ACTEX

PUBLICATIONS

The Experts In Actuarial Career Advancement

Product Preview

Preface

In this guide, interest theory is covered in Modules 1-7 and financial mathematics is covered in Modules 8-15. It is important to discuss Modules 8-15, since the syllabus has been updated to allow the use of only the third edition of the official text *Derivatives Markets*. Previously both the second and third editions were listed as official textbooks. This update should not change too much since the two editions are quite similar.

Modules 8-15 contain lecture notes on the required chapters of the financial mathematics text *Derivatives Markets* (Third Edition) and solutions to relevant odd-numbered homework problems in the text. (Answers to even-numbered problems are available in the student solution manual which you can purchase.)

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A note about Errors:
If you find a possible error in this manual, please let us know at the "Customer Feedback" link on the ACTEX homepage (www.actexmadriver.com). We will review all comments and respond to you with an answer. Any confirmed errata will be posted on the ACTEX website under the "Errata & Updates" link.

September, 2014   Matt Hassett, PhD ,Toni Garcia, MS, Amy Steeby, MBA, MEd

Study Tips

1) Come up with a schedule to complete your studying in time for the exam. Divide your schedule into time for each section and time at the end to review and to do final practice problems. This may vary depending on how much time you have before the exam. A reasonable amount may be one chapter per week.

2) If possible, join a study group of peers studying for the same exam.

3) For each chapter:
   a) Read the chapter in the FM manual.
   b) Make sure that you can compute the examples in the text correctly as you're reading through them.
   c) Recite or summarize each concept learned in the margins or in a notebook.
   d) Understand the main idea of each concept and be able to summarize in your own words. Imagine that you are trying to teach somebody else this concept.
   e) While reading, create flash cards for formulas to start memorization.
   f) Learn the calculator skills and know all of your calculator functions.
   g) Do a review of the corresponding chapter in the recommended text.
   h) Do the Basic Review Problems and review your solutions.
   i) Do the Sample Exam Problems and review your solutions.
      i) If you have been stuck on a problem for more than 20 minutes, it is OK if you need to refer to the solutions. Just make sure that when you are finished with the problem, you can recite the concept that you missed and summarize it in your own words. If you get stuck on a problem, think about what principles were used in this question and see if you could rewrite a different problem with similar structure, as if you were the exam writer.
      ii) Mark each sample exam problem as an Easy, Medium, or Hard problem.

4) After learning each chapter, it is a good idea to go back to previous chapters and do a quick half-hour to hour review, so that information isn't forgotten.

5) Go back and redo the sample exam problems that you have marked as Medium or Hard when you looked through them the first time.

6) At the end of modules 3, 7 and 15 we have included practice exams that are like midterms and will help you to take a test that integrates your knowledge.

7) After learning each chapter and reviewing past chapters and midterms, go to the practice exams.
   a) The first six practice exams are more straightforward to enable review of basics. You can attempt them in a non-timed environment and evaluate your skills.
   b) The final 5 practice exams attempt to introduce more difficult questions to parallel the exam experience. You may wish to attempt them in a timed environment.
   c) Please keep in mind that the actual exam questions are confidential and there is no guarantee that the exams you take will look exactly like the ones here.

On passing exams

How to Learn Actuarial Mathematics and Pass Exams

On the next page we will give a list of study tips for learning the material in the exam FM syllabus –and passing exam FM. First it is important to state the basic learning philosophy that we are using in this guide:

1. **You must master the basics before you proceed to the more difficult problems**

   Think about your basic calculus course. There are some very challenging applications in which you use derivatives to solve hard max-min problems.

   It is important to work on these hard problems, but if you do not have the basic skills of taking derivatives and doing algebraic simplification you cannot do the more advanced problems. Thus every calculus book has you practice derivative skills before the tougher section on applied problems.

2. **To pass exams, you must be able to answer all of the easier problems before you can answer the hard ones.**

   An exam is graded on percentage terms, and a multiple choice exam like exam FM will have a mix of problems at different difficulty levels.

   If an exam has ten problems and three are very hard, getting the right answers to only the three hard problems and missing the others gets you a score of 30%. This is actually a possibility if the very hard problems are the first problems on the exam and you try to solve them first.

   Remember that a classic exam strategy is to go through the exam and quickly solve all the more basic problems before spending extra time on the hard ones.

   One of our students is a former professional baseball player and he was advised by a friend who is a very successful exam taker –you always need to be able to hit the fastball down the middle when you are lucky enough to get it.

3. **This guide is designed to progress from simpler problems to harder ones.**

   In each chapter we start with basics and simpler examples, but progress to the point where you can attack sample exam problems by the end of the chapter.

The same philosophy is used in our sample exams at the end of the guide. The numbered practice tests are simpler as you begin and become more difficult as you progress through them.

This is very important not just in passing exams but in mastering the subject. The material in exam FM is used extensively in financial work. You need it on the job and it is important to know the basics as well as the advanced ideas.

We will leave you with two quotes from Albert Einstein:

"*Any intelligent fool can make things bigger, more complex, and more violent. It takes a touch of genius -- and a lot of courage -- to move in the opposite direction.*"

"*Everything should be made as simple as possible, but not simpler.*"

Module 1

Interest Rates and Time Value of Money

Section 1.1

Time Value of Money

Interest theory deals with the **time value of money**. For example, a dollar invested at 6% per year is worth $1.06 one year from today. What happens after the first year depends on whether you are earning **compound interest** or **simple interest**. We illustrate this with an example. Suppose you invest 100 at 6% interest for two years.

a) Compound interest. You earn interest on the total amount in your account at the beginning of each year. The amounts in your account at the end of year 1 and year 2 are:

Year 1: $100 + 0.06(100) = 100(1.06) = 106$

Year 2: $106 + 0.06(106) = 106(1.06) = 100(1.06)^2 = 112.36$

b) Simple interest. You earn interest only on the original 100 each year. The amounts in your account at the end of year 1 and year 2 are:

Year 1: $100 + 0.06(100) = 100(1.06) = 106$

Year 2: $106 + 0.06(100) = 100 * (1 + 2(0.06)) = 112$

Compound interest is the most widely used method, especially for multi-period investments. Simple interest is more commonly used for shorter term investments. There are other ways to calculate interest, and we will see some of these later.

Section 1.2

Present Value and Future Value with Compound Interest

We will start with a look at compound interest, since it is so widely used, The value today is the **present value** [PV] and the value n periods from today is the **future value** [FV]. If funds are invested at a periodic compound interest rate i for n periods, the basic relationships are:

(1.1)

$$FV = PV(1+i)^n \qquad\qquad PV = \frac{FV}{(1+i)^n}$$

Example (1.2)

Let $n = 10$ and $i = 0.06$.

a) If PV = 1,000, then $FV = 1,000(1.06)^{10} = 1,790.85$

b) If FV = 1,000, then $PV = \dfrac{1,000}{(1.06)^{10}} = 558.39$

Calculator Note

The BA II Plus calculator has five time value of money keys.

PV	Present value
FV	Future value
N	Number of periods
PMT	Periodic payment
I/Y	Periodic interest rate

In this module we will not look at any problems that involve periodic payments. The PMT key will be used starting in Module 2. The other four keys can be used to solve compound interest problems like Example (1.2), as we illustrate next.

To begin any new problem, it is wise to clear the time value of money [TVM] registers to erase any numbers left over from prior problems. Note that the legend CLR TVM appears above the FV key on the BA II Plus calculator. To clear the TVM registers use the keystrokes 2ND CLR TVM.

Before we do the actual calculation, we must point out an important BA II Plus convention for signs on answers: **money that you receive is positive, but money that you pay out is negative.** Thus, if you put 1000 into an account now, you should enter it into the calculator as –1000 to indicate that it is out of pocket. You can make an entry negative using the $\boxed{+/-}$ key.

Now, let's rework Example (1.2) using the calculator:

To find the future value of 1000 in 10 years at 6% compound interest per year, use the keystrokes

> 1000 $\boxed{+\text{-}}$ $\boxed{\text{PV}}$ 6 $\boxed{\text{I/Y}}$ 10 $\boxed{\text{N}}$ $\boxed{\text{CPT}}$ $\boxed{\text{FV}}$.

You will see the display FV = 1790.85. **Note that the answer is positive since this is money that is paid back to you.**

To find the present value at 6% compound interest of 1000 paid 10 years in the future use the keystrokes

> 1000 $\boxed{\text{FV}}$ 6 $\boxed{\text{I/Y}}$ 10 $\boxed{\text{N}}$ $\boxed{\text{CPT}}$ $\boxed{\text{PV}}$.

You will see the display PV = −558.39. Note that the answer is negative for money that you put into the account.

Exercise (1.3)

Using an interest rate of 5% compounded annually, find a) the present value of 20,000 payable in 15 years and b) the future value of 5,000 in 6 years.

Answer a) -9620.34 b) 6700.48

Section 1.3

Functions of Investment Growth Notation

A long-term investor might wish to plot growth of an invested amount over time. There are two functions of interest:

> $a(t)$, which is the amount an initial investment of 1 grows to by time t,
> - and-
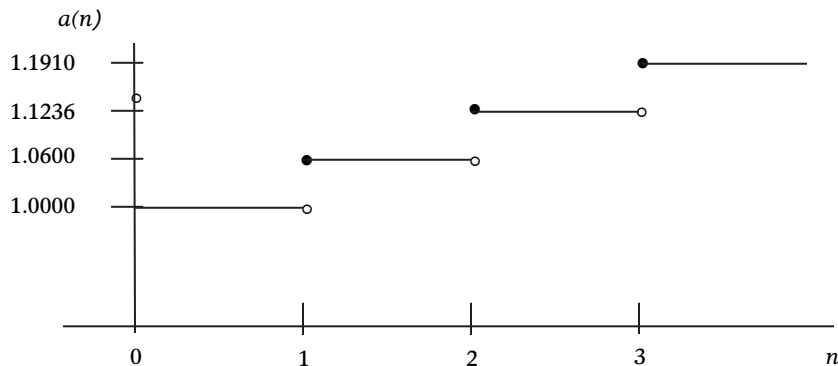> $A(t)$, the amount an initial investment of A(0) grows to by time t.

For compound interest applied on a per year basis, amounts change only at year end when interest is paid. For positive integer values of n,

(1.4) $$a(n) = (1+i)^n \qquad A(n) = A(0)(1+i)^n$$

For i = 0.06 compound interest per year paid at year end, the first four values of a(n) are:

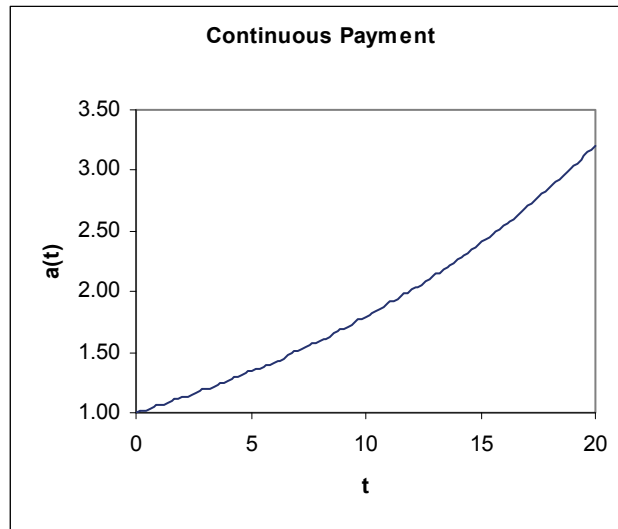| N	0	1	2	3
$a(n)$	1	1.060	1.1236	1.1910

The graph of a(n) is a step function:

Interest may also be paid on a continuous basis, which is an advantage for the investor who wishes to get his money before year-end. The accumulated amount under continuous compound interest at time t is:

(1.5) $$a(t) = (1+i)^t = e^{t\ln(1+i)} \qquad A(t) = A(0)(1+i)^t$$

In this case, the graph of a(t) is a smooth, continuous function. For i = 0.06:

Continuous Payment

We will look at continuous interest in more detail later.

For simple interest the accumulation functions are

(1.6) $a(t) = (1 + it)$ $A(t) = A(0)(1 + it)$

For i = 0.06 simple interest per year paid at year end, the first four values of $a(n)$ are:

N	0	1	2	3
a(n)	1	1.06	1.12	1.18

Section 1.4

Effective Rate of Interest for a Specified Period

We can use the accumulation functions to find an effective rate of interest for any time period. For the time period $[t, t+1]$, the beginning amount is $A(t)$, and the amount earned over the interval is $A(t+1) - A(t)$. The effective rate of interest over this period is

(1.7)

$$i_{t+1} = \frac{\text{amount earned}}{\text{beginning amount}} = \frac{A(t+1) - A(t)}{A(t)} = \frac{a(t+1) - a(t)}{a(t)}$$

Example (1.8)

Let the interest rate be 6% and the time interval be [1,2].

For compound interest

$$i_2 = \frac{a(2) - a(1)}{a(1)} = \frac{.0636}{1.06} = .06$$

For simple interest

$$i_2 = \frac{a(2) - a(1)}{a(1)} = \frac{.06}{1.06} = .0566$$

Exercise (1.9)

Let the interest rate be 6% and the time interval be [2,3]. Find i_3 for a) compound interest and b) simple interest.

Answer a) .06 b) .0536

Note that over multi-year periods compound interest gives a constant effective rate of 6% over each year, while simple interest leads to declining effective rates over time.

Section 1.5

Nominal Rates of Interest

In many instances where payments are made for a period less than a year (e.g., monthly, quarterly, or semi-annually), the period interest rate is stated as a **nominal annual rate**, which is the interest rate per period multiplied by the number of periods per year.

For example, if you are to earn interest at 2% compounded quarterly, you could multiply 2% by 4 and refer to a nominal rate of 8%, converted quarterly. This gives a simple way of referring to the quarterly rate on an annual scale, but it is not the rate you actually earn. In the example of a 2% rate compounded quarterly, one actually earns more than 8%. One dollar actually accumulates to $(1.02)^4 = 1.0824$, so that the nominal rate of 8% actually leads to a true annual earning rate of 8.24%. This true annual earning rate is referred to as the **effective rate**.

Many students find this confusing, so we will go over again for reinforcement:

1.	**The given rate is your starting point**
	<u>Example:</u> 2% per quarter.
2.	**Calculate the annual nominal rate.**
	<u>Nominal Rate</u> (Rate/period)(Number of periods per year)
	<u>Example</u> $2\% \times 4 = 8\%$
3.	**The effective rate is what you really earn with compound interest**
	<u>Example.</u> Compound accumulation is $(1.02)^4 = 1.0824$
	Effective rate is 8.24%.

The nominal rate is an artificial rate that gives you a way of talking about a periodic rate (such as a quarterly or monthly) in familiar annual terms. The effective rate is not artificial. It tells you what you really earn with compounding in a year.

Exercise (1.10)

Suppose you are earning 1% interest compounded monthly.
        a) What is your annual nominal rate?
        b) What is your annual effective rate?

a) Nominal 12% b) Effective 12.6825%

In the general case for which there are m payment periods per year, we denote the nominal rate by $i^{(m)}$. The periodic interest rate is $\dfrac{i^{(m)}}{m}$, and the effective rate is:

(1.11)

$$\left(1 + \frac{i^{(m)}}{m}\right)^{m} - 1$$

This has the important consequence that
(1.12)

$$1 + i = \left(1 + \frac{i^{(m)}}{m}\right)^{m}$$

You will often see the terminology the interest is **credited** or **convertible m times per year.**

Example (1.13)

Suppose interest is credited monthly and the nominal rate is $i^{(12)} = 0.09$. Then, the effective rate is:

$$\left(1 + \frac{0.09}{12}\right)^{12} - 1 = 1.0075^{12} - 1 = 0.0938.$$

This process can easily be reversed to find the nominal rate given the effective rate

Example (1.14)

Interest is paid semi-annually and results in an effective rate of 10.25%. Find the nominal annual rate.

Solution:
$m = 2$, so we need to find $i^{(2)}$.
By **(1.11)**,

$$\left(1 + \frac{i^{(2)}}{2}\right)^{2} - 1 = 0.1025$$

$$\left(1 + \frac{i^{(2)}}{2}\right)^{2} = 1.1025$$

$$\left(1 + \frac{i^{(2)}}{2}\right) = \sqrt{1.1025} = 1.05$$

Thus, $i^{(2)}$= 10%, and the periodic interest rate is 5% per semi-annual period.

Note that you can derive a formula that solves for $i^{(m)}$ given i and m. It is

$$i^{(m)} = m\left[(1+i)^{\frac{1}{m}} - 1\right].$$

We did not use this formula above. Formula **(1.11)** is intuitive and easy to remember, and we can always substitute given values into it to solve for $i^{(m)}$ given i and m. This approach is what we used in Example **(1.14)**, and leaves us with one less formula to memorize.

Calculator Note

The BA II Plus calculator has an interest conversion worksheet that can be used to solve these problems. The legend above the 2 key is ICONV, which stands for interest conversion. You can get into the worksheet using the keystrokes. 2ND ICONV. The worksheet has three variables:

NOM for nominal rate
EFF for effective rate
C/Y for number of conversion periods per year.

You can scroll between these variables using the ↑ and ↓ keys.

In **(1.13)** we found the effective rate corresponding to a nominal rate of 9% credited monthly. To do this on the BA II Plus calculator, enter the ICONV worksheet and scroll to the line for NOM. Key in 9 and hit the Enter key. Then scroll ↑ to the line for C/Y and key in 12 and hit the Enter key. Then scroll ↑ to the line for EFF and use the CPT key to compute the effective rate. The rate displayed is EFF = 9.38 (to two decimal places).
In **(1.14)** we found the nominal rate corresponding to an effective rate of 10.25% convertible semiannually. To do this on the BA II Plus calculator, enter the ICONV worksheet and scroll to the line for EFF. Key in 10.25 and hit the Enter key. Then scroll ↓ to the line for C/Y and key in 2 and hit the Enter key. Then scroll ↓ to the line for NOM and use the CPT key to compute the effective rate. The rate displayed is NOM = 10.00.

To exit the ICONV worksheet, press the C/E key. This will also allow you to exit any other BAII Plus worksheet.

Exercise (1.15)

a)   Given $i^{(12)} = 6\%$, find the effective rate i.
b)   Given an effective rate of $i = 5\%$, find $i^{(12)}$.

Answers: a) 6.168%   b) 4.889%

Section 1.6

Interest Rate v. Discount Rate

Investments can be structured in many ways. Consider an investor who would like to earn 6% for one year. Two common approaches are:

a) <u>Invest a given sum at the beginning of the year</u>. If you invest $1,000 at the beginning of the year at 6% per year, you would require a payment of $1,060 at year end.

b) <u>Require a given sum at the end of the year, but take a **discount** on the amount invested</u>. Suppose that you require $1,000 at year end. The present value of $1,000 at 6% is $\dfrac{\$1,000}{1.06} = \$943.40$.

You would invest $943.40 and be repaid $1,000. The difference of $56.60 is referred to as a **discount**. This is really only a present value problem, but the discount is quoted instead of the present value. United States Treasury bills are quoted on a discount basis.

The **rate of discount**, d, is used extensively in interest theory and actuarial mathematics. We can easily derive an expression for d in terms of i. If you wish to obtain a future value of 1, the present value to invest is:

$$PV = \frac{1}{(1+i)}$$

Thus, the rate of discount d is:

(1.16)
$$\boxed{d = 1 - \frac{1}{(1+i)} = \frac{i}{(1+i)}}$$

This yields the key relationship:

(1.17)
$$\boxed{d = \frac{i}{(1+i)}}$$

Example (1.18)

For $i = 0.06$, $d = \dfrac{0.06}{1.06} = 0.0566$

Exercise (1.19)

Given $i = .10$, find d.

Answer : 0.0909

Section 1.7

Essential Interest Theory Notation

A critical notation for actuarial interest problems is:

(1.20)
$$v = \frac{1}{1+i}$$

From the definition of d in **(1.17)**, we see:

(1.21)
$$d = iv$$

Note also that $1 - v = 1 - \dfrac{1}{1+i} = \dfrac{i}{1+i} = d$. Thus

(1.22)
$$d = 1 - v \text{ and } 1 - d = v$$

The difference $i - d$ simplifies nicely:

$$i - d = i - \frac{i}{(1+i)} = \frac{i^2}{(1+i)} = id$$

(1.23)
$$i - d = id$$

The preceding relationships are often used in actuarial examination solutions.

Example (1.24)

> Given $d = 0.07$, find v and i.
>
> *Solution.*
>
> $v = 1 - d = 0.93$. Then $1 + i = \dfrac{1}{v} = 1.0753$. It follows that $i = 0.0753$.

Exercise (1.25)

> Given $d = 0.05$, find v and i.
>
> Answer $v = 0.95$, $i = 0.0526$:

Note that we can now write

$$PV = \frac{FV}{(1+i)^n} = v^n \, FV$$

The use of the v notation is common in actuarial texts and essential for the actuarial exams. Many other financial professionals do not use it.

Section 1.8

Nominal Rates of Discount

You can also quote a discount rate per period as a nominal annual rate. If you were using a discount rate of 2% per quarter, you could refer to this as a nominal discount rate of 8% convertible quarterly. The effective annual rate of discount would not be 8%, as we shall see below.

The nominal discount rate convertible m-thly is denoted by $d^{(m)}$. For example, a nominal discount rate convertible quarterly would be denoted $d^{(4)}$. It is related to the effective annual discount rate d by the equation

(1.26)

$$\underbrace{1-d}_{v} = \underbrace{\left(1 - \frac{d^{(m)}}{m}\right)^{m}}_{\substack{v \text{ for a} \\ m^{th}\text{-ly period} \\ \text{raised to } m^{th} \text{ power}}}$$

This equation can be remembered by noting that the left side represents v and the right side represents the v for an m-thly period raised to the m-th power.

Example (1.27)

Find the effective annual discount rate for a nominal rate of 8% convertible quarterly.

Solution.

$$1-d = \left(1 - \frac{0.08}{4}\right)^{4} = (0.98)^{4} = 0.9224 \qquad d = 0.0776$$

Example (1.28)

Find the nominal discount rate convertible semiannually corresponding to an annual effective discount rate of 6%.

Solution.

$$1 - 0.06 = 0.94 = \left(1 - \frac{d^{(2)}}{2}\right)^{2}$$

$$\sqrt{0.94} = 0.9695 = \left(1 - \frac{d^{(2)}}{2}\right)$$

$$d^{(2)} = 0.0609$$

Calculator Note

If you use the ICONV worksheet with either EFF or NOM entered as a negative number, the BA II Plus will interpret the negative number as a discount rate and solve for the other discount rate as a negative.

For example, in **(1.27)** we found the effective rate discount rate corresponding to a nominal discount rate of 8% credited quarterly. To do this on the BA II Plus calculator, enter the ICONV worksheet and scroll to the line for NOM. Key in 8 +l- and hit the Enter key. Then scroll ↑ to the line for C/Y and key in 4 and hit the Enter key. Then scroll ↑ to the line for EFF and use the CPT key to compute the effective rate. The rate displayed is EFF = -7.76 (to two decimal places). That is the correct discount rate for **(1.27)**.

You can clear out the computed values in the worksheet by keying in 2ND CLR WORK (above the CE/C key.) The value of C/Y will remain but will be changed as soon as you enter a new value for it.

Exercise (1.29)

Find
  a)   The effective discount rate for a nominal rate of 7.5% convertible every 4 months (m=3), and

  b)   the nominal discount rate convertible monthly corresponding to an annual effective discount rate of 6%.

Answer a) 7.31% b) 6.17%

Occasional problems require conversion of a nominal interest rate convertible m times per year to an equivalent nominal discount rate convertible p times per year. The equation for this problem is

$$\underbrace{\left(1+\frac{i^{(m)}}{m}\right)^{m}}_{1+i} = \underbrace{\left(1-\frac{d^{(p)}}{p}\right)^{-p}}_{\frac{1}{v}=1+i}$$

In this equation the left hand side represents $1+i$ and the right hand side represents $\dfrac{1}{v}=1+i$.

Example (1.30)

Find the rate of discount convertible semi-annually that is equivalent to a nominal rate of interest of 8% convertible monthly.

Solution.

$$\left(1 + \frac{0.08}{12}\right)^{12} = 1.083 = \left(1 - \frac{d^{(2)}}{2}\right)^{-2}$$

$$\sqrt{1.083} = 1.0407 = \left(1 - \frac{d^{(2)}}{2}\right)^{-1}$$

$$d^{(2)} = 0.0782$$

Section 1.9

Continuous Interest and Force of Interest

We have already noted that interest may also be paid on a continuous basis. The accumulated amount under continuous compound interest at time t is:

(1.31)

$$a(t) = (1+i)^t = e^{t\ln(1+i)} \qquad A(t) = A(0)(1+i)^t$$

For example, if interest is paid continuously at 6%, we have

$$a(t) = (1.06)^t = e^{t\ln(1.06)}$$

There is an important distinction here. Under continuous interest at 6% at time 2, $a(2) = 1.06^2$, which is the same amount you would have under annual interest. However, there is difference at fractional times like $t = 1.5$. For continuous interest, you would have $1.06^{1.5}$ at $t = 1.5$, but for annual interest, you still only have 1.06 at $t = 1.5$, because interest is not compounded until $t = 2$.

The accumulation function above implies continuous growth at a continuously compounded rate of ln(1.06) = 0.0583. The constant continuous growth rate is denoted by δ. In general, for the compound growth model $a(t) = (1+i)^t$, we can write

(1.32)

$$a(t) = e^{\delta t} \qquad \delta = \ln(1+i)$$

The next two formulas are very important variations of **(1.32)**

(1.33)

$$(1+i)^n = e^{n\delta}$$

(1.34)

$$v^n = (1+i)^{-n} = e^{-n\delta}$$

In actuarial textbooks, δ is referred to as a **constant force of interest**. Continuous interest has been applied in practice – some banks paid continuous interest in the 1980's.

We can define the instantaneous **force of interest** for any accumulation function *a(t)* at time *t* by

(1.35)

$$\delta(t) = \frac{a'(t)}{a(t)}$$

This has a natural interpretation. The percentage rate at which you are earning at time t is the rate of change of $a(t)$ expressed as a percentage of $a(t)$.

For the constant force model, $a(t) = e^{\delta t}$, this definition yields:

$$\delta(t) = \frac{\delta e^{\delta t}}{e^{\delta t}} = \delta$$

Thus definition **(1.35)** is consistent with our original reference to δ as the constant force of interest.

The force of interest does not have to be constant as the following example shows:

Example (1.36)

> Let $a(t) = (t+1)^2$.
>
> (*Note: This is an unrealistic accumulation function, but easy to compute.*)
>
> Then $\quad \delta(t) = \dfrac{2(t+1)}{(t+1)^2} = \dfrac{2}{t+1}$.

Note that as n, the number of compounding periods per year becomes large, the resulting interest paid approximates continuous interest with a constant force of interest equal to the nominal rate.

For example, if we have a constant force of interest, δ = 8%, we have an effective annual rate of $(e^{0.08} - 1) = 8.32871\%$. The table below shows the effective rate for a nominal rate of 8% with increasing numbers of compounding periods per year.

Nominal Rate:	8%	
Conversion	n	**Effective Rate**
Semi-annual	2	8.16000%
Quarterly	4	8.24322%
Monthly	12	8.29995%
Daily	365	8.32776%
Hourly	8,760	8.32867%
Per Minute	525,600	8.32871%

Note the result of compounding every minute matches the continuous effective rate to five decimal places.

This approximation is based on the useful identity **(1.37)** below. which you should memorize.

(1.37)
$$\lim_{n \to \infty}\left(1 + \frac{r}{n}\right)^n = e^r$$

There is another useful relationship which enables us to find $a(t)$ if only $\delta(t)$ is given. Note that $\dfrac{d}{dt}\ln[a(t)] = \dfrac{a'(t)}{a(t)} = \delta(t)$.

Thus, $\displaystyle\int_0^k \delta(t)dt = \ln[a(t)]\Big|_0^k = \ln[a(k)] - \ln(1) = \ln[a(k)]$

This implies that:
(1.38)
$$e^{\int_0^t \delta(u)du} = a(t)$$

Example (1.39)

Given $\delta(t) = \dfrac{2}{(t+1)}$. Find $a(t)$.

Solution.
To find $a(t)$, we first need to integrate $\delta(t)$:
$$\int_0^t \delta(u)du = \int_0^t \frac{2}{(u+1)}du = 2\ln(u+1)\Big|_0^t = 2\ln(t+1).$$

Thus,

$$a(t) = e^{2\ln(t+1)} = (e^{\ln(t+1)})^2 = (t+1)^2$$

Note:
If your calculus is rusty, you may need to review to do these problems.

Exercise (1.40)

Given $\delta(t) = \dfrac{6}{(2t+1)}$. Find $a(t)$.

Answer: $e^{3\ln(2t+1)} = (2t+1)^3$

Section 1.10

Relating Discount, Force of Interest and Interest Rate

A very important relationship is:

(1.41)
$$\boxed{d < d^{(m)} < \delta < i^{(m)} < i \,,\, i > 0, m > 1}$$

This relationship is good to know for plausibility checking of results. It is not hard to see that $d < \delta < i$, since for $i > 0$

$$\frac{i}{1+i} < \ln(1+i) < i$$

For a concrete example, let $i = 0.05$ and $m = 4$. Then

$$\delta = \ln(1.05) = 0.0488 \qquad d = \frac{.05}{1.05} = 0.0476$$

$$i^{(4)} = 0.049089 \qquad\qquad d^{(4)} = 0.048494$$

Section 1.11

Solving for PV, FV, *n*, and *i* with Compound Interest

Time value of money calculations can be done easily with the BAII Plus. This section will show you how to solve for PV, FV, *n,* and i,. First, we will solve the problems without the calculator to establish the logic, and then show you how to use the calculator's time value of money [TVM] keys to save time.

Example (1.42)

You want to have 80,000 in a college fund in 18 years. How much should you deposit now into an account earning 6%?

Solution.
You need to have FV = 80,000. Thus

$$PV = 80,000\,v^{18} = \frac{80,000}{1.06^{18}} = 28,027.50$$

For the BA II Plus the keystrokes are

80000 FV 6 I/Y 18 N CPT PV

The equation used in the last example is called an **equation of value.** It equates the unknown PV with a mathematical expression for it. In each of the next problems we will refer to the appropriate equation of value in order to solve.

Exercise (1.43)

How much should you deposit in the fund described in **(1.42)** if you wanted 100,000 in 16 years?

Answer 39,364.63

Example (1.44)

You deposit 1,000 in an account earning 5.75%. How much will you have in 5 years?

Solution.
The equation of value is
$$FV = 1000(1.0575)^{5} = 1,322.52$$

For the BA II Plus the keystrokes are

1000 +|- PV 5.75 I/Y 5 N CPT FV

Exercise (1.45)

How much will be in the account in the last problem in 10 years?

Answer 1,749.06

Example (1.46)

You deposit 1000 in an account earning a force of interest of 6%. How long will it take to double your money?

Solution.
Doubling your money gives FV = 2000. The equation of value is

$$2000 = 1000e^{.06t}$$

It follows that

$$2 = e^{.06t}$$

$$\ln(2) = 0.06t$$

$$t = 11.5525$$

Exercise (1.47)

For the account in Example **(1.46)** how long would it take to triple your money?

Answer 18.3102

Now, let's look at a variation on the preceding example that requires careful thinking:

Example (1.48)

You deposit 1000 in an account earning 6% compounded annually. How long will it take to have at least 2000 dollars?

Solution.
In this case interest is only paid at year end. Since 2000 would be reached exactly with continuous interest in 11.8957 years, you will have less than 2000 at the end of 11 years and more at the end of the 12th year. The answer here is 12 years.

Example (1.49)

> You make an investment where you pay 1000 now and get 1500 back
> in 5 years. What interest rate did you earn?
>
> *Solution.*
> The equation of value is
> $$1000(1+i)^5 = 1500$$
> Thus
> $$(1+i)^5 = 1.5$$
> $$5\ln(1+i) = \ln(1.5)$$
> $$\ln(1+i) = .0811$$
> $$1+i = e^{.0811} = 1.0845 \ \rightarrow \ i = .0845$$
>
> For the BA II Plus the keystrokes are
> $$1000 \ \boxed{+|-} \ \boxed{PV} \ \ 1500 \ \boxed{FV} \ 5 \ \boxed{N} \ \ \boxed{CPT} \ \ \boxed{I/Y}$$
>
> The calculator saves a bit of time here.

Exercise (1.50)

> You make an investment where you pay 1000 now and get 2000 back in
> 12 years. What interest rate did you earn?
>
> <div align="right">Answer 5.9463</div>

The problems can be made more complex, as you will see when you move to the
exam problems at the end of this module. One way to make a problem a bit
more complex is to state it using a nominal interest rate.

Example (1.51)

> You deposit 1,000 in an account earning 5.75% convertible
> semiannually. How much will you have in 5 years?
>
> *Solution.*
> Now we have a interest rate of $\dfrac{0.0575}{2} = 0.02875$ per semiannual
>
> period for $2 \times 5 = 10$ periods. The equation of value is
> $$FV = 1000(1.02875)^{10} = 1327.70$$
> The BA II Plus keystrokes are
> $$1000 \ \boxed{+|-} \ \boxed{PV} \ \ 2.875 \ \boxed{I/Y} \ \ 10 \ \boxed{N} \boxed{CPT} \ \ \boxed{FV}$$
>
> The BA II Plus also has an option under which you can set the number
> of payments per year to 2 using the P/Y option. **We advise against this**
> since it is more complicated to use and it is easy to forget to reset the
> number of payments per year which can cause trouble on later
> problems.

Example (1.52)

You make an investment where you pay 1000 now and get 1500 back in 5 years. What nominal interest convertible quarterly did you earn?

Solution.
Here we will give only the calculator solution. The interest is paid over 20 quarters.

To find the quarterly interest rate, the BA II Plus the keystrokes are

1000 +l- PV   1500 FV 20 N   CPT   I/Y

The quarterly rate is 2.048%. The required nominal rate is $4 \times 2.048\% = 8.192\%$ convertible quarterly.

Other problems may have a few different future amounts or an unknown amount at some point. We see this in the next two examples.

Example (1.53)

How much should you deposit now in a bank account earning 5% annually to be able to withdraw 1000 in 2 years and 2000 in 4 years?

Solution.
The equation of value is

$$PV = 1000v^2 + 2000v^4 = \frac{1000}{1.05^2} + \frac{2000}{1.05^4} = 2552.43$$

Example (1.54)

You deposit 1000 in an account now and an amount X in one year. The account pays 6% annually. What amount X is required to have 2000 in the account at the end of two years?

Solution.
The equation of value is

$$1000(1.06)^2 + X(1.06) = 2000 \quad \rightarrow \quad X = 826.79$$

Another type of problem that requires more thought is one in which the interest rates change over time.

Example (1.55)

You deposit 5000 to an account that earns 5% compounded annually for two years and 7% in all subsequent years. What has the account grown to in 5 years?

Solution.

$$FV = 5000(1.05)^2 (1.07)^3 = 6753.05$$

You could easily do the problem on the BA II Plus in two steps.

Amount in 2 years 5000 $\boxed{+/-}$ $\boxed{PV}$ 5 $\boxed{I/Y}$ 2 $\boxed{N}$ $\boxed{CPT}$ $\boxed{FV}$
(Answer 5512.50)

Amount in 5 years 5512.5 $\boxed{+/-}$ $\boxed{PV}$ 7 $\boxed{I/Y}$ 3 $\boxed{N}$ $\boxed{CPT}$ $\boxed{FV}$
(Answer 6753.05)

Example (1.56)

What constant rate of interest is equivalent to the 5 year return above?

Solution.

The BA II Plus can be used to get this quickly. We accumulated FV = 6753.05 in 5 years from an initial investment of 5000. Solve for the interest rate using

5000 $\boxed{+/-}$ $\boxed{PV}$ 6753.05 $\boxed{FV}$ 5 $\boxed{N}$ $\boxed{CPT}$ $\boxed{I/Y}$ (Answer 6.20%)

To solve mathematically, denote the unknown interest rate by i.

$$(1+i)^5 = 1.05^2 (1.07^3) = 1.3506$$
$$5\ln(1+i) = \ln(1.3506)$$
$$\ln(1+i) = .0601$$
$$1+i = e^{.0601} = 1.062 \rightarrow i = .062$$

Example (1.57)

You deposit 5000 to an account that earns 5% compounded annually for two years and continuous interest with $\delta(t) = \dfrac{2}{(t+1)}$ in subsequent years. What has the account grown to in 5 years?

Solution.
At the end of two years the account contains 5512.50. [See (**1.55**), above]. Note that the force of interest must be applied from time 2 through time 5. The equation of value for the final amount in 5 years is

$$5512.50 \left(e^{\int_{2}^{5} \frac{2}{t+1} dt} \right)$$

Note the limits on the integral. A common mistake is to integrate from 0 to 3.

We now calculate

$$\int_{2}^{5} \frac{2}{t+1} dt = 2\ln(t+1)\Big|_{2}^{5} = 2\big[\ln(6) - \ln(3)\big]$$

$$e^{\int_{2}^{5} \frac{2}{t+1} dt} = e^{2\ln(6) - 2\ln(3)} = 4$$

The final answer is

$$5512.50 \left(e^{\int_{2}^{5} \frac{2}{t+1} dt} \right) = 5512.50(4) = 22{,}050$$

Section 1.12

Formula Sheet

$$FV = PV(1+i)^n \qquad\qquad PV = \frac{FV}{(1+i)^n}$$

$$d = \frac{i}{(1+i)} \qquad\qquad v = \frac{1}{1+i} \qquad\qquad v = 1-d \qquad\qquad d = 1-v$$

$$i - d = id \qquad\qquad d = iv$$

a(t): the amount an initial investment of 1 grows to by time t
A(t): the amount an initial investment of $A(0)$ grows to by time t

$$a(t) = (1+i)^t = e^{t\ln(1+i)} \qquad\qquad A(t) = A(0)(1+i)^t = A(0)e^{t\ln(1+i)}$$

$$\delta = \ln(1+i) \qquad\qquad a(t) = e^{\delta t} \qquad\qquad v^n = (1+i)^{-n} = e^{-n\delta}$$

$$\delta(t) = \frac{a'(t)}{a(t)} \qquad\qquad e^{\int_0^t \delta(u)\,du} = a(t)$$

Effective interest rate with nominal rate $i^{(m)}$ convertible m-thly.

$$\left(1+\frac{i^{(m)}}{m}\right)^m - 1$$

Effective discount rate d with nominal rate $d^{(m)}$ convertible m-thly.

$$1 - d = \left(1 - \frac{d^{(m)}}{m}\right)^m$$

Nominal rate equivalence

$$\left(1+\frac{i^{(m)}}{m}\right)^m = \left(1 - \frac{d^{(p)}}{p}\right)^{-p}$$

Note the negative exponent, $-p$, above.

Section 1.13

Basic Review Problems

1. Let the annual interest rate be 5% and the time interval be [3,4]. Find i_4 for (a) annual compound interest and (b) simple interest.

2. (a)   Given $i^{(2)} = 5\%$, find the effective rate i.
   (b)   Given an effective rate of $i = 5.26\%$, find $i^{(6)}$.

3. Given $d = 0.056$, find v and i.

4. Given $i^{(4)} = 0.07$. Find $d^{(2)}$.

5. Find the effective annual discount rate for a nominal discount rate of 9% convertible monthly.

6. Find the rate of interest convertible quarterly that is equivalent to a nominal rate of interest of 6% convertible semiannually.

7. Let $a(t) = (t+1)^3$. Find $\delta(t)$.

8. Given $\delta(t) = \dfrac{4}{(t+3)}$. Find $a(t)$.

9. You deposit 1800 in an account earning a force of interest of 5%. How long will it take to accumulate to 2,700?

10. You make an investment where you pay 10,500 now and get 12,500 back in 3 years. What nominal interest convertible monthly did you earn?

11. You deposit 1,500 to an account that earns a nominal 6% convertible monthly for one year and a nominal 8% convertible quarterly for the next two years. a) How much is in the account in 3 years? b) Find an equivalent level nominal rate convertible semiannually for this account.

Section 1.14

Basic Review Problem Solutions

Calculator solutions will be given whenever possible.

1. (a) $a(t) = 1.05^t$. $i_4 = \dfrac{a(4) - a(3)}{a(3)} = \dfrac{1.05^4 - 1.05^3}{1.05^3} = 0.05$.

   Note that for compound interest the periodic rate is the always the effective rate.

   (b) $a(t) = 1 + 0.05t$. $i_4 = \dfrac{a(4) - a(3)}{a(3)} = \dfrac{1.20 - 1.15}{1.15} = 0.0435$.

2. Calculator solutions using the ICONV feature:
   (a)   Set NOM = 5 and C/Y = 2. CPT EFF = 5.0625
   (b)   Set EFF = 5.26 and C/Y = 6. CPT NOM = 5.1483

3. $v = 1 - d = 0.944$, $\dfrac{1}{v} = 1 + i = 1.0593 \rightarrow i = 0.0593$

4. $\left(1 + \dfrac{0.07}{4}\right)^4 = 1.07186 = \left(1 - \dfrac{d^{(2)}}{2}\right)^{-2} \rightarrow 0.9659 = \left(1 - \dfrac{d^{(2)}}{2}\right) \rightarrow d^{(2)} = 0.0682$

5. We are trying to solve

   $$1 - d = \left[1 - \dfrac{0.09}{12}\right]^{12}$$

   but we think it is easier to use the calculator's ICONV feature. Set NOM $= -9$ and C/Y = 12. CPT EFF $= -8.6379$. Answer 8.6379%.

6. We want to solve

   $$(1 + i) = \left(1 + \dfrac{i^{(4)}}{4}\right)^4 = \left(1 + \dfrac{0.06}{2}\right)^2.$$

   You can do this by hand, but we think it is easier to use the calculator in steps:

   First find the annual effective rate using the given nominal semiannual rate. Set NOM = 6, C/Y = 2 and CPT EFF = 6.09. Then use this effective rate to find the quarterly nominal rate. You already have EFF = 6.09. Set C/Y = 4 and CPT NOM = 5.9557. Answer 5.9557%.

7. $\delta(t) = \dfrac{a'(t)}{a(t)} = \dfrac{3(t+1)^2}{(t+1)^3} = \dfrac{3}{t+1}$

8. $\int_0^t \delta(u)du = \int_0^t \dfrac{4}{(u+3)}du = 4\ln(u+3)\Big|_0^t = 4\ln\left(\dfrac{t+3}{3}\right).$

$a(t) = e^{4\ln((t+3)/3)} = \left(\dfrac{t+3}{3}\right)^4$

9. $A(t) = 1800e^{.05t}$. We need

$2700 = 1800e^{.05t} \to e^{.05t} = 1.5 \to .05t = \ln(1.5)$, thus

$t = \dfrac{\ln(1.5)}{0.05} = 8.1093$

10. First, we need to get the annual effective rate, then we can use this to solve for the nominal rate. Formulaic version:

$$(1+i) = \left(1 + \dfrac{i^{(m)}}{m}\right)^m$$

Using the calculator: 10500 +|- PV 12500 FV 3 N CPT I/Y
Answer 5.984

Now use ICONV to get the nominal rate convertible monthly. Set EFF = 5.984 , C/Y = 12 and CPT NOM = 5.826.

11. (a) $1500\left(1 + \dfrac{0.06}{12}\right)^{12}\left(1 + \dfrac{0.08}{4}\right)^{4(2)} = 1,865.89$

(b)   We are trying to find $i^{(2)}$.

$$\left(1 + \dfrac{0.06}{12}\right)^{12}\left(1 + \dfrac{0.08}{4}\right)^{4(2)} = 1.2439 = \left(1 + \dfrac{i^{(2)}}{2}\right)^6$$

$$1.2439^{1/6} = 1.037 = \left(1 + \dfrac{i^{(2)}}{2}\right)$$

$$i^{(2)} = 0.074$$

Section 1.15

Sample Exam Problems

1.   (2005 Exam FM Sample Questions #1)

Bruce deposits 100 into a bank account. His account is credited interest at a nominal rate of interest of 4% convertible semiannually.

At the same time, Peter deposits 100 into a separate account. Peter's account is credited interest at a force of interest of δ.

After 7.25 years, the value of each account is the same. Calculate δ.

(A) 0.0388   (B) 0.0392   (C) 0.0396   (D) 0.0404   (E) 0.0414

2.   (2005 Exam FM Sample Questions #3)

Eric deposits 100 into a savings account at time 0, which pays interest at a nominal rate of i, compounded semiannually.

Mike deposits 200 into a different savings account at time 0, which pays simple interest at an annual rate of i.

Eric and Mike earn the same amount of interest during the last 6 months of the 8th year. Calculate i.

(A) 9.06%   (B) 9.26%   (C) 9.46%   (D) 9.66%   (E) 9.86%

3.   (2005 Exam FM Sample Questions #12)

Jeff deposits 10 into a fund today and 20 fifteen years later. Interest is credited at a nominal discount rate of d compounded quarterly for the first 10 years, and at a nominal interest rate of 6% compounded semiannually thereafter. The accumulated balance in the fund at the end of 30 years is 100.

Calculate d.

(A) 4.33%   (B) 4.43%   (C) 4.53%   (D) 4.63%   (E) 4.73%

4.   (2005 Exam FM Sample Questions #13)

Ernie makes deposits of 100 at time 0, and X at time 3. The fund grows at a force of interest

$$\delta_t = \frac{t^2}{100}, t > 0$$

The amount of interest earned from time 3 to time 6 is also X. Calculate X.

(A) 385   (B) 485   (C) 585   (D) 685   (E) 785

5.   (2005 Exam FM Sample Questions #20)

David can receive one of the following two payment streams:

   (i) 100 at time 0, 200 at time n, and 300 at time $2n$
   (ii) 600 at time 10

At an annual effective interest rate of i, the present values of the two streams are equal. Given $v^n = 0.76$, determine i.

(A) 3.5%   (B) 4.0%   (C) 4.5%   (D) 5.0%   (E) 5.5%

6.   (2005 Exam FM Sample Questions #27)

Bruce and Robbie each open up new bank accounts at time 0. Bruce deposits 100 into his bank account, and Robbie deposits 50 into his. Each account earns the same annual effective interest rate.

The amount of interest earned in Bruce's account during the 11th year is equal to X. The amount of interest earned in Robbie's account during the 17th year is also equal to X. Calculate X.

(A) 28.0   (B) 31.3   (C) 34.6   (D) 36.7   (E) 38.9

7.   (May 05, #13)

At a nominal interest rate of i convertible semi-annually, an investment of 1000 immediately and 1500 at the end of the first year will accumulate to 2600 at the end of the second year. Calculate i.

(A) 2.75%   (B) 2.77%   (C) 2.79%   (D) 2.81%   (E) 2.83%

8.   **(May 05, #18)**

A store is running a promotion during which customers have two options for payment.

   Option one is to pay 90% of the purchase price two months after the date of sale.

   Option two is to deduct X% off the purchase price and pay cash on the date of sale.

A customer wishes to determine X such that he is indifferent between the two options when valuing them using an effective annual interest rate of 8%.

Which of the following equations of value would the customer need to solve?

A) $\left(\dfrac{X}{100}\right)\left(1+\dfrac{0.08}{6}\right)=.90$ B) $\left(1-\dfrac{X}{100}\right)\left(1+\dfrac{0.08}{6}\right)=.90$

C) $\left(\dfrac{X}{100}\right)(1.08)^{1/6}=.90$ D) $\left(\dfrac{X}{100}\right)\left(\dfrac{1.08}{1.06}\right)=.90$

E) $\left(1-\dfrac{X}{100}\right)(1.08)^{1/6}=.90$

9.   **(May 05, #19)**

Calculate the nominal rate of discount convertible monthly that is equivalent to a nominal rate of interest of 18.9% per year convertible monthly.

(A) 18.0%   (B) 18.3%   (C) 18.6%   (D) 18.9%   (E) 19.2%

10.   **(Nov 05, #7)**

A bank offers the following choices for certificates of deposit:

Term (in years)	Nominal annual interest rate convertible quarterly
1	4.00%
3	5.00%
5	5.65%

The certificates mature at the end of the term. The bank does NOT permit early withdrawals. During the next 6 years the bank will continue to offer certificates of deposit with the same terms and interest rates.

An investor initially deposits 10,000 in the bank and withdraws both principal and interest at the end of 6 years. Calculate the maximum annual effective rate of interest the investor can earn over the 6-year period.

(A) 5.09%   (B) 5.22%   (C) 5.35%   (D) 5.48%   (E) 5.61%

11.   **(Nov 05, #25)**

The parents of three children, ages 1, 3, and 6, wish to set up a trust fund that will pay X to each child upon attainment of age 18, and Y to each child upon attainment of age 21.

They will establish the trust fund with a single investment of Z.

Which of the following is the correct equation of value for Z ?

(A) $\dfrac{X}{v^{17} + v^{15} + v^{12}} + \dfrac{Y}{v^{20} + v^{18} + v^{15}}$ (B) $3\left[Xv^{18} + Yv^{21} \right]$

(C) $3Xv^3 + Y\left[v^{20} + v^{18} + v^{15} \right]$ (D) $\left(X + Y \right) \dfrac{v^{20} + v^{18} + v^{15}}{v^3}$

(E) $X\left[v^{17} + v^{15} + v^{12} \right] + Y\left[v^{20} + v^{18} + v^{15} \right]$

Section 1.16

Sample Exam Solutions

1.
We look at the future value in 7.25 years for each person.

<u>Bruce</u>. He is credited interest for 29 quarters. We are given that his interest rate per semiannual period is 2%. Thus his interest rate per quarter is $\sqrt{1.02}-1$, and his future value is $FV = \left(\sqrt{1.02}\right)^{29} 100 = 133.26$

<u>Peter.</u> He earns continuous interest at a rate of δ for 7.25 years. His future value is $FV = 100e^{7.25\delta}$.

To finish the problem we equate the two future values and solve.
$$133.26 = 100e^{7.25\delta}$$
$$1.3326 = e^{7.25\delta}$$
$$\ln(1.3326) = 7.25\delta$$
$$\delta = \frac{\ln(1.3326)}{7.25} = .0396$$
Answer C

Please note: There has been some confusion about the solution to this problem.
In solving the equation we assumed that Bruce gets one quarter's worth of interest for the 29th quarter of his 7.25 years of investment. This gives an equation for δ :
$$100 \times 1.02^{14.5} = 100e^{7.25\delta}$$

The solution is = .0396. This matches the official SOA solution, so the writers clearly intended that this assumption be made.

If you assume that "convertible semiannually" means that interest is only paid semiannually, then you could assume that Bruce forfeits the interest on his final quarter. (This is what actually happens if you cash out a CD early. You forfeit some interest.) Then the equation for δ becomes:
$$100 \times 1.02^{14} = 100e^{7.25\delta}$$

The solution then changes to .0382, but this is not one of the multiple choices -so you can rule out this assumption.

One of the official texts by Kellison discusses the meanings of "compounded", "payable" and "convertible". After discussing the first two terms, Kellison says "The term 'convertible' does not seem to possess either connotation. The reader is advised not to rely on such connotations when encountering these terms, but to ascertain exactly how the interest is computed and paid." In other words, convertible is ambiguous and you need to clarify how it is used in each situation. In Matt's work in finance, there were legal documents to review to clarify the intent. In the exam world the clarification is simpler: **Does your assumption match a multiple choice answer?**

2.

The last 6 months of the eighth year are in the time interval from time 7.5 to time 8. For each of the two savers we will find the find the half year interest on that interval.

<u>Eric.</u> At time 7.5 he has a balance of $100\left(1+\dfrac{i}{2}\right)^{15}$. His interest on this balance over the next half year is $100\left(1+\dfrac{i}{2}\right)^{15}\left(\dfrac{i}{2}\right)$.

<u>Mike</u> . Since Mike only earns simple interest on the original amount, his interest earned in any half year is $200\left(\dfrac{i}{2}\right)$.

Since these interest amounts are equal

$$100\left(1+\frac{i}{2}\right)^{15}\left(\frac{i}{2}\right)=200\left(\frac{i}{2}\right)$$

$$\left(1+\frac{i}{2}\right)^{15}=2 \;\rightarrow\; \frac{i}{2}=.0473 \;\rightarrow\; i=.0946\,.$$

Answer C

3

First we will deal with the initial 10 years during which a discount rate d was quoted. For each of these 10 years the relevant values of v and i are

$$v = 1 - d = \left(1 - \frac{d^{(4)}}{4}\right)^4 \text{ and } 1 + i = \frac{1}{v} = \left(1 - \frac{d^{(4)}}{4}\right)^{-4}.$$

Thus after 10 years the initial deposit of 10 grows to

$$10(1+i)^{10} = 10\left(1 - \frac{d^{(4)}}{4}\right)^{-40}.$$

The accumulated balance on this 10 after 20 more years (or 40 semiannual periods) at 3% per semiannual period is

$$10\left(1 - \frac{d^{(4)}}{4}\right)^{-40}(1.03)^{40} = 32.62\left(1 - \frac{d^{(4)}}{4}\right)^{-40}$$

The second deposit of 20 accumulates after 30 semiannual periods at a rate of 3% to a value of

$$20(1.03)^{30} = 48.55$$

The total accumulated balance is

$$100 = 32.62\left(1 - \frac{d^{(4)}}{4}\right)^{-40} + 48.55$$

Thus

$$\left(1 - \frac{d^{(4)}}{4}\right)^{-40} = 1.577 \rightarrow \left(1 - \frac{d^{(4)}}{4}\right)^{40} = .634 \rightarrow \left(1 - \frac{d^{(4)}}{4}\right) = .98867 \rightarrow d^{(4)} = .0453$$

Answer C

4.

The amount of interest earned from time 3 to time 6 is the difference between the ending amount at time 6 and the starting amount in the account at time 3.

We will start by looking at the original deposit of 100. At time 3 it has grown to

$$100e^{\int_0^3 \delta_t dt} = 100e^{\int_0^3 \frac{t^2}{100}dt} = 100e^{\frac{27}{300}} \approx 109.42\,.$$

At time 3 deposit of X is made, so that the beginning amount at time 3 is

$$A(3) = 109.42 + X$$

At time 6 the account grows to

$$A(6) = (109.42 + X)e^{\int_3^6 \delta_t dt} = (109.42 + X)e^{\int_3^6 \frac{t^2}{100}dt} = $$
$$(109.42 + X)1.8776 = 205.45 + 1.8776X$$

The interest earned between time 3 and time 6 is

$$A(6) - A(3) = 96.03 + .8776X\,.$$

This interest must equal X, so that

$$X = 96.03 + .8776X \;\rightarrow\; X = 784.56$$

Answer E

5.

<u>Present value of stream (i):</u>

$$100 + 200v^n + 300v^{2n} = 100 + 200(.76) + 300(.76^2) = 425.28$$

<u>Present value of stream (i):</u>

$$600v^{10}$$

Since the present values are equal

$$600v^{10} = 425.28 \rightarrow v = .9662 \rightarrow 1 + i = 1.035$$

Answer A

6.

The interest earned during a year equals

(Balance at the start of the year)×(Interest Rate)

Let i denote unknown interest rate.

For Bruce the interest during year 11 is $X = i100(1+i)^{10}$

For Robbie the interest during year 17 is $X = i50(1+i)^{16}$

It follows that

$$i50(1+i)^{16} = i100(1+i)^{10} \rightarrow 50(1+i)^{16} = 100(1+i)^{10}$$
$$(1+i)^6 = 2 \rightarrow (1+i) = 1.12246$$
$$X = i100(1+i)^{10} = .12246(100)(1.12246)^{10} = 38.88$$

Answer E

7.
The equation of value is

$$1000\left(1+\frac{i^{(2)}}{2}\right)^4 + 1500\left(1+\frac{i^{(2)}}{2}\right)^2 = 2600$$

This is a problem that can be reduced to a quadratic –a common exam trick. Set

$$x = \left(1+\frac{i^{(2)}}{2}\right)^2.$$

Then the above equation becomes

$$1000x^2 + 1500x = 2600 \text{ or } x^2 + 1.5x - 2.6 = 0$$

The positive root of the quadratic (quadratic formula) is $x = 1.0283 = 1+i$. Thus

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = 1.0283$$

$$\left(1+\frac{i^{(2)}}{2}\right) = 1.0141 \rightarrow i^{(2)} = .0281$$

Answer D

8.
The customer has two options. Let P be the purchase price.

<u>Pay cash on the date of the sale with X% taken off the price.</u> The amount paid is
$P\left(1-\frac{X}{100}\right)$ immediately.

<u>Pay 90% of the purchase price in two months.</u> The amount paid in two months
(1/6 of a year) is $.90P$. The present value on the date of sale is

$$\frac{.90P}{1.08^{1/6}}.$$

The equation of value is $P\left(1-\frac{X}{100}\right) = \frac{.90P}{1.08^{1/6}}$. This is equivalent to

$$\left(1-\frac{X}{100}\right)(1.08)^{1/6} = .90.$$

Answer E

9.

$$\left(1+\frac{0.189}{12}\right)^{12} = 1.2063 = \left(1-\frac{d^{(12)}}{12}\right)^{-12} \rightarrow 0.9845 = \left(1-\frac{d^{(12)}}{12}\right) \rightarrow d^{(12)} = 0.186$$

Answer C

10.

Since the one year rate of 4% is the lowest, we can immediately eliminate the possibility of investing in six consecutive one year CDs or three consecutive one year CDs coupled with a three year CD. The two possible choices for maximum yield are A) two consecutive 3 year CDs or B) a 5 year CD coupled with a one year CD. Note that our CDs will earn at quarterly rates.

Term	Nominal Annual Rate	Quarterly Rate
1	4.00%	1.0000%
3	5.00%	1.2500%
5	5.65%	1.4125%

The total accumulation factors under the two options are:

Option A)   Two consecutive 3 year CDs for n=12 quarters each.

$$\left(1.0125\right)^{24} = 1.34735$$

Option B)   A 5 year CD coupled with a one year CD.

$$\left(1.014125\right)^{20}\left(1.01\right)^{4} = 1.377575$$

Option B) is better. It gives a total accumulation of 1.377575 over 24 quarters (which is six years). Therefore, the maximum annual effective rate an investor can earn over the six-year period is

$$1.377575^{1/6} - 1 = .0548$$

Answer D

11.

Below we tabulate the years remaining to ages 18 and 21 for each child.

Age now	1	3	6
Years to age 18 (X)	17	15	12
Years to age 21 (Y)	20	18	15

The present value of the age 18 payments is $X\left(v^{17} + v^{15} + v^{12}\right)$

The present value of the age 21 payments is $Y\left(v^{20} + v^{18} + v^{15}\right)$

The present value of the total fund required is

$$Z = X\left(v^{17} + v^{15} + v^{12}\right) + Y\left(v^{20} + v^{18} + v^{15}\right)$$

Answer E

Section 1.17

Supplemental Exercises

1. Given $d^{(4)} = 0.05$, find $i^{(6)}$, v and δ.

2. Money accumulates at a simple interest rate of 6.5% per year. For the interval [4, 5], find i_5.

3. If $a(t) = (2t + 1)^4$, find $\delta(t)$.

4. A deposit is made into a fund. For the first 5 years interest is credited at an annual nominal rate of 6% convertible quarterly. For the next 5 years interest is credited at an annual discount rate of 7% convertible semiannually. What is the equivalent constant force of interest for 10 year period?

5. A man deposits 500 into an account. At the end of 5 years the account has grown to 650. Find the annual nominal rate of interest convertible quarterly for this account.

6. Tom and Jerry deposit money into accounts at the same time. Tom's account earns at an annual effective rate of r. Jerry's account earns at a simple rate of r. For year 8, Tom's effective rate of interest is 1.5 times Jerry's effective rate for year 8. Find r.

7. An amount X is deposited into an account that pays 8% simple interest. At the same time $\dfrac{X}{2}$ is deposited into an account that accumulates at a constant force of interest δ. The total interest earned in each account after 10 years is the same. Find δ.

8. A bank pays an annual effective interest rate of i. A man deposits 1000 today and 1500 in one year. At the end of two years his account is at 2800. Find i.

9. A woman makes deposits into an account of 100 today and 300 12 years later. For the first 12 years interest is credited at an annual nominal rate of 6% convertible quarterly. For the next 8 years the account earns at a force of interest of δ. At the end of 20 years the accumulated amount is 802. Find δ.

10. Elmer deposits 1000 into a bank account. The bank credits interest at an annual nominal rate of i convertible quarterly for the first 8 years and an annual nominal rate of 1.5i convertible bimonthly thereafter. The amount in his account at the then end of 5 years is 1516. What is the amount in his account at the end of 10 years?

Section 1.18

Supplemental Exercise Solutions

1. We have the following equivalences:
   $(1 - d^{(4)}/4)^{-4} = (1 + i^{(6)}/6)^6 = 1 + i = v^{-1} = e^\delta$
   The common value is $(1 - 0.05/4)^{-4} = 1.05160$.

   $i^{(6)} = 6(1.0516^{1/6} - 1) = 0.0505$.
   $v = 1/1.0516 = 0.9509$
   $\delta = \ln(1.0516) = 0.0503$

2. $i_5 = [a(5) - a(4)]/a(4)$ $a(t) = 1 + 0.065t$ $a(4) = 1.260$ $a(5) = 1.325$.
   $i_5 = (1.325 - 1.26)/1.26 = 0.0516$

3. $\delta(t) = a'(t)/a(t) = 4(2t + 1)^3(2)/(2t + 1)^4 = 8/(2t + 1)$

4. The accumulation factor $a(10)$ for the 10-year period is
   $(1.015)^{20}(1 - 0.035)^{-10} = 1.9233 = e^{10\delta}$

   $\delta = (1/10)\ln(1.9233) = 0.0654$

5. $500(1 + i^{(4)}/4)^{20} = 650$
   $i^{(4)} = 4[(650/500)^{1/20} - 1] = 0.0528$

6. Tom's effective rate of interest for year 8 is r.
   Jerry's effective rate of interest for year 8 is
   $(1 + 8r - 1 - 7r)/(1 + 7r) = r/(1 + 7r)$.

   Therefore $r = 1.5r/(1 + 7r) \Rightarrow 1 + 7r = 1.5 \Rightarrow r = 0.5/7 = 0.0714$

7. The total interest earned on the first account is $(0.08)(10)X = 0.8X$. The total
   interest earned on the second account is $(X/2)(e^{10\delta} - 1)$

   $0.8X = (X/2)(e^{10\delta} - 1) \Rightarrow 1.6 = e^{10\delta} - 1$
   $10\delta = \ln(2.6) \Rightarrow \delta = 0.0956$

8. The accumulated amount in the account is
   $1000(1 + i)^2 + 1500(1 + i) = 2800$.   Let $x = 1 + i$.

   This yields the quadratic equation $10x^2 + 15x - 28 = 0$.
   The positive root of the equation is $x = 1 + i = 1.084$. $\Rightarrow i = 0.084$

9. The accumulated amount is $[100(1.015)^{48} + 300]e^{8\delta} = 802$.
   Thus $504.35e^{8\delta} = 802 \Rightarrow 8\delta = \ln(1.59) \Rightarrow \delta = 0.058$

10. At the end of 5 years the accumulation is $1000(1 + i/4)^{20} = 1516$.
    At the end of 10 years the accumulation is

    $1000(1 + i/4)^{32}(1 + 1.5i/6)^{12} = 1000(1 + i/4)^{44}$ $(i/4 = 1.5i/6)$
    $(1 + i/4)^{44} = 1.516^{44/20} = 2.49769$

    Accumulation is 2497.69

Practice Exam 8

Exam FM

Questions

1. Alex invests 50,000 today in a 10-year project. At the end of the 5th, 7th, and 9th year he receives a payout of 22,000. Rather then keeping the payouts, Alex reinvests them into a fund that earns a 6% annual effective rate and receives a lump sum payout ten years from today. What is the net present value of this project today valued at an annual rate of interest of 5%?

   A) 1,523   B) 4,267   C) –5,907   D) –1,523   E) –4,267

2. A stock has a current price of 40. The continuous annual risk-free rate is 2%. An investor wishes to create a zero-cost collar using options with maturity $T = .25$. Which of the following strike prices cannot be used for the put in the collar?

   A) 38.37   B) 39.93   C) 40.01   D) 40.20   E) 41

3. Tim makes four payments of 200 at four-year intervals starting today. Interest is credited at a nominal interest rate of 5% compounded semiannually for the first 9 years and at a nominal discount rate of 3% compounded monthly thereafter. Calculate the accumulated value of the four payments 25 years from today.

   A) 1,394   B) 1,554   C) 1,538   D) 1,311   E) 1,449

4. An executive is given 100 shares of his company's stock at no cost. He cannot sell for three months. He hedges his position using a zero cost collar constructed using 3-month options. He buys 100 puts with a strike price of 50 and writes 100 calls with a strike price of 53.74. What is the range of possible values of his position in three months?

   A) 4800 to 5300   B) 5000 to 5300
   C) 4800 to 5374   D) 5000 to 5374
   E) None of these

5. Brian borrows 40,000 where the principal must be repaid in 20 years, but interest payments are due at the end of each year at an effective rate of 4%. Brian pays the interest each year but also deposits X into a fund at the end of each month to save up to repay the loan principal in 20 years. This fund earns an annual effective rate of 5%. How much is Brian's total contribution towards repaying this loan?

    A) 55,360   B) 59,800   C) 55,660   D) 56,190   E) 72,000

6. Ruth deposits 50 into a fund at the beginning of every 6-month period starting today for 28 years.   At an annual effective rate of j ($j > 0$), the amount in 28 years from today is four times the amount of money in the fund 14 years from today.   Calculate the total amount of interest that Ruth will earn from this investment 28 years from today.

    A) 7,596   B) 3,900   C) 10,396   D) 7,196   E) 7,797

7. Tony buys two bonds. One bond is bought to yield 4% effective annually and is a 3-year zero coupon bond that can be redeemed for 1000.   The second bond is a 1000 par value 6% semi-annual coupon bond that matures in 6 years.   Both of the bonds have the same purchase price. Calculate the effective annual yield of the coupon bond.

    A) 6.4%   B) 6.9%   C) 4.2%   D) 8.6%   E) 8.4%

8. Brenda just turned 20 and wants to set up a fund that will pay her X when she is 40 and 50 years old, and that will also pay her 1000 when she is 45 and 55 years old. Brenda is setting up the fund with a lump sum deposit today of 875 in order to match the future payments.   If the fund earns an annual effective rate of 4.5%, calculate X?

    A) 650   B) 1672   C) 1274   D) 481   E) 2,293

9. Austin bought an annual perpetuity due in 2002 that pays 4 on "even" years (i.e. 2002, 2004, 2006, etc.) and 7 on "odd" years (i.e. 2003, 2005, etc.). Calculate the present value of the perpetuity at an annual effective rate of 5%.

    A) 107.3   B) 118.3   C) 104.1   D) 114.7   E) 200.0

10. Michael deposits 300 at time 4 and a deposit of 200 at time t into an account growing at a force of interest of

$$\delta(t) = \frac{6}{(t+4)}$$

If the total future value of these two deposits at time 8 is 4500, which of the following is closest to t?

A) 4   B) 5   C) 6   D) 7   E) 8

11. The current yield curve is

Maturity	Spot rate
1	4.00%
2	4.40%
3	5.00%

The forward prices for oil per barrel are

Year	Forward Price
1	20
2	21
3	22

Which of the following sequences is acceptable for a commodity swap for delivery of one barrel of oil per year for the next 3 years? Present value final answers can be rounded to two places.

1. 20.9644, 20.9644, 20.9644
2. 19.6851, 20.6851, 22.6851
3. 21.9288, 20.9288, 19.9288

A) 1 only
B) 1 and 2
C) 1 and 3
D) 2 and 3
E) All

12. The S&R index pays no dividends, and is currently priced at 800. The annual
effective risk free rate is 3%. You wish to create a forward to sell the index
in one year for 840. You will create the forward using one year put and call
options with strike price 840. Which of the following will you do (per share)?

    A) Buy the put and sell the call, at a cost of 16.
    B) Buy the put and sell the call, at a cost of 15.53.
    C) Sell the put and buy the call at a cost of 16.
    D) Sell the put and buy the call at a cost of 15.53
    E) None of these

13. Alex and Rudy are both going to buy bonds at the same time.   Alex is buying
a 15 year 1000 bond redeemable at par with semiannual coupons.   Rudy is
buying a 5 year 800 bond redeemable at par with semiannual coupons.
Alex's bond is sold to yield 4% convertible semiannually and the yield rate
of Rudy's bond is 10% convertible semiannually.   The price of Alex's bond
is twice that of Rudy's bond and their coupon rates are equal.   Calculate
their nominal coupon rate convertible semiannually.

    A) 4.28%   B) 8.57%   C) 7.26%   D) 8.67%   E) 7.51%

14. Jeanette takes out a 50-year loan of 30,000.   Instead of making level
payments, Jeanette's annual payments at the end of each year are all equal
to 600 plus the amount of interest owed.   The lender charges an annual
effective interest rate of 7%.   How much total interest will Jeanette pay
towards this loan?

    A) 52,500   B) 53,550   C) 78,690   D) 52,000   E) 59,200

15. A fund with 10 quarterly payments of 300 has a present value of 2,700.
What would be the quoted nominal discount rate convertible monthly for
this fund?

    A) 7.85%   B) 7.28%   C) 7.67%   D) 7.80%   E) 7.75%

16. You are given the following term structure of spot interest rates:

Term (in years)	Spot interest rate
1	4.00%
2	4.50%
3	5.25%
4	5.75%
5	6.00%

Using forward rates, calculate the future value at time 5 of a four-year annuity due issued today with four annual payments of 3500.

A) 16,964   B) 16,555   C) 16,733   D) 17,494   E) 15,920

17. Calculate the Modified Duration, at i = 0.045, of payments of 400 at the end of the 2nd year, 300 at the end of the 3rd year, 200 at the end of the 4th year, and 100 at the end of the 6th year.

A) 2.54   B) 3.30   C) 3.00   D) 2.91   E) 3.22

18. You believe that the volatility of a stock will be lower than market expectation as measured by option prices. Which of the following would enable you to speculate on your expectation by trading in combinations of options on the stock?

1) Buy a straddle
2) Sell a straddle
3) Buy a straddle and sell a strangle
4) Sell a straddle and buy a strangle

A) 1 and 3   B) 2 and 4   C) 2 only
D) 4 only   E) All

19. Seth borrows X for 9 years at an annual effective rate of 5.75%.   He will be repaying the loan with level payments at the end of each 6-month period. The principal repaid in the 2nd payment is 580.   Calculate the outstanding balance immediately after the 7th payment.

A) 9,000   B) 1,670   C) 8,710   D) 8,470   E) 9,380

20. John is going to pay off a 10,000 loan that is being charged an annual effective interest rate of 9% with 15 annual payments.   Instead of level payments, John is going to have payments that are each 13% smaller than the previous payment, with an initial payment of X.   Solve for X.

   A) 1240.59   B) 2277.42   C) 2297.86   D) 9600.00   E) 3853.97

21. A stock for a company sells for 50 per share assuming an annual effective rate of 12%.   Annual dividends will be paid at the end of each year forever. The first dividend is 5, with each subsequent dividend X% greater than the previous year's dividend.   Calculate X.

   A) 3.5   B) 2.8   C) 2.4   D) 2.0   E) Does Not Exist

22. The current price of a non-dividend paying stock is 40. European put and call options are available with strikes of 35, 40 and 45. A spread on the stock has a payoff diagram of the following shape.
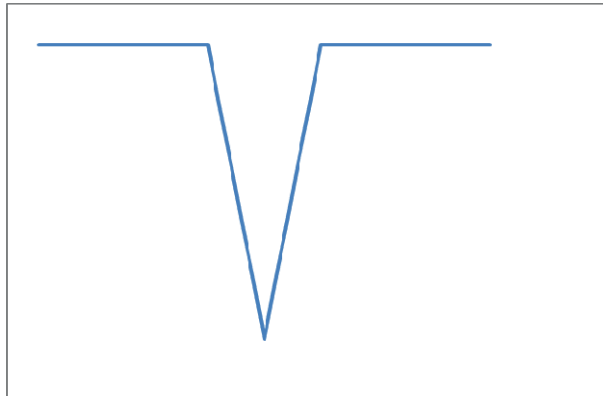
Which of the following will have a payoff diagram of the same shape?

   1) Sell a 35 put, sell a 45 call, buy a 40 put and a 40 call.
   2) Buy a 35 put, buy a 45 call, sell a 40 put and a 40 call.
   3) Sell short one share of the stock, sell a 35 put, sell a 45 call and buy two 40 calls.

   A) 1 only   B) 2 only   C) 1 and 3
   D) 2 and 3   E) None of these

23. Rafael bought a 10-year 1000 par value bond for a price of 1025. The bond is callable in six years at par.   The purchase price guaranteed a yield rate of at least 5% convertible semiannually.   Calculate the amount of each semi-annual coupon.

    A) 54.93   B) 53.24   C) 52.82   D) 27.44   E) 26.60

24. A house is worth 300,000. A three-year fire insurance policy with a deductible of 30,000 is purchased for the house. The following three-year call options to buy the house may be purchased at the time the insurance is written.

Strike price K	Call Option price
270,000	$74,825
300,000	$58,168

    The continuously compounded risk-free rate is 4%. What is the price of the insurance?

    A) 14,293   B) 21,820   C) 24,244   D) 25,386   E) 34,825

25. Dora borrows 5000 at a nominal interest rate of 7% convertible semiannually and will repay all interest and principal in 8 years.

    Dora takes the borrowed amount and buys five 1000 par value bonds with semiannual coupons of 30 and priced to yield 6% convertible semi-annually. The bonds are redeemed at par in 8 years immediately before Dora repays her original loan.   Dora also invests all coupons after each one is received into a fund earning a nominal interest rate of 5% convertible semi-annually.

    Calculate Dora's net gain or loss in 8 years.

    A) 684   B) 763   C) -763   D) -5,763   E) -684

26. The current price of a non-dividend-paying stock is 51. The forward price for a sale at time T=0.25 is $F_{0,T} = 51.77$. The price of a European call expiring at time .25 with strike price equal to that forward price is $2.03. The risk-free rate is 6%. What is the price of a European put with strike price 51.77?

    A) 1.98   B) 2.00   C) 2.03   D) 2.06   E) 2.08

27. Sara invests 75 at the beginning of each quarter for 15 years into a fund earning an nominal interest rate of 6% convertible quarterly at the end of each quarter. Sara immediately reinvests each interest payment into another fund earning an annual effective rate of 4.5%. At the end of the 15 years Sara's total accumulated value is X. Calculate X.

   A) 6,994   B) 4,585   C) 7,099   D) 6,442   E) 7,089

28. You are given the following information about the activity in an investment account:

Date	Fund Value Before Activity	Activity Deposit	Activity Withdrawal
January 1, 2008	1000		
April 1, 2008	1100		300
T	850	400	
December 31, 2008	1300		

If the time weighted yield is 1% more than the dollar weighted yield for this account, calculate the Date T.

   A) May 1, 2008   B) June 1, 2008   C) July 1, 2008
   D) September 1, 2008   E) October 1, 2008

29. The non-dividend-paying S&R index has a current value of 1000. Three month put and call options are available at the following prices:

Strike	Call	Put
950.00	120.405	51.777
1,000.00	93.809	74.201
1,020.00	84.47	84.47
1,050.00	71.802	101.214

Which combination of puts and calls has a profit function of the following form?

A) Sell a 1000 put and a 1000 call, buy a 950 put and a 1020 call.
B) Buy a 1000 put and a 1000 call, sell a 950 put and a 1020 call
C) Buy a 950 call and a 1020 call, sell a 1000 call and 1050 call
D) Sell a 950 call and a 1020 call, buy a 1000 call and a 1050 call.
E) None of these

30 Adam took out a loan for 36,000 with level quarterly payments, and financed it at an effective annual rate of 6.75% over 10 years. Adam decides to send an extra 1000 with the 5th and 10th payment. What is the interest portion of the 11th payment?

A) 456.16   B) 478.60   C) 444.30   D) 468.9   E) 462.00

31. The following is a table reporting investment year rates and portfolio rates in %.

Calendar Year of Investment	Investment Year Rates in %				Portfolio Rates in %
y	i_1^y	i_2^y	i_3^y	i_4^y	i^{y+4}
2002	8.30	8.30	8.85	8.90	8.70
2003	9.00	9.00	9.20	9.29	9.30
2004	9.00	9.20	9.30	9.40	9.30
2005	9.35	9.45	9.50	9.65	
2006	9.60	9.60	9.70		
2007	X	10.00			
2008	9.70				

The two-year accumulation factor for an investor starting in 2007 under the investment year method is equal to the two-year accumulation factor for an investor starting in 2007 under the portfolio method.   Calculate X.

A) 8.40   B) 8.60   C) 9.00   D) 9.30   E) 9.70

32. A premium bond is purchased to yield i convertible semiannually.   The amount of premium amortized in the fourth payment is 7.32.   The amount of premium amortized in the eighth payment is 10.45.   Find the amount of premium amortized in the 11th payment.

A) 13.65   B) 12.80   C) 14.92   D) 13.11   E) 12.50

33. Amy wants to purchase an item on January 1st 2023 that will cost 15,000. Today is January 1st 2010 and to finance the item, she deposits X into an account each January 1st, starting now, and 2X into the account every July 1st.

The account earns an annual effective interest rate of 7%, however interest is compounded every 6 months.

Calculate X. No payment will be made on 1/1/2023.

A) 232   B) 237   C) 243   D) 251   E) 278

34. You are given the following price information for one-year European options.

Strike Price	Call Price	Put price
50	5.10	4.61
55	3.20	7.56

Find the effective one-year risk-free rate.

A) 2%   B) 2.5%   C) 3%   D) 3.5%   E) 4%

35. The current price of the commodity actuarium is $40 per pound. A mining company is considering hedging this price for a sale in 3 months. Two options are proposed.

Option A: Buy a 3-month European put with K=40 for a price of 1.8355.

Options B: Buy two 3-month puts for 1.8355 each and sell one 3-month put with K=43.014 for 3.671.

Which of the following is true?

A) Option B is always has a higher profit because its cost is 0.
B) Option A is always more profitable.
C) Option B is more profitable for values of actuarium less than 40 because its cost is 0, but it is less profitable for high prices of actuarium.
D)   Option B is less profitable for values of actuarium less than 40, but it is more profitable for high prices of actuarium.
E) The company does not need to hedge.

Solutions

1. There are only two cash flows in this NPV calculation, a cash flow of –50,000 at time zero and a positive cash flow at time 10.   The second cash flow is the future value of the reinvested payouts.   So, calculate those first:

   $$FV = 22,000(1.06)^5 + 22,000(1.06)^3 + 22,000(1.06) = 78,963.3147$$

   In the NPV calculation, there are the two cash flows: $CF_0 = -50,000$ and $CF_{10} = 78,963.3147$.   Evaluate these cash flows at a rate of 5%.

   $$NPV = -50,000 + 78963.3147 v_{0.05}^{10} = -1,523.3746$$

   Answer: D

2. We know that the put price in the collar must be less than or equal to the forward price of $40e^{.02(.25)} = 40.2005$.

   Answer: E

3. First, we can find the effective annual interest rates for the first nine years, and for the remaining 16 years.
   For the first 9 years, $i = (1 + \dfrac{0.05}{2})^2 - 1 = 0.050625$
   For the remaining 16 years, $j = (1 - \dfrac{0.03}{12})^{-12} - 1 = 0.030493241$

   The future value of the four payments of 200 is then,
   $$200(1+i)^9(1+j)^{16} + 200(1+i)^5(1+j)^{16} + 200(1+i)(1+j)^{16} + 200(1+j)^{13}$$
   $$= 504.4080 + 413.9911 + 339.7818 + 295.5404 = 1,553.7213$$

   Answer: B

4. The value of his position is
    $$100\left[S_{.25} + \max(0, 50 - S_{.25}) - \max(0, S_{.25} - 53.74)\right]$$

    For $S_{.25} < 50$ the value of the position is 50 and for $S_{.25} > 53.74$ the value of the position is 53.74. The graph of the value of the position is

Position
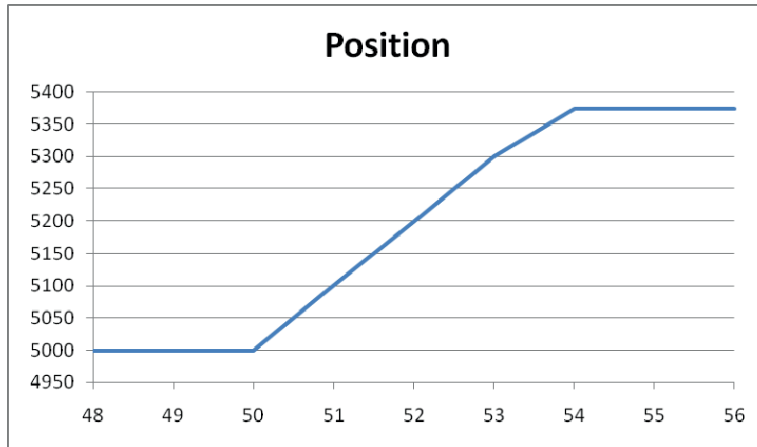
Answer: D

5. Brian's interest only payments each year are 40,000(0.04)=1600. He will be making 20 such payments for a total of 32,000.

    Brian's sinking fund payments can be found by setting up a future value of an annuity equation. The effective monthly interest rate is
    $$1.05^{\frac{1}{12}} - 1 = 0.0040741$$

    $$40,000 = Xs_{\overline{240}|i} = X\frac{(1+i)^{240} - 1}{i}$$
    $$X = 98.5696$$

    He will be making 240 such payments for a total of 23,656.7124.
    Therefore, Brian's total contribution is 23,656.7124+32,000=55,656.7124.
    The closest choice is choice C.

    Answer: C

6. Set up two future value equations and set the amount in 28 years equal to four times the amount in 14 years.

Let i stand for the semiannual rate. First, since the payments are made semiannually,

In 28 years, the future value will be: $50\ddot{s}_{\overline{56}|} = 50\dfrac{(1+i)^{56}-1}{d}$

In 14 years, the future value will be: $50\ddot{s}_{\overline{28}|} = 50\dfrac{(1+i)^{28}-1}{d}$

Setting the 28-year future value equal to four times the 14 year value, we have:

$$50\frac{(1+i)^{56}-1}{d} = 4(50)\frac{(1+i)^{28}-1}{d}$$
$$(1+i)^{56}-1 = 4[(1+i)^{28}-1]$$
$$(1+i)^{56}-4(1+i)^{28}+3 = 0$$
$$(1+i)^{28} = \frac{4\pm\sqrt{16-4(1)(3)}}{2}$$
$$(1+i)^{28} = 3 \quad\text{or}\quad (1+i)^{28} = 1$$

Since i cannot be zero, i= 0.040016

The future value after 28 years is then: 10,395.96
Ruth only contributed 56(50) = 2,800.
So, the interest she earned is 7,595.96.

Answer: A

7. First, calculate the price of the zero coupon bond.

$$P = 1000v_{0.04}^{3} = 888.9963$$

Now, use a finance calculator to solve for the interest yield rate of a coupon bond with 12 coupons, a coupon rate of 3%, a future value of 1000, and a purchase price of 888.9963

N=12,   PV = -888.9963,   PMT = 1000(0.03) = 30,   FV = 1000
Compute I/Y = 4.196278

Remember, that this is the effective semi-annual yield.   And it asks for the effective annual yield. $i = (1.041963)^{2} - 1 = 0.085686$

Answer: D

8. Set up an equation of value for the present values
    $$PV = 875 = Xv^{20} + 1000v^{25} + Xv^{30} + 1000v^{35}$$
    $$875 - 1000v^{25} - 1000v^{35} = X(v^{20} + v^{30})$$
    $$X = \frac{875 - 1000v^{25} - 1000v^{35}}{v^{20} + v^{30}} = 481.2123$$

    Answer: D

9. Think of these as two separate two-year payment perpetuities due. The "even" one starts at the present value and the "odd" one is deferred by one year. First, calculate the two-year effective rate. $i = (1.05)^2 - 1 = 0.1025$

    The present value of the "even" perpetuity is

    $$PV_{even} = 4\left(\frac{1}{d}\right) = 4\left(\frac{1}{\dfrac{i}{1+i}}\right) = 4\left(\frac{1.1025}{0.1025}\right) = 43.0244$$

    The present value of the "odd" perpetuity is similar, but it is deferred by one **year.**

    $$PV_{even} = \frac{1}{1.05}7\left(\frac{1}{d}\right) = \frac{7}{1.05}\left(\frac{1}{\dfrac{i}{1+i}}\right) = \frac{7}{1.05}\left(\frac{1.1025}{0.1025}\right) = 71.7073$$

    The total present value is the sum.   43.0244 + 71.7073 = 114.7317

    Answer: D

10. The accumulation function using force of interest is:

$$a(t) = e^{\int_0^t \delta(u)\,du}$$

The two deposits at time 8 accumulate to 4500, so we are trying to solve:

$$300e^{\int_4^8 \frac{6}{u+4}du} + 200e^{\int_t^8 \frac{6}{u+4}du} = 4500$$

$$300e^{6\ln(u+4)\big|_4^8} + 200e^{6\ln(u+4)\big|_t^8} = 4500$$

$$e^{6\ln(u+4)\big|_t^8} = \frac{4500 - 300e^{6\ln 12 - 6\ln 8}}{200}$$

$$6\ln(u+4)\big|_t^8 = \ln\left(\frac{4500 - 300e^{6\ln 12 - 6\ln 8}}{200}\right)$$

$$6\ln 12 - 6\ln(t+4) = \ln\left(\frac{4500 - 300e^{6\ln 12 - 6\ln 8}}{200}\right)$$

$$\frac{6\ln 12 - \ln\left(\dfrac{4500 - 300e^{6\ln 12 - 6\ln 8}}{200}\right)}{6} = \ln(t+4)$$

$$e^{\frac{6\ln 12 - \ln\left(\frac{4500-300e^{6\ln 12-6\ln 8}}{200}\right)}{6}} = t+4$$

$$t = e^{\frac{6\ln 12 - \ln\left(\frac{4500-300e^{6\ln 12-6\ln 8}}{200}\right)}{6}} - 4 = 5.0558$$

Answer: **B**

11. This is based on problem 4 of the sample question set FM-09-07. The present value of the 3 forward prices is 57.50 to two places. Each of the sequences given has present value of 57.50 to two places. Thus each is legitimate for a swap.

Answer: **E**

12. The correct forward price is $800(1.03) = 824$. You can create a synthetic short forward with forward price of 840 by selling the call and buying the put with strike 840.

    The synthetic forward is priced 16 too high. This means that you must pay a premium for it. The premium is the present value of the mispricing of 16, $16/1.03 = 15.53$. This is based on problem 5 of the sample question set FM-09-07.

    Answer: B

13. The price of a bond formula is: $P = (Fr)a_{\overline{n}|i} + Fv_i^n$

    Plug in what we know for Alex and Rudy's bond

    $$P_{Alex} = (1000r)a_{\overline{30}|0.02} + 1000v_{0.02}^{30}$$

    $$P_{Rudy} = (800r)a_{\overline{10}|0.05} + 800v_{0.05}^{10}$$

    They tell us that Alex's Price is twice of Rudy's price:

    $$P_{Alex} = (1000r)a_{\overline{30}|0.02} + 1000v_{0.02}^{30} = 2P_{Rudy} = 2[(800r)a_{\overline{10}|0.05} + 800v_{0.05}^{10}]$$
    $$(1000r)a_{\overline{30}|0.02} + 1000v_{0.02}^{30} = (1600r)a_{\overline{10}|0.05} + 1600v_{0.05}^{10}$$
    $$r(1000a_{\overline{30}|0.02} - 1600a_{\overline{10}|0.05}) = 1600v_{0.05}^{10} - 1000v_{0.02}^{30}$$
    $$r = \frac{1600v_{0.05}^{10} - 1000v_{0.02}^{30}}{1000a_{\overline{30}|0.02} - 1600a_{\overline{10}|0.05}} = \frac{430.1903}{10,041.6798} = 0.0428405$$

    This is the effective semiannual rate, so the nominal convertible semiannual rate would be $2(0.0428405) = 0.085681$, or 8.568%

    Answer :   B

14. The amount of 600 is the amount of each payment that will be paying down the principal of the loan. The amount of interest owed with the first payment is 30,000(0.07), the second payment (30,000-600)(0.07), etc.   The amount of interest owed with the nth payment is (30,000-(n-1)600)(0.07)

So, total interest paid is an arithmetic series calculation:

$$\begin{aligned} Int &= [(30,000(0.07) + (30,000 - 600)(0.07) + ... + (30,000 - 49(600))(0.07)] \\ &= 0.07[30,000 + 29,400 + 28,800 + ... + 600] \\ &= 0.07\left[\frac{50}{2}(30,000 + 600)\right] = 53,550 \end{aligned}$$

We end up with a sum of a finite arithmetic sequence

Answer: B

15. First, calculate the interest rate of the fund using the present value:
$$2700 = 300a_{\overline{10}|i}$$

The easiest way to do this would be to use the finance calculator.
PMT = 300   N = 10   FV = 0   PV = -2700

Compute I/Y = 1.96299798

This is the effective quarterly rate.   Multiply it by 4 and you have the nominal rate compounded quarterly of 7.85199%

To change this to a nominal discount rate, use the nominal rate equivalence formula

$$\left(1 + \frac{i^{(m)}}{m}\right)^m = \left(1 - \frac{d^p}{p}\right)^{-p}$$

$$\left(1 + \frac{0.0785199}{4}\right)^4 = \left(1 - \frac{d^{12}}{12}\right)^{-12}$$

$$\left(\left(1 + \frac{0.0785199}{4}\right)^4\right)^{-1/12} = 1 - \frac{d^{12}}{12}$$

$$-12\left[\left(\left(1 + \frac{0.0785199}{4}\right)^4\right)^{-1/12} - 1\right] = d^{12} = 0.0775078$$

Answer: E

16. There are four payments of 3500 made at time zero, one, two and three. We want the value of these payments at time 5. If we first find the present value at time 0 we can get the future value at time 5 using the 5 year spot rate.

Present value $3500\left(1 + \dfrac{1}{1.04} + \dfrac{1}{1.045^2} + \dfrac{1}{1.0525^3}\right) = 13072.38$

Future value: $13072.38\left(1.06^5\right) = 17493.79$

This could also be done using forward rates. So, we need four forward interest rates:

$i_{(0,5)}$, $i_{(1,5)}$, $i_{(2,5)}$, and $i_{(3,5)}$.

$$\left(1 + i_{(0,5)}\right)^5 = \frac{(1 + s_5)^5}{1} = 1.06^5$$

$$\left(1 + i_{(1,5)}\right)^4 = \frac{(1 + s_5)^5}{(1 + s_1)^1} = \frac{1.06^5}{1.04}$$

$$\left(1 + i_{(2,5)}\right)^3 = \frac{(1 + s_5)^5}{(1 + s_2)^2} = \frac{1.06^5}{1.045^2}$$

$$\left(1 + i_{(3,5)}\right)^2 = \frac{(1 + s_5)^5}{(1 + s_3)^3} = \frac{1.06^5}{1.0525^3}$$

The future value of these payments will then be:
$$FV = 3500(1 + i_{0,5})^5 + 3500(1 + i_{1,5})^4 + 3500(1 + i_{2,5})^3 + 3500(1 + i_{3,5})^2$$

$$= 3500(1.06^5) + 3500\left(\frac{1.06^5}{1.04}\right) + 3500\left(\frac{1.06^5}{1.045^2}\right) + 3500\left(\frac{1.06^5}{1.0525^3}\right)$$

$$= 17,493.7900$$

Answer: D

17. First, calculate the Macaulay duration of the cash flows

$$D = \frac{2(400v^2) + 3(300v^3) + 4(200v^4) + 6(100v^6)}{400v^2 + 300v^3 + 200v^4 + 100v^6} = 3.0364$$

Next, calculate the modified duration:
$$DM = \frac{D}{1 + i} = \frac{3.0364}{1.045} = 2.9056$$

Answer: D

18. Selling a straddle is clearly a bet on lower volatility. Choice 4 is equivalent to the sale of a butterfly spread, which is also a bet on lower volatility.

Answer: B

19. The effective semi-annual interest rate is: $i = (1.0575)^{1/2} - 1 = 0.0283482$
Use the principal paid in payment 2 to solve for the level payment amount:

$$PR = 580 = PMTv^{18-2+1}$$
$$PMT = \frac{580}{v^{17}} = 580(1+i)^{17} = 932.8487$$

Now, use this payment amount to solve for the outstanding balance after the 7$^{\text{th}}$ payment using the prospective method, which will be the present value of the remaining 11 payments.

$$PV = 932.8487a_{\overline{11}|i} = 8,710.8041$$

Answer: C

20. This will be solving a geometric series problem.

$$10,000 = \frac{X}{1.09} + \frac{X(0.87)}{1.09^2} + \frac{X(0.87)^2}{1.09^3} + ... + \frac{X(0.87)^{14}}{1.09^{15}}$$
$$10,000 = \frac{X}{1.09}\left(1 + \frac{0.87}{1.09} + \frac{0.87^2}{1.09^2} + ... + \frac{0.87^{14}}{1.09^{14}}\right)$$
$$10,000 = \frac{X}{1.09}\left(\frac{1 - \left(\frac{0.87}{1.09}\right)^{15}}{1 - \frac{0.87}{1.09}}\right)$$
$$X = 2277.4165$$

Answer: B

21. Price of a stock using the constant growth model is

$$P = \frac{D}{i - g} .$$

We have a Price of 50, a dividend of 5, and interest rate of i =0.12. Solve for g:

$$50 = \frac{5}{0.12 - g}$$

$g = 0.02$

Answer: D

22. Choice 1 consists of a purchase straddle centered at 40 and a written strangle from 35 to 45. This is a written butterfly spread, which has the form above. Choice 2 is the opposite of choice 1 and can thus be eliminated. The payoff of choice C is

$$-S - \max(0, 35 - S) - \max(0, S - 45) + 2\max(0, S - 40)$$

$$= \begin{cases} S < 35, & -S + S - 35 = -35 \\ 35 \le S < 40, & -S \\ 40 \le S < 45, & -S + 2(S - 40) = S - 80 \\ S > 45, & -S - (S - 45) + 2(S - 40) = -35 \end{cases}$$

This does have a graph of the above form. This is based on problem 9 of the sample question set FM-09-07.

Answer: C

23. First, because of the purchase price, we know that the bond is a Premium bond. Premium callable bonds are priced at the earliest possible redemption date. So, set up a price equation calculated with a time frame of 6 years

$$P = 1025 = (1000r)a_{\overline{12}|0.025} + 1000v_{0.025}^{12}$$

$r = 0.027437$

The amount of each coupon is then 1000(0.027437) = 27.44

Answer: D

24. The insurance on the house is equivalent to a put option on the house with a strike price of the value of the house less the deductible, or 270,000. Using put-call parity

$$Put(270000,3) = Call(270000,3) - S_0 + PV(270,000)$$
$$= 74,825 - 300,000 + 270,000e^{-0.04(3)}$$
$$= 14,293$$

Answer: A

25. First, calculate the amount that Dora will have to repay for the loan in 8 years.

$$FV = 5000(1 + \frac{0.07}{2})^{16} = 8669.9302$$

Next, figure out the future value of the reinvested coupons that Dora is receiving from her bonds.   Notice that since the coupon rate is equal to the yield rate, the purchase price of the bonds is the same as the redemption values.   She will be investing five coupons of amount 30 every 6-month period for 8-years.   The future value of the coupons is then,

$$FV = 150s_{\overline{16}|0.025} = 2907.0337$$

At the end of the 8-years, Dora will have the 2,907.0337 from the reinvested coupons and 5000 from the redemption value of the coupons for a total of 7,907.0337.   However, she owes 8,669.9302 from her original loan. Therefore, Dora has a net loss of 762.8965.

Answer: C

26. The put-call parity equation shows that if the strike price of an option is the forward price $F_{0,T}$, then the put and call with that strike price and the same maturity have the same premium.

Answer: C

27. First, make sure that you have the interest rates translated to the appropriate quarterly rate.   Her payments of 75 are earning a quarterly effective rate of $i = \dfrac{0.06}{4} = 0.015$.   Her reinvested interest payments are earning a quarterly effective rate of $j = (1.045)^{\frac{1}{4}} - 1 = 0.011065$.

Let's organize the payments and interest into a table:

Time	0	0.25	0.5	$\cdots$	14.5	14.75	15
Payment	75	75	75	$\cdots$	75	75	0
Total payments	75	150	225	$\cdots$	4,425	4,500	4,500
Interest earned		75(0.015)	150(0.015)	$\cdots$	4,350(0.015)	4,425(0.015)	4,500(0.015)

Sara's total accumulated value is the 75 payments of 60 = 4500 plus the future value of the reinvested interest payments.

The reinvested interest payments have a future value calculation of

$$
\begin{aligned}
FV &= 75(0.015) I s_{\overline{60}|j} \\
&= 75(0.015) \frac{\ddot{s}_{\overline{60}|} - 60}{j} \\
&= 75(0.015) \frac{\dfrac{(1+j)^{60} - 1}{d} - 60}{j} \\
&= 2{,}588.7282
\end{aligned}
$$

Add the 4,500 from Sara's payments and we have a total accumulated value of 2,588.7282+4500=7,088.73

Answer: E

28. You don't need T to calculate the time weighted yield:

$$1 + i = \frac{1100}{1000}\left(\frac{850}{800}\right)\left(\frac{1300}{1250}\right) = 1.2155$$

So, i=0.2155, and the time weighted yield is 21.55%.
Therefore, the dollar weighted yield would be 20.55%
Set this equal to a dollar weighted yield calculation to solve for T.

Interest earned is : $1300 - 1000 - (400 - 300) = 200$

The dollar weighted calculation would be:

$$0.2055 = \frac{200}{1000 - 300(1 - 3/12) + 400(t)}$$

$$1000 - 300(1 - 3/12) + 400(t) = \frac{200}{0.2055}$$

$$400t = \frac{200}{0.2055} + 300(3/4) - 1000$$

$$t = 0.4956$$

The closest date would be July 1st.

Answer: C

29. You can quickly eliminate A and B. A is a butterfly spread and B is the
negative of A. D can also be eliminated, because it is constant up to 950 and
then decreases up to 1000, while the graph here increases at the first point
where it is not constant. Thus we need only look at C to see if that works.
The profit function has the same shape as the payoff function, so we need
only look at the payoff of C.

$$\max(0, S - 950) - \max(0, S - 1000) + \max(0, S - 1020) - \max(0, S - 1050)$$

$$= \begin{cases} S < 950, & 0 \\ 950 < S < 1000, & S - 950 \\ 1000 < S < 1020, & S - 950 - (S - 1000) = 50 \\ 1020 < S < 1050, & 50 + S - 1020 = S - 970 \\ 1020 < S < 1050, & S - 970 - (S - 1050) = 80 \end{cases}$$

This has the proper form.

Answer: C

30. First, calculate the amount of the level payments for the original loan. The effective quarterly rate is

$$i = (1.0675)^{1/4} - 1 = 0.016464$$
$$36,000 = Ka_{\overline{40}|}$$
$$K = 1235.7762$$

Now, find the outstanding balance immediately after the 5th payment.   The easiest way to do this would be on the finance calculator:

PV = 36,000   I/Y = 1.6464   N = 5   PMT = -1235.7762
Compute FV = 32,677.0058

Subtract the extra $1000 that was added with the 5th payment and get an outstanding balance of 31,677.0058

Now, we need to find the outstanding balance immediately after the 10th payment, or 5 payments after this balance of 31,677.0058.   The easiest way to do this would be to use the finance calculator:

PV = 31,677.0058   I/Y = 1.6464   N = 5   PMT = -1235.7762
Compute FV = 27,986.2304

Subtract the extra $1000 that was added with the 10th payment and get an outstanding balance of 26,986.2304.

The interest of the 11th payment will be (26,986.2304*0.016464) = 444.30

Answer: C

31. Let j = X/100

The two-year accumulation factor for an investor starting in 2007 under the investment year method would be equal to:
$$(1 + i_1^{2007})(1 + i_2^{2007}) = (1 + j)(1.10)$$

The two-year accumulation factor for an investor starting in 2007 under the portfolio method would be equal to:
$$(1 + i^{2007})(1 + i^{2008}) = (1.093)(1.093) = 1.19465$$

Set this equal to the investment year method and solve for j.

$$1.19465 = (1 + j)(1.10)$$
$$j = 0.08604$$

Answer B

32. The easiest way to solve this problem is to remember the relationship between amortized premiums of payments.

The amortized amount in period k: $F(r - i)v^{n-k+1}$

The amortized amount in period k+m: $F(r - i)v^{n-k+1}(1 + i)^m$

We just need to multiply the amortized amount of the fourth payment by $(1 + i)^4$ to get the amortized amount of the eighth payment and then solve for i.

$$7.32(1 + i)^4 = 10.45$$
$$(1 + i)^4 = 1.4275956$$
$$i = 0.0930784$$

Then, multiply the amortized amount of the eighth payment by $(1 + i)^3$ to get the amortized amount of the 11th payment.

$$10.45(1 + i)^3 = 13.6480$$

Answer A

33. The deposits of X are an annual annuity due with the future value calculated at 1/1/2023.

$X\ddot{s}_{\overline{13}|0.07}$

The deposits of $2X$ can be thought of an annual annuity immediate with the future value calculated at 7/1/2022. So, the future value will need another six-month interest accumulation to bring it to 1/1/2023.

$2Xs_{\overline{13}|0.07}(\sqrt{1.07})$

The sum of these two terms should equal 15,000

$$2Xs_{\overline{13}|0.07}(\sqrt{1.07}) + X\ddot{s}_{\overline{13}|0.07} = 15,000$$
$$41.6673X + 21.5505X = 15,000$$
$$X = 237.27499$$

Answer B

34. You can create a risk-free investment by entering into a forward purchase at 50 and a forward sale at 55. This will always pay 5 in one year. You can use the options to create the forwards.

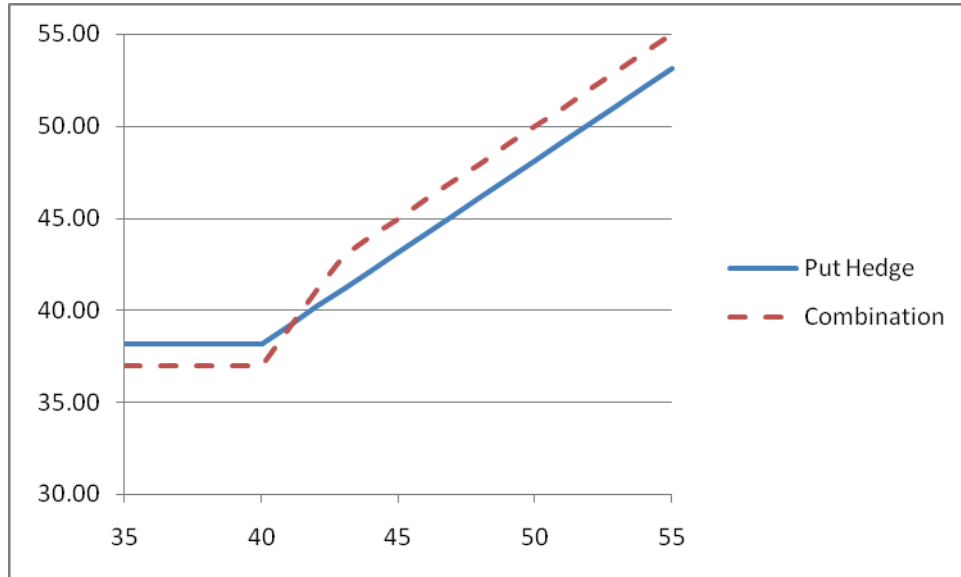Forward purchase at 50: Buy the 50-strike call and sell the 50-strike put. Cost: 5.10 – 4.61 = 0.49

Forward sale at 55: Buy the 55-strike put and sell the 50-strike call. Cost: 7.56 – 3.20 = 4.36

Total cost: 0.49 + 4.36 = 4.85

Effective interest rate: $\dfrac{5}{4.85} - 1 = .03$

Answer C

35. Option B is the pay later strategy given in Chapter 4. The graphs of the pay
    later and the put are shown below.

AnswerD

- You have Reached the Last Page -

We wish you exam success!