\frac{1}{1+i}=\frac{1}{1.21}$.

$$P=.50J\left[\left(\frac{1.10}{1.21}\right)^6+\left(\frac{1.10}{1.21}\right)^7+\cdots\right]$$

Thus we have $$=.50J\left[\left(\frac{1}{1.10}\right)^6+\left(\frac{1}{1.10}\right)^7+\cdots\right]$$

$$=\frac{.50J}{(1.10)^6}\left[\frac{1}{1-\frac{1}{1.10}}\right]=\frac{(.50J)(1.10)}{.10(1.10)^6}=\frac{5J}{(1.10)^5},$$ Answer B.

2-34. The price of the bond is $P = 1000a_{\overline{2|}.10} = 1735.54$. An investment of this size provides a return of 1000 at $t = 2$ at an 8% 2-year spot rate and another 1000 at $t = 1$ at a one-year spot rate of i. Thus we have

$$1735.54 = 1000(1.08)^{-2} + 1000(1+i)^{-1},$$

so

$$(1+i)^{-1} = \frac{1735.54 - 1000(1.08)^{-2}}{1000} = .87820,$$

and

$$i = (.87820)^{-1} - 1 = .13869, \text{ Answer D.}$$

2-35. 700 is the present value of an increasing annuity, so we have

$700 = 10(Ia)_{\overline{n|}}$, *or* $\dfrac{\ddot{a}_{\overline{n|}} - nv^n}{.05} = 70$. Since this equation cannot be solved explicitly for n, we must use trial and error. Eventually we find $(Ia)_{\overline{14|}} = 66.452$ and $(Ia)_{\overline{15|}} = 73.680$. Thus we have 14 regular payments and a drop payment at $t = 15$. Then

$700 = 10(Ia)_{\overline{14|}} + R \cdot v^{15} = 10(66.452) + R \cdot v^{15}$, so $R = \dfrac{700 - 664.52}{v^{15}} = (35.48)(1.05)^{15} = 73.76$,

Answer D.

Solutions to Practice Exam 3

Solutions

3-1. The monthly effective rate is $\frac{.05}{12}$, and there are 8.5 years, so 102 months in the term of the investment. The accumulated value is

$$8800\left(1+\frac{.05}{12}\right)^{102}=13,448.52\text{, Answer E.}$$

3-2. Unit payments at the end of each period for *n* periods accumulate as an immediate annuity to value $s_{\overline{n}|i}$. The closed-form expression for $s_{\overline{n}|}$ is $\frac{(1+i)^n-1}{i}$, Answer A.

3-3. After the deposit of \$1800, the amount financed is \$7200. The effective weekly rate is $j=\frac{.06}{52}$, and the equation of value is

$$\begin{aligned}7200 &= 4500\,(1+j)^{-156}+X\,(1+j)^{-208}\\ &= (4500)\,(.83536)+X\,(.78674),\end{aligned}$$

so

$$X=\frac{7200-(4500)\,(.83536)}{.78674}=4373.59\text{, Answer D.}$$

3 - 4. We are given $P = 40a_{\overline{n}|} + M \cdot v^n$, $Q = 30a_{\overline{n}|} + M \cdot v^n$, and $X = 80a_{\overline{n}|} + M \cdot v^n$, where all interest functions are at the same rate. Thus $P - Q = 10a_{\overline{n}|}$, and $X - Q = 50a_{\overline{n}|}$. Then

$a_{\overline{n}|} = \dfrac{P-Q}{10}$, so $X = Q + 50\left(\dfrac{P-Q}{10}\right) = Q + 5(P-Q) = 5P - 4Q$, Answer D.

3-5. Correct answer D

3-6. The Redington requirement regarding convexity is that $\bar{c}_A > \bar{c}_L$, so the second statement is false.

3-7. Because the contracts are short, George can lose more than his initial investment. Since market prices have risen, George will most likely have to buy the forward contracts at a higher price than he expected. Thus, the correct answer is D.

3-8. Stand-alone options do not require that the underlying asset be owned at issue. Thus, the correct answer is B.

4-16. Each semiannual coupon is 30, and the semiannual yield rate is .025. The Macauley duration is the weighted average of the payment time points, using the present value of the payment as the weight. We have

$$D = \frac{\sum_{t=1}^{20} t \cdot 30v^t + (20)(1000)v^{20}}{\sum_{t=1}^{20} 30v^t + 1000v^{20}}$$

$$= \frac{30(Ia)_{\overline{20|}} + 20,000v^{20}}{30a_{\overline{20|}} + 1000v^{20}}.$$

Here

$$(Ia)_{\overline{20|}} = \frac{\ddot{a}_{\overline{20|}} - 20v^{20}}{.025} = 150.93960,$$

$$a_{\overline{20|}} = 15.58916,$$

and

$$v^{20} = .61027,$$

$$D = \frac{(30)(150.93960) + (20,000)(.61027)}{(30)(15.58916) + (1000)(.61027)} = 15.524$$

measured in half-years, or 7.762 years, Answer D.

5-13. $R = \int_0^4 mt^3 \cdot \exp\left[-\int_0^t kr^3\, dr\right] dt = \int_0^4 mt^3 \cdot e^{-kt^4/4}\, dt = -\frac{m}{k} \cdot e^{-kt^4/4} \Big|_0^4 = \frac{m}{k}(1-e^{-64k}),$
Answer E.

5-14. The outstanding balance after 9 payments is

$$OB_9 = L(1.05)^9 - X \cdot s_{\overline{9}|.05},$$

so the principal repaid in the 10th payment is

$$PR_{10} = X - i \cdot OB_9 = X - L(.05)(1.05)^9 + X\left[(1.05)^9 - 1\right].$$

Similarly, the principal repaid in the 11th payment is

$$PR_{11} = .50X - i \cdot OB_{10} = .50X - L(.05)(1.05)^{10} + X\left[(1.05)^{10} - 1\right].$$

Note that

$$L = X \cdot a_{\overline{10}|} + .50X \cdot v^{10} \cdot a_{\overline{10}|} = .50X(a_{\overline{20}|} + a_{\overline{10}|}),$$

so we have

$$\frac{PR_{10}}{PR_{11}} = \frac{1 - .50(a_{\overline{20}|} + a_{\overline{10}|})(.05)(1.05)^9 + (1.05)^9 - 1}{.50 - .50(a_{\overline{20}|} + a_{\overline{10}|})(.05(1.05)^{10} + (1.05)^{10} - 1}$$

$$= \frac{-.50(12.46221 + 7.72173)(.05)(1.05)^9 + (1.05)^9}{-.50(12.46221 + 7.72173)(.05)(1.05)^{10} + (1.05)^{10} - .50}$$

$$= \frac{-.50(20.18394)(.05)(1.55133) + (.55133)}{-.50(20.18394)(.05)(1.62889) + 1.62889 - .50}$$

$$= \frac{.76835}{.30695} = 2.50314\text{, Answer C.}$$