A PROBLEM-SOLVING APPROACH TO PENSION FUNDING AND VALUATION

Second Edition

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The ankh was the ancient Egyptian symbol for life.
Its use on the cover of this text symbolizes the payment of pension income for life.

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Chapter 2

Increasing Cost Individual Cost Methods

In this chapter we learn how to calculate the cost *each year* of a piece of pension benefit. Younger participants have low costs because we discount at interest for many years. Older participants have higher costs because we discount for fewer years. (In the following two chapters we calculate the *levelized* annual cost of the expected pension.)

2.1 Traditional Unit Credit

Traditional Unit Credit (TUC) is the first actuarial cost method we will study. The TUC actuarial liability is the value, at the valuation date, of the pension benefit accrued from the date of entry into the plan to the date of valuation. This method is also called the **Unit Credit Cost Method** by ERISA and by Anderson [2], and the **Accrued Benefit Cost Method** by McGill and Grubbs [7]. A salary scale assumption for salaries in all future years is *not* made with this method; with other methods, a salary scale assumption is often made.

If a participant is to retire at age r with an expected annual pension of B_r, with one-twelfth of B_r payable at the beginning of each month, then $B_r \cdot \ddot{a}_r^{(12)}$ is sufficient to fund this pension at age r. B_r denotes the benefit accrued (or earned) during the participant's active years of service, from age e to age r.

Let us consider a pension fund valuation at time 0 for a participant who is age x at time 0. The annual pension benefit which has accrued from age e to age x is usually a certain number of dollars per month for each year of service. The annual benefit that has accrued to age x is denoted by B_x. For example, if the pension benefit is \$30 per month for each of 20 years of service, then $B_x = 30 \times 12 \times 20 =$ \$7200. The actuarial liability at age x is the value of the pension benefit accrued from age e to age x. It is given by

$$AL_x = B_x \cdot \frac{D_r^{(\tau)}}{D_x^{(\tau)}} \cdot \ddot{a}_r^{(12)},$$

where $D_r^{(\tau)}/D_x^{(\tau)}$ is computed from a service table. The calculation includes interest and decrements such as mortality, withdrawal, disability, and discharge. Under the Unit Credit method, we obtain the total actuarial liability (TAL) at time 0 for all active participants in a pension plan by summing the individual liabilities for each active participant, obtaining

$$TAL_0 = \sum AL_x = \sum B_x \cdot \frac{D_r^{(\tau)}}{D_x^{(\tau)}} \cdot \ddot{a}_r^{(12)}.$$

The liability for each participant increases with age. It follows that, if no participants leave the group and no new participants enter the group, the total liability will increase with time.

The TUC cost method is most often used with pension plans that provide a flat pension benefit, such as \$30 per month for each year of service. If the entry age is 35 and the retirement age is 65, the annual pension benefit commencing at retirement will be $30 \times 30 \times 12 =$ \$10,800. At age 45, after ten years of service, the accrued pension benefit will be $B_{45} = 30 \times 10 \times 12 =$ \$3600.

b_x denotes the piece of the total pension benefit that is earned in the year following age x. The simplest case is the one in which the same benefit is earned each year, so that

$$b_x = \frac{B_r}{r - e}.$$

This case will be assumed to apply unless otherwise stated. Therefore in our example of a benefit of \$30 per month for each year of service, we have $b_x = 30 \times 12 =$ \$360.

The normal cost at the beginning of each year is the cost of the pension benefit that is earned (accrues) in that year. It is given by

$$NC_x = b_x \cdot \frac{D_r^{(\tau)}}{D_x^{(\tau)}} \cdot \ddot{a}_r^{(12)}.$$

The normal cost for younger participants is lower due to the greater effect of discounting in the $D_r^{(\tau)}/D_x^{(\tau)}$ term. The total normal cost for the plan each year is the sum of the normal costs for all participants receiving benefit accruals. The total normal cost in future years will be affected by aging, the size of the unit benefit, withdrawals, retirements, deaths, and new entrants.

In single-decrement situations, we will often use the simple variations

$$NC_x = b_x \cdot \frac{N_r^{(12)}}{D_x} = b_x \cdot v^t \, {}_tp_x \cdot \ddot{a}_r^{(12)},$$

where $t = r - x$, and

$$AL_x = NC_x(x-e).$$

DISCUSSION QUESTIONS

2-1 Write an expression for NC_{44}, where the benefit is \$35 per month for each year of service.

2-2 Write an expression for the total actuarial liability for all active, terminated, and retired participants.

2-3 What is the normal cost for a retired or terminated participant?

EXAMPLE 2.1

Plan effective date: 1/1/84
Normal retirement benefit: \$30 per month for each year of service
All employees were hired at age 25.
Retired or terminated vested participants: None
Preretirement terminations other than by death: None
Selected annuity value: $\ddot{a}_{65}^{(12)} = 10$
Census data on 1/1/94, and commutation functions:

Age x	Participants	D_x
25	8	16
35	0	8
45	2	4
55	0	2
65	0	1

What is the TUC actuarial liability and normal cost as of 1/1/94?

SOLUTION Let 1/1/94 be time 0. The participants who are age 25 have just been hired, so $B_{25} = 0$ since no benefit has yet accrued. For each of the two participants who are attained age 45, we have a benefit accrual of $B_{45} = 30 \times 12 \times 20 = 7200$. Then

$$AL_{45} = 7200 \cdot \frac{D_{65}}{D_{45}} \cdot \ddot{a}_{65}^{(12)} = 18{,}000$$

and the total liability is

$$TAL_0 = 2 \cdot AL_{45} = \$36{,}000.$$

The normal cost for the plan is the sum of the several NC_x, so

$$TNC_0 = \sum b_x \cdot \frac{D_r}{D_x} \cdot \ddot{a}_r^{(12)},$$

where $b_x = 30 \times 12 = 360$. Thus we have

$$\begin{aligned} TNC_0 &= 8\left[360 \cdot \frac{D_{65}}{D_{25}} \cdot \ddot{a}_{65}^{(12)}\right] + 2\left[360 \cdot \frac{D_{65}}{D_{45}} \cdot \ddot{a}_{65}^{(12)}\right] \\ &= 360\ddot{a}_{65}^{(12)}\,[8(1/16) + 2(1/4)] = \$3600. \end{aligned}$$ □

Note that because there are no decrements other than death, we can use an appropriate single-decrement table (mortality only). This explains the use of D_x rather than $D_x^{(\tau)}$.

EXAMPLE 2.2

Normal retirement benefit: $10 per month for each year of service
Actuarial cost method: Unit Credit
Actuarial assumptions:

Interest: 6%
Preretirement terminations other than deaths: None
Retirement age: 65
Participants as of 1/1/93: 100 active employees, all age 60
Normal cost for 1993 as of 1/1/93: $100,000
Selected mortality value: $q_{60} = .04$

Calculate the normal cost for 1994 as of 1/1/94 (a) per survivor, (b) for the total group if 92 participants are alive at 1/1/94, (c) if 96 participants are alive, and (d) if all participants are alive.

SOLUTION The normal cost per participant at age 60 is

$$NC_{60} = (10 \times 12)v^5 \, {}_5p_{60} \, \ddot{a}_{65}^{(12)},$$

and the normal cost per survivor at age 61 is

$$NC_{61} = (10 \times 12)v^4 \, {}_4p_{61} \, \ddot{a}_{65}^{(12)}.$$

Note that $\frac{NC_{60}}{NC_{61}} = vp_{60}$. Then we have the following results.

(a) $$NC_{61} = \frac{NC_{60}}{vp_{60}} = \frac{100{,}000/100}{.96/1.06} = \frac{1060}{.96} = \$1104.17.$$

(b) $$TNC_{61} = 92\left(\frac{1060}{.96}\right) = \$101{,}583.$$

(c) $$TNC_{61} = 96\left(\frac{1060}{.96}\right) = \$106{,}000.$$

(d) $$TNC_{61} = 100\left(\frac{1060}{.96}\right) = \$110{,}417.$$

Note that in (a) the normal cost per survivor does not depend on the actual mortality experience; in (b) more than expected die so there is a gain from mortality (see Chapter 5); in (c) the expected mortality is exactly realized, so the *TNC* at 1/1/94 is $100{,}000(1+i)$; in (d) there is a loss from mortality.

□

If F_0 is the amount of the pension fund at time 0 and TAL_0 represents the plan's total actuarial liability for all active, retired and terminated vested participants at time 0, then the surplus at that time is $F_0 - TAL_0$.

Traditionally, most plans had, and many plans still have, a *negative* sur-plus, called the unfunded actuarial liability, where

$$UAL_0 = TAL_0 - F_0.$$

The disposition of an unfunded actuarial liability is an important actuarial matter; it is discussed in Section 3.6.

EXAMPLE 2.3
Refer to the data given in Example 2.1. Under the Unit Credit cost method, what is the unfunded actuarial liability as of 1/1/94 if the plan assets amount to $5000 at that time?

SOLUTION We need the total actuarial liability for the plan since the unfunded liability at time 0 is

$$UAL_0 = TAL_0 - F_0.$$

From Example 2.1 we have $TAL_0 = \$36,000$, so

$$UAL_0 = 36,000 - 5,000 = \$31,000. \qquad \square$$

The fund balance at the beginning of the year (BOY), which we have denoted by F_0, will increase during the year by actual investment income and contributions to the fund. It will be diminished by amounts withdrawn from the fund as benefits. At time 0, we can calculate what we *expect* the unfunded actuarial liability to be at time 1 as

$$^{exp}UAL_1 = (UAL_0 + NC_0)(1+i) - {}^{i}C,$$

where ${}^{i}C$ is the contribution plus the interest earned during the year on the contribution using the actuarial interest assumption. If the contribution is made at the end of the year (EOY), then ${}^{i}C = C$, and if it is made at BOY, then ${}^{i}C = C(1+i)$. A total experience gain, ${}^{tot}G$, will result if the *actual* unfunded actuarial liability (${}^{act}UAL$) is less than ${}^{exp}UAL$. In other words

$$^{tot}G_1 = {}^{exp}UAL_1 - {}^{act}UAL_1.$$

A negative gain is called a loss. The relationship between UAL_0 and UAL_1 is developed, and the subject of gains and losses is discussed, in Chapter 5.

EXAMPLE 2.4

Actuarial cost method: Unit Credit
Assumed interest rate: 6%
Valuation results as of 1/1/93:

Actuarial liability	\$100,000
Actuarial value of assets	50,000
Normal cost as of 12/31/93	10,000

Valuation results as of 1/1/94:

Actuarial liability	\$115,000
Actuarial value of assets	70,000

Contributions:
\$13,910 at 12/31/93
\$15,587 at 12/31/94

What is the experience gain for 1993?

SOLUTION Let 1/1/93 be time 0 and 1/1/94 be time 1. There is a gain when the actual unfunded liability turns out to be less than the expected. Since the normal cost is at 12/31/93 and ${}^{i}C = C$, then the expected unfunded liability at time 1 is

$$\begin{aligned} {}^{\text{exp}}UAL_1 &= UAL_0(1+i) + NC - C \\ &= (100{,}000-50{,}000)(1.06) + 10{,}000 - 13{,}910 \\ &= 49{,}090. \end{aligned}$$

The actual unfunded liability at time 1 is the difference between AL_1 and F_1, so

$$ {}^{\text{act}}UAL_1 = 115{,}000 - 70{,}000 = 45{,}000. $$

Then

$$\begin{aligned} {}^{\text{tot}}G &= {}^{\text{exp}}UAL - {}^{\text{act}}UAL \\ &= 49{,}090 - 45{,}000 = \$4{,}090. \end{aligned}$$ □

Note that the normal cost is usually at BOY but occasionally, as in Example 2.4, it is at EOY; the contribution is usually at EOY but occasionally it is at BOY or mid-year.

EXAMPLE 2.5

Which of the following statements concerning the Unit Credit cost method are true?

I. Under this method, the assumption must be made that each participant will remain in the plan until retirement or prior death.

II. If the benefit accrual in each year is constant for any given participant, the normal cost for that participant will also remain constant, provided actual experience is in accordance with actuarial assumptions.

III. The actuarial liability of a newly established plan is equal to the present value of the benefits attributable to credited service prior to the effective date of the plan.

SOLUTION

I. Not true. We could use other decrements such as withdrawal or disability.

II. Not true. The effect of mortality and interest discount is reducing with age.

III. True. □

EXAMPLE 2.6

Normal retirement benefit: $10 per month for each year of service
Vesting eligibility: 100% after 5 years of service
Preretirement death benefit: None
Actuarial cost method: Unit Credit
Actuarial assumptions:
  Interest rate: 7% per year
  Preretirement terminations other than deaths: EOY
  $q_x^{(d)} = q_x^{\prime(d)}$
  $q_x^{(\tau)} = q_x^{(d)} + q_x^{(w)}$
  Retirement age: 65
  Selected annuity value: $\ddot{a}_{65}^{(12)} = 8.736$
Data for sole participant:

Date of birth 1/1/31
Date of hire 1/1/89
Status as of 1/1/94 Active

Selected probabilities:

x	$q_x^{(\tau)}$	$q_x^{(d)}$
63	.069	.019
64	.081	.021
65	.023	.023

What is the normal cost for 1994 as of 1/1/94?

SOLUTION The participant is 63 years old on 1/1/94. Then

$$NC_{63} = 120 \times 8.736 \times v^2 \, {}_2p_{63}^{(\tau)}$$

$$= 1048.32 \times v^2(1-.019)(1-.021) = \$879.38.$$

- In this defined benefit plan with no participant contributions, 100% **vesting** means that the *participant* is entitled to 100% of the retirement benefit accrued to the date of withdrawal. If withdrawal occurs before five years of service, no retirement benefit is payable because there is no vesting. 60% vesting, for example, would mean that the participant is entitled to 60% of the accrued benefit.

- Since deaths occur at EOY, $q_x^{(d)} = q_x^{\prime(d)}$, and we discount for mortality because the death benefit is zero. If, alternatively, the terminal reserve were paid out on death, we would not discount for mortality. Similarly, we do not discount for withdrawal because vesting is 100%, and the unit credit liability is not released. The student may wish to review these important aspects of contingency theory. (See, for example, Chapter 9 of Bowers, *et al.* [5].) □

Plan experience and changes in plan benefits will affect plan liabilities and costs. These are discussed in Section 3.6 and Chapters 5 and 6. If death and withdrawal benefits are not stated, they are assumed to be zero; they are discussed in more detail in Chapter 5.

2.2 Benefits with a Salary Scale

The benefit formulas in Section 2.1 did not make use of projected future salary, but those of Section 2.3 will. This section will introduce the idea of **salary scales** themselves.

If a salary scale is not being used in the benefit projection, it would appear that the actuary is assuming that salaries are not expected to increase. The effect of such an assumption is normally to shift a portion of the costs from the present to the future. This is often not appropriate.

If the actuary does use a salary scale, it may be a scale independent of age, such as 5% per year, or it may be a more sophisticated scale that depends on age. In any event, it will make due allowance for inflation, actual past salaries, and expected future salaries which are also discussed in Sections 3.2 and 3.3.

We will illustrate the use of a 5% scale by calculating the pension benefits for (a) a 2% final salary plan, (b) a 2% final three-year average plan, and (c) a 2% career average plan.

(a) For a person currently age x with a salary of S_x, the expected (or projected) final salary, at age $r-1$, is

$$S_{r-1} = (1.05)^{r-1-x} S_x,$$

and the pension benefit accrued to age x is

$$B_x = .02(1.05)^{r-1-x} S_x \cdot (x-e).$$

(b) The expected final three-year average salary is

$$\begin{aligned} FAS &= \frac{1}{3}\left[S_{r-3}+S_{r-2}+S_{r-1}\right] \\ &= \frac{1}{3}\left[(1.05)^{r-3-x}+(1.05)^{r-2-x}+(1.05)^{r-1-x}\right]S_x \\ &= \frac{1}{3}(1.05)^{r-1-x} S_x \cdot \ddot{a}_{\overline{3}|.05}, \end{aligned}$$

and the accrued pension benefit is