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NOTES

Questions and parts of some solutions have been taken from material copyrighted by the Casualty Actuarial Society and the Society of Actuaries. They are reproduced in this study manual with the permission of the CAS and SoA solely to aid students studying for the actuarial exams. Some editing of questions has been done. Students may also request past exams directly from both societies. I am very grateful to these organizations for their cooperation and permission to use this material. They are, of course, in no way responsible for the structure or accuracy of the manual.

Exam questions are identified by numbers in parentheses at the end of each question. CAS questions have four numbers separated by hyphens: the year of the exam, the number of the exam, the number of the question, and the points assigned. SoA or joint exam questions usually lack the number for points assigned. W indicates a written answer question; for questions of this type, the number of points assigned are also given. A indicates a question from the afternoon part of an exam. MC indicates that a multiple choice question has been converted into a true/false question.

Page references refer to Robert L. McDonald, *Derivatives Markets* (2006).

Although I have made a conscientious effort to eliminate mistakes and incorrect answers, I am certain some remain. I am very grateful to students who discovered errors in the past and encourage those of you who find others to bring them to my attention. Please check our web site for corrections subsequent to publication.

Hanover, NH 7/1/11
PJM

PAST CAS AND SoA EXAMINATION QUESTIONS ON MCDONALD 14

1. Barrier call option prices are shown in the table below. Each option has the same underlying asset and the same strike price.

Type of Option	Price	Barrier
Down and out	\$25	30,000
Up and out	15	50,000
Down and in	30	30,000
Up and in	X	50,000
Down rebate	25	30,000
Up rebate	20	50,000

Calculate X, the price of the up-and-in option.

A. \$20 B. \$25 C. \$30 D. \$35 E. \$40 (07S–3–34–2)

2. Consider a model with two stocks. Each stock pays dividends continuously at a rate proportional to its price. $S_j(t)$ denotes the price of one share of stock j at time t. Consider a claim maturing at time 3. The payoff of the claim is maximum $[S_1(3), S_2(3)]$. You are given:

i) $S_1(0) = \$100$
ii) $S_2(0) = \$200$
iii) Stock 1 pays dividends of amount $(.05)S_1(t)\,dt$ between time t and time (t + dt).
iv) Stock 2 pays dividends of amount $(.1)S_2(t)\,dt$ between time t and time (t + dt).
v) The price of a European option to exchange stock 2 for stock 1 at time 3 is \$10.

Calculate the price of the claim.

A. \$96 B. \$145 C. \$158 D. \$200 E. \$234 (07S–MFE–6)

3. Let S (t) denote the price at time t of a stock that pays dividends continuously at a rate proportional to its price. Consider a European gap option with expiration date T, T > 0. If the stock price at time T is greater than \$100, the payoff is S(T) – 90; otherwise, the payoff is zero. You are given:

i) S(0) = \$80
ii) The price of a European call option with expiration date T and strike price \$100 is \$4.
iii) The delta of the call option in ii) is .2.

Calculate the price of the gap option.

A. \$3.60 B. \$5.20 C. \$6.40 D. \$10.80
E. There is not enough information to solve the problem. (07S–MFE–17)

4. Which one of the following statements is true about exotic options?

A. Asian options are worth more than European options.
B. Barrier options have a lower premium than standard options.
C. Gap options cannot be priced with the Black-Scholes formula.
D. Compound options can be priced with the Black-Scholes formula.
E. Asian options are path-independent options. (07F–3–26–2)

1. Ordinary Option =
   Down-and-Out Option + Down-and-In Option = Up-and-Out Option + Up-and-In Option

   $25 + 30 = 15 + X \qquad X = 40$, pp. 450–51.

   Answer: E

2. Price = Prepaid Forward Price for Delivery of S_2 at Time 3 + Price of European Exchange Option

   Price $= S_2(0)e^{-(.1)(3)} + 10 = (200)(.74082) + 10 = 158.16$, p. 459.

   Answer: C

3. $\Delta = .2 = e^{-\delta T} N(d_1)$

   $$e^{-rT} N(d_2) = \frac{Se^{-\delta T} N(d_1) - C}{K_2} = \frac{(80)(.2) - 4}{100} = .12$$

   $C(S, K_1, K_2, \sigma, r, T, \delta) = Se^{-\delta T} N(d_1) - K_1 e^{-rT} N(d_2) = (80)(.2) - (90)(.12) = 5.20$,
   pp. 383, 457–58.

   Answer: B

4. A. F, p. 445 – Substitute "less" for "more."

   B. T, p. 449

   C. F, p. 457 – Substitute "can" for "cannot."

   D. F, p. 454 – Substitute "cannot" for "can."

   E. F, p. 444 – Substitute "path-dependent" for "path-independent."

   Answer: B

5. At the start of the year, a speculator purchases a six-month geometric average price call option on a company's stock. The strike price is 3.5. The payoff is based on an evaluation of the stock price at each month's end. Based on the stock prices below, calculate the payoff of the option.

Date	1/31	2/28	3/31	4/30	5/31	6/30
Stock price	1.27	4.11	5.10	5.50	5.13	4.70

A. < .3 B. ≥ .3 but < .75 C. ≥ .75 but < 1.00 D. ≥ 1.00 but < 1.75 E. ≥ 1.75 (07F–3–27–2)

6. You are given the following information on a compound CallOnPut option:

i) The continuously compounded risk-free rate is 5%.
ii) The strike price of the underlying option is 43 and that of the compound option is 3.
iv) The compound option expires in six months.
v) The underlying option expires six months after the compound option.
vi) The underlying option is American.

Today	6 Months	12 Months
		$S_{HH} = 64.52$
	$S_H = 50.80$	
$S_0 = 40.00$		$S_{HL} = 42.16$
	$S_L = 33.20$	
		$S_{LL} = 27.56$

Based on the above binomial stock price tree, calculate the value of the compound option.

A. < 3.00 B. ≥ 3.00 but < 3.50 C. ≥ 3.50 but < 4.00 D. ≥ 4.00 but < 4.50 E. ≥ 4.50 (07F–3–28–2)

7. A market-maker sells 1.000 one-year European gap call options, and delta-hedges the position with shares. You are given:

i) Each gap call option is written on one share of a non-dividend-paying stock.
ii) The current price of the stock is \$100.
iii) The stock's volatility is 100%.
iv) Each gap call option has a strike price of \$130 and a payment trigger of \$100.
vi) The risk-free interest rate is 0%.

Under the Black-Scholes framework, determine the initial number of shares in the delta hedge.

A. 586 B. 594 C. 684 D. 692 E. 797 (Sample–MFE–18)

8. Consider a forward start option which, one year from today, will give its owner a one-year European call option with a strike price equal to the stock price at that time. You are given:

i) The European call option is on a stock that pays no dividends.
ii) The stock's volatility is 30%.
iii) The forward price for delivery of one share of the stock one year from today is \$100.
iv) The continuously compounded risk-free interest rate is 8%.

Under the Black-Scholes framework, determine the price today of the forward start option.

A. \$11.90 B. \$13.10 C. \$14.50 D. \$15.70 E. \$ 16.80 (Sample–MFE–19)

5. $G(t) = [(1.27)(4.11)(5.10)(5.50)(5.13)(4.70)]^{1/6} = 3.90$
Payoff $= \max[0, G(t) - K] = \max[0, 3.90 - 3.50] = .40$, p. 446.

Answer: B

6. 1) Calculate the probability of a rise:
$u = 50.80/40 = 1.27 \qquad d = 33.20/40 = .83$
$$p^* = \frac{e^{rT} - d}{u - d} = \frac{e^{(0.05)(.5)} - .83}{1.27 - .83} = .44$$
2) Calculate the values of the put at the end of one year:
$f_{uu} = 0 \qquad f_{du} = f_{ud} = 43 - 42.16 = .84 \qquad f_{dd} = 43 - 27.56 = 15.44$
3) Calculate the value of the put at the end of six months if the stock decreases during that period if the option is exercised and if it is not:
f_d if Exercised $= 43 - 33.20 = 9.80$
f_d if Not Exercised $= [(.44)(.84) + (.56)(15.44)]/e^{(.05)(.5)} = 8.79$
4) Calculate the value of the put at the end of six months if the stock increases during that period if the option is exercised and if it is not:
f_u if Exercised $= 0$
f_u if Not Exercised $= [(.44)(0) + (.56)(.84)]/e^{(.05)(.5)} = .46$
5) Calculate the outcome of the CallOnPut option at the end of six months:
$f'_u = 0 \qquad f'_d = 9.80 - 3 = 6.80$
6) Calculate the current value of the CallOnPut option:
$f = e^{-rT}[p^*f_u + (1 - p^*)f_d] = e^{-(.05)(.5)}[(.44)(0) + (.56)(6.80)] = 3.71$, pp. 313–30, 453–54.

Answer: C

7. $C_{gap}(S, K_1, K_2, \sigma, r, T, \delta) = Se^{-\delta T} N(d_1) - K_1e^{-rT} N(d_2) = Se^{-0T} N(d_1) - 130e^{-0T} N(d_2)$
$C_{gap} = [SN(d_1) - 100N(d_2)] - 30N(d_2)$
$$d_1 = \frac{\ln(Se^{-\delta T}/K_2e^{-rT}) + .5\sigma^2 T}{\sigma\sqrt{T}} = \frac{\ln(100e^{-0T}/100e^{-0T}) + .5(1)^2(1)}{(1)\sqrt{1}} = .5$$
$d_2 = d_1 - \sigma\sqrt{T} = .5 - (1)(1) = -.5$
$$\Delta_{gap} = N(d_1) - 30N'(d_2)\frac{\partial d_2}{\partial S} = N(d_1) - 30N'(d_2)\frac{\partial d_2}{\partial S} = N(d_1) - 30N'(d_2)(1/S\sigma\sqrt{T}\,))$$
$\Delta_{gap} = N(.5) - \{30N'(-.5)\}\{1/[(100)(1)(1)]\} = .69146 - (.3)(.35207) = .58584$
$1{,}000\Delta_{gap} = (1{,}000)(.58584) = 586$, pp. 383, 457–58.

Answer: A

8. 1) Calculate the price of a call option at time 1:
$$d_1 = \frac{\ln(S_1/S_1) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} = \frac{\ln 1 + [.08 + (.3)^2/2][1]}{.3\sqrt{1}} = .417$$
$d_2 = d_1 - \sigma\sqrt{T} = .41667 - .3\sqrt{1} = .117$
$C(S_1) = S_1N(d_1) - S_1e^{-rT} N(d_2) = S_1N(.417) - S_1e^{-.08}N(.117)$
$C(S_1) = (S_1)(.66166) - (S_1)(.92312)(.54657) = .15711S_1$
2) Calculate the price today of the forward start option:
Price Today $= .15711\, F^P_{0,1}(S) = .15711\, e^{-r} F_{0,1}(S) = (.15711)(e^{-.08})(100) = 14.50$,
pp. 128–29, 377, 465.

Answer: C

9. Consider a chooser option (also known as an as-you-like-it option) on a non-dividend-paying stock. At time 1, its holder will choose whether it becomes a European call option or a European put option, each of which will expire at time 3 with a strike price of \$100. The chooser option price is \$20 at time $t = 0$. The stock price is \$95 at time $t = 0$. Let $C(T)$ denote the price of a European call option at time $t = 0$ on the stock expiring at time T, $T > 0$, with a strike price of \$100. You are given that the risk-free interest rate is 0 and $C(1) = \$4$. Determine $C(3)$.

   A. \$9 B. \$11 C. \$13 D. \$15 E. \$17 (Sample–MFE–25)

10. Assume the Black-Scholes framework. You are given:

   i) $S(t)$ is the price of a non-dividend-paying stock at time t.
   ii) $S(0) = 10$
   iii) The stock's volatility is 20%.
   iv) The continuously compounded risk-free interest rate is 2%.

   At time $t = 0$, you write a one-year European option that pays 100 if $[S(1)]^2$ is greater than 100 and pays nothing otherwise. You delta-hedge your commitment. Calculate the number of shares of the stock for your hedging program at time $t = 0$.

   A. 20 B. 30 C. 40 D. 50 E. 60 (Sample–MFE–28)

11. You have observed the following monthly closing prices for stock XYZ:

1/31/08	105	2/29/08	120
3/31/08	115	4/30/08	110
5/31/08	115	6/30/08	110
7/31/08	100	8/31/08	90
9/30/08	105	10/31/08	125
11/30/08	110	12/31/08	115

   The following are one-year European options on stock XYZ. The options were issued on 12/31/2007.

   i) An arithmetic average Asian call option with a strike of 100. (The average is calculated based on monthly closing stock price.)
   ii) An up-and-out call option with a barrier of 125 and a strike of 120.
   iii) An up-and-in call option with a barrier of 120 and a strike of 110.

   Calculate the difference in payoffs between the largest and the smallest payoffs of the three options.

   A. 5 B. 10 C. 15 D. 20 E. 25 (09S–MFE–2)

12. You own one share of a non-dividend-paying stock. You worry that its price may drop over the next year and decide to employ a rolling insurance strategy, which entails obtaining one three-month European put option on the stock every three months, with the first one being bought immediately. You are given:

   i) The continuously compounded risk-free interest rate is 8%.
   ii) The stock's volatility is 30%.
   iii) The current stock price is 45.
   iv) The strike price for each option is 90% of the then-current stock price.

   Your broker will sell you the four options but will charge you for their total cost now. Under the Black-Scholes framework, how much do you now pay your broker?

   A. 1.59 B. 2.24 C. 2.85 D. 3.48 E. 3.61 (Sample–MFE–33)

9. See exercise 14.20b. At the choice date, the payoff equals:
$\text{Max}[C(S_1, 100, 2), P(S_1, 100, 2)] = C(S_1, 100, 2) + \text{Max}[0, P(S_1, 100, 2) - C(S_1, 100, 2)]$
But since the interest rate is 0%, according to the put-call parity;
$P(S_1, 100, 2) - C(S_1, 100, 2) = K - S_1$, we get:
$\text{Max}[C(S_1, 100, 2), P(S_1, 100, 2)] = C(S_1, 100, 2) + \text{Max}[0, K - S_1]$
Since the second term is the payoff for a European put option, the value of the chooser option at t = 0 equals:
$20 = \text{Chooser Option} = C(S_0, 100, 3) + P(S_0, 100, 1)$
But since the interest rate is 0%, according to the put-call parity:
$P(S_0, 100, 1) = C(S_0, 100, 1) + K - S_0 = C(S_0, 100, 1) + 100 - 95 = 4 + 5 = 9$
$20 = \text{Chooser Option} = C(S_0, 100, 3) + 9 \qquad C(S_0, 100, 3) = 20 - 9 = 11$, pp. 283, 465.
Answer: B

10. $$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} = \frac{\ln(10/10) + [.02 + (.2)^2/2][1]}{.2\sqrt{1}} = .2$$
$d_2 = d_1 - \sigma\sqrt{T} = .2 - .2\sqrt{1} = 0 \qquad K = \sqrt{100} = 10$
Option Price = [Payoff][PV Factor][Probability (Asset Price > Strike Price)] = $100e^{-.02}N(d_2)$
$$\text{Shares Needed for Hedging} = \frac{\partial}{\partial S}100e^{-.02}N(d_2) = 98.02\,N'(d_2)\frac{\partial d_2}{\partial S}$$
$$\text{SNFH} = \frac{98.02\,N'(0)}{S\sigma\sqrt{T}} = \frac{(98.02)(.39894)}{(10)(.2)\sqrt{1}} = 19.55$$, pp. 377, 383, 457–58.

Answer: A

11. i) $(105 + 120 + 115 + 110 + 115 + 110 + 100 + 90 + 105 + 125 + 110 + 115)/12 = 110$
$\text{Payoff}_A = \text{Average} - \text{Strike Price} = 110 - 100 = 10$, pp. 446–47.
ii) Since the price reaches 125 on 10/31/08, the option ends at that time and does not exist at the end of the year and there is no payoff, p. 450.
iii) Since the price reaches 120 on 2/29/08, it comes into existence at that point and exists at the end of the year.
$\text{Payoff}_{U\&I} = \text{Ending Price} - \text{Strike Price} = 115 - 110 = 5$, p. 450.
$\text{Difference} = \text{Largest Payoff} - \text{Smallest Payoff} = 10 - 0 = 10$

Answer: B

12. See exercise 14.22.
1) Calculate the price of a three-month put;
$$d_1 = \frac{\ln(S/.9S) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} = \frac{\ln(10/9) + [.08 + (.3)^2/2][.25]}{.3\sqrt{.25}} = .911$$
$d_2 = d_1 - \sigma\sqrt{T} = .911 - .3\sqrt{.25} = .761$
$P = Ke^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1) = .9Se^{-.08/4}N(-.761) - SN(-.911)$
$P = (.9S)(.98020)(.22333) - .18115S = .01587S$
2) Calculate the total price of four rolling three-month puts. This is the sum of their forward prices at time 0. Since when there are no dividends, the price of the prepaid forward contract is the stock price today, we get:
$\text{Payment} = 4P = (4)(.01587S_0) = (.06352)(45) = 2.8566$, pp. 129, 377–78, 465.

Answer: C

13. Prices for six-month 60-strike European up-and-out call options on a stock S are available. Below is a table of option prices with respect to various values of H, the level of the barrier. Here S(0) = 50.

H	60	70	80	90	∞
Price of up-and-out call	0	.1294	.7583	1.6616	4.0861

Consider a special six-month 60-strike European knock-in, partial knock-out call option that knocks in at H_1 = 70, and partially knocks out at H_2 = 80. The strike price of the option is 60. The following table summarizes the payoff at the exercise date.

H_1 not hit	0
H_1 hit and H_2 not hit	2 max[S(.5) – 60, 0]
H_1 hit and H_2 hit	max[S(.5) – 60, 0]

Calculate the price of the option.

A. .6289 B. 1.3872 C. 2.1455 D. 4.5856
E. It cannot be determined from the information given above. (Sample–MFE–42)

14. Assume the Black-Scholes framework. Consider two non-dividend-paying stocks whose time-t prices are denoted by $S_1(t)$ and $S_2(t)$. You are given:

i) $S_1(0) = 10$ and $S_2(0) = 20$.
ii) Stock 1's volatility is .18.
iii) Stock 2's volatility is .25.
iv) The correlation between the continuously compounded returns of the two stocks is .40.
v) The continuously compounded risk-free interest rate is 5%.
vi) A one-year European option with payoff $\max\{\min[2S_1(1), S_2(1)] - 17, 0\}$ has a current (time–0) price of 1.632.

Consider a European option that gives its holder the right to sell either two shares of stock 1 or one share of stock 2 at price of 17 one year from now. Calculate the current (time-0) price of this option.

A. .66 B. 1.12 C. 1.49 D. 5.18 E. 7.86 (Sample–MFE–54)

13. Buying the knock-in partial knock-out call is equivalent to buying two up-and-in calls with strike 60 and barrier 70 and selling one up-and-in call with strike 60 and barrier 80. Since an up-and-in call equals a ordinary call less an up-and-out call and a ordinary call is the same as an up-and-out call with a barrier of infinity, we get:

KI-KO = 2(Ordinary with Strike 60 – Up & Out with Strike 60 and Barrier 70) –
(Ordinary with Strike 60 – Up & Out with Strike 60 and Barrier 80)

KI-KO = 2(4.0861 – .1294) – (4.0861 – .7583) = 4.5856, pp. 450–51.

Answer: D

14. The described option is equivalent to a 17-strike put on $\min[2S_1(1), S_2(1)]$ defined as $M(1)$, since its payoff is the difference between the lower of the prices of $2S_1(1)$ and $S_2(1)$. Since $M(T) = 2S_1(T) - \max[2S_1(T), S_2(T)]$, and $\max[2S_1(T), S_2(T)]$ is the payoff on an exchange option, we can calculate the price of the exchange option and the prepaid forward price of $M(1)$ and use the put-call parity to calculate the price of the described option.

1) Calculate the variance of the two stocks:

$$\sigma = \sqrt{\sigma^2{}_1 + \sigma^2{}_2 + 2\rho\sigma_1\sigma_2} = \sqrt{(.18)^2 + (.25)^2 + (2)(.4)(.18)(.25)} = .36180$$

2) Calculate the price of the exchange option:

$$d_1 = \frac{\ln[2S_1(0)/S_2(0)] + .5\sigma^2 T}{\sigma\sqrt{T}} = \frac{\ln[(2)(10)/20] + [(.3618)^2/2][1]}{.3618\sqrt{1}} = .181$$

$$d_2 = d_1 - \sigma\sqrt{T} = .181 - .3618\sqrt{1} = -.181$$

$$C = 2S_1(0)\,N(d_1) - S_2(0)\,N(d_2) = 20N(.181) - 20N(-.181)$$

$$C = (20)(.57181) - (20)(.42819) = 2.87$$

3) Calculate the prepaid forward price of M(1):

$$F^P_{0,1} = 2S_1(0) - C = (2)(10) - 2.87 = 17.13$$

4) Calculate the price of the described option:

$$P(K, T) = C(K, T) - F^P_{0,1} + Ke^{-rT} = 1.632 - 17.13 + 17e^{-(.05)(1)} = .67290,$$
pp. 283, 459–60.

Answer: A

