Guo's QuickStart Online Course

This online course is right for you if
1. **Have roughly 12 weeks before your exam date**
2. **You failed Exam P at least once. You need an alternative way to study.**
3. **You are a career switcher, too busy to take a class in the classroom.**
4. **You are a college student, but your major is not statistics or math.**
5. **You want to learn how to really pass Exam P.**
6. **You find Exam P overwhelming and you need help.**

Key Features:
- **You can enroll at any time and study anywhere**
- **Ideal for career switchers and college students**
- **Over 70 videos lessons ranging from calculus review to probability theories to various probability distributions.**
- **Designed to help you pass Exam P in one try**
- **10 full length computer based exams simulating CBT**
- **Intensive drills sharpening your understanding and problem solving skills**

Prior coursework in statistics and probability is NOT required. If you have strong calculus, I'll teach you probability.

My P manual and this online course is all you need to pass Exam P. You don't need to buy another textbooks recommended by SOA. Nor do you need to buy any other study manuals.

For detailed description of this online class, turn to the next page.

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Guo's Online Course Exam P

Overview of the online course

This online course is designed to help you pass the SOA Exam P/CAS 1. It explains, in simple English and down-to-earth examples, the complex concepts and intimidating formulas commonly tested in Exam P. It uses practice drills to help you develop the essential problem solving skills for Exam P.

To help you get used to the computer based testing (CBT), this online course offers 10 full-length timed online mock exams that simulate CBT.

The SOA syllabus for Exam P contains a vast amount of knowledge, from abstract concepts (such as sample space) to difficult probability distributions (such as lognormal and Pareto). This online course will help you navigate through the Exam P jungle, teaching you the core concepts and problem solving skills necessary to pass Exam P.

Curriculum philosophy

This online class is designed on the philosophy that to pass an actuary exam an exam candidate needs to do things: (1) to gain an in-depth knowledge of the core concepts (i.e. "deeper understanding"), and (2) to learn how to quickly solve a thorny problems with tricks and shortcuts ("faster calculation").

Deeper understanding of the core concepts boots candidates' confidence and helps them formulate solutions to new types of problems in the exam. Shortcuts enable candidates to quickly solve a frequently tested types of problems. With both deeper understanding and faster calculation, an exam candidate can pass exam P with a wide margin.

Curriculum structure

The class is designed for about 12 weeks, but you'll be given 100 days access to the website.

Before enrolling in this class, make sure you have roughly 12 weeks to prepare for Exam P. However, if you are an advanced student, you may need less than 12 weeks to fully take advantage of this online class.

Week	Lesson
1	**Class starts. Calculus Review.**
2	**Probability basics.**
3	**Counting, conditional probability, Bayes' Theorem**
4	**Independence & Random Variables**
5	**Discrete & continuous random variables**
6	**Multivariate distribution**
7	**Transformation & common distributions**
8	**Order statistics, moment generating function, risk & insurance**
9	**Marathon session starts. Practice Exam 1,2,3**
10	**Practice Exam 4,5,6**
11	**Practice Exam 7,8**
12	**Practice Exam 9,10**

Enroll at any time. Study now. Study anywhere. Study at your own pace

This online class offers great flexibility. You can enroll at any time, study anywhere, and study at your own pace. As long as you have a computer and internet connect, you can travel anywhere in the world and prepare for Exam P.

Due to its flexibility, this online class can fit into your schedule no matter you are college student or a career switcher.

What you do each week

Each week, you'll learn a core concept and solve a series of problems. Specifically, you'll do the following 4 tasks:

1. **Read a specific chapter of the Guo manual and work through the problems in that chapter**. I'll tell you what chapter you need to read for that week.

2. **Read the supplemental lecture note and work through the problems in this supplemental lecture note**. Each week, I will send you a supplemental lecture note (a PDF file) to further clarify the core concept. The supplemental lecture note has worked out problems. You'll need to read the supplemental lecture note and work through the problems in the lecture note.

3. **Solve offline problems**. Each week, I'll post problems in my website. These problems center on the core concept you're learning that week. A few days later, I'll post detailed solutions.

4. **Solve online timed problems**. Each week, I'll have some timed online problems for you to solve. After you submit your answers online, my test engine automatically grades your answers. I will post the detailed solutions to the online problems later that week.

Final exams – 10 full length timed online mock exams

In the last 4 weeks, you'll do 10 full-length timed online exams (5 are previous SOA exams and 5 are Guo's original exams) testing your overall command of the core concepts and your problem solving skills for Exam P. Each exam is 3 hour long and contains 30 problems.

In addition to these 10 full length CBT exams, this online class has many off-line quizzes with detailed solutions.

Cancellation

Cancellation is permitted (hence a refund is given) if you meet BOTH of the following conditions

- You decide to cancel your enrollment during the <u>first 5 days after your password is sent to you. Beyond these 5 days, no refund is given.</u>

- Your access to the online course materials is limited to the first two units "Unit 1 Calculus Review" and "Unit 2 Probability Basics." The content in these two units should give you a good feel of this online course. If you want to cancel your enrollment, you should not open any other modules. <u>If you have opened or otherwise accessed any other study modules, no refund is given.</u>

If you meet both conditions and decide to cancel your enrollment, your tuition will be refunded to you. However, **you'll be charged a $5 processing fee**. In addition, your purchase of the Guo manual (an e-manual) can not be refunded.

If you purchased a discount package at $275 tuition + $75 Guo manual =$350 (a $10 saving off the regular $275 tuition + $85 Guo manual =$360), your tuition refund will be $275 -$10 package discount repayment - $5 processing fee =$260.

FAQ

Q1   What types of students enrolled in your online course?
I started teaching online classes for Exam P in 2005.

Students of Guo's online Exam P prep class are from all walks of life: college students majoring in statistics or non-math majors; career switchers (high school math teachers, IT workers, stay-home moms); full time actuary trainees working in consulting firms or insurance companies. Most of the students are located in U.S. and Canada. Some students are from other countries. Though from different backgrounds, students of Guo's online Exam P class have one thing in common: they are interested in becoming actuaries and want to pass Exam P.

Q2   How is your online course different from a course in a classroom?
First, convenience. You don't have to commute to a school; instead, you study at your home or office. Second, CBT practice. This online course offers timed online quizzes and 4 full length timed online mock exams that simulate CBT. By taking this online course, you'll start preparing for CBT from Day One.

Q3   Does your online class have videos?
A   Yes. My online class contains links to over 70 video lessons that explain various concepts in Exam P, ranging from calculus review, basic probability theories, random variables, to probability distributions (such as normal distribution, Poisson distribution). When you take this online class, you'll be able to immediately watch these videos. However, I didn't produce these videos. These video are from YouTube.

Why didn't I produce my own videos (instead of using YouTube videos)? YouTube contains a large number of video lessons on probability. Many of the people who posted probability video lessons in YouTube are excellent teachers who have been teaching math and probability for many years. Their lessons are clear and to the point. Some YouTube lessons are inspiring. One of my favorite YouTube videos is http://www.youtube.com/watch?v=Uk4t0E0ZkZk. Here is video lesson on probability http://www.youtube.com/watch?v=3ER8OkqBdpE.

Since YouTube contains many high quality videos on general math and probability, I decided not to reinvent the wheel.

Q4   If I don't understand a concept or can't solve a problem, how can I get help?
You can email me directly or post a question in the discussion forum.

Q5   Who is this online course for?
This course aims at both college students and career switchers. Many college students with previous coursework in statistics tend to be strong on theories and weak in solving SOA problems. They tend to be "books mart, exam poor." The practice problems in this online course will help them bridge the gap between knowing what's in the book and what's on the exam. Career switchers, working on their current jobs while preparing for

Exam P, have no time to waste on fluff. They want to cut to the chase and learn what really matters for the exam. This course will help them quickly get up to speed and develop the essential problem solving techniques for Exam P.

Q6   I didn't take any class on probability. Can I still take your online course?
You do NOT need to know any probability theories to take this course. Prior coursework in probability, statistics, or data analysis is NOT required. You'll learn probability theories by taking this online course. However, you do need to know some calculus. See the next question.

Q7   Is this online course right for me?
Not every exam candidate needs to take a prep course. To determine whether this online course is right for you, ask yourself three questions:
- Do I know enough calculus?
- Am I determined to become an actuary?
- Can I do it myself?

Calculus. To get the full benefit of this online course (and to pass Exam P), you need to know some basic math operations and calculus. To check whether you have the basic math knowledge, go to Page 22 and do the 1st assignment. If you understand my solutions, you are probably ready for this online course. If my solutions read like a foreign language to you, you are not ready to take this online course or take Exam P. In that case, you'll need to learn basic calculus before attempting Exam P.

Determination. You must desperately want to become an actuary. Preparing for Exam P takes hundreds of study hours. If your heart is not set on the actuary career, you'll find studying for Exam P is the most boring thing in the world. Then you'll have millions of excuses not to study.

To pass Exam P and succeed in the actuary profession, you must feel that your life is incomplete without being an actuary. If you can live happily without being an actuary, you shouldn't take this online course or attempt Exam P.

Do-it-yourself vs. taking a prep course. Preparing for the exam by yourself is cheaper, but you need to discipline yourself to study. When not put in a structured study program such as this online course, many tend to procrastinate.

If you want to take Exam P in the next sitting and don't know where to start, you might consider taking this course. Taking a course is taking a guided tour to a foreign city. You pay more, but a good tour guide can quickly get you up to speed.

Q8   I'm a career switcher. I learned calculus at college. That was 8 years ago. I haven't touched calculus since then. What advice can you give me?
First, congratulate yourself for finding the actuary career. I found it when I was in mid 30's. Actuary career is one of the best jobs in terms of salary and job security. If you have strong math skills, actuary career is an excellent choice. Next, study hard and relearn

calculus. Here is the good news. Though calculus problems and formulas look intimidating, the idea behind integration and derivative is simple. After you do many practice problems, calculus formulas will become your 2nd nature.

Q9   Can you recommend a book for me so I can quickly learn calculus?
I recommend **Calculus the Easy Way (Easy Way Series)** by Douglas Downing. I looked this book at my local Borders bookstore. It's an excellent tutorial with clear explanations and plenty of practice problems (with solutions). You can buy this book at your local bookstore or Amazon. The price is reasonable; about $10 at Amazon.

When studying calculus, please ignore trigonometry functions $\sin(x)$, $\cos(x)$, $\tan(x)$, $\cot(x)$, $\sec(x)$, and $\csc(x)$. Please also ignore the inverse trigonometry functions $\sin^{-1}(x)$, $\cos^{-1}(x)$, $\tan^{-1}(x)$, $\cot^{-1}(x)$, $\sec^{-1}(x)$, and $\csc^{-1}(x)$. The trigonometric and inverse trigonometric functions are NOT tested in Exam P; you don't need to learn them.

In addition, download Sample Exam P problems officially released by SOA. You can find the sample P problems from the soa website. Since SOA constantly changes its website, I can't give you a download link that always works. However, go to http://soa.org/education/exam-req/edu-exam-p-detail.aspx and look around. See whether you can download the Sample P problems and solutions.

Read the SOA solutions and get a feel of what calculus knowledge is really needed to pass Exam P. If you understand SOA solutions, you know enough calculus for the purpose of passing Exam P.

Q10   What equipment do I need?
A computer and internet connection (DSL preferred). In addition, you'll need to have PDF reader installed to open PDF files from my website.

Where to get the free PDF reader (either option is fine):
- Option 1   Download the free Foxit reader at http://www.foxitsoftware.com/download.htm

  Make sure you download the Foxit Reader (free), not the Foxit Reader Pro (not free)

- Option 2   Download the free Adobe Reader from Adobe http://www.adobe.com/products/acrobat/readstep2.html.

Q11   How long is the online course?
The class is set up for 12 weeks. However, each student is given 100 days' access to Guo's Exam P website. If you enroll in this online course, you'll have 100 days' access to the online course content. Your access will expire after 100 days.

If you need additional access time beyond the given 100 days, you can buy additional access directly from me at $15 per month. For example, if you need 2 additional months beyond the 100 days, you just pay $30 per month. You can send your payment to me through paypal.

Q12   What final advice can you give me?

<u>First, don't just dream about becoming an actuary</u>. Take Exam P as soon as you are reasonably prepared. Unless you have passed Exam P, you have not convinced yourself or your potential employer that you are serious about becoming an actuary. <u>Second, don't underestimate the amount of study needed for one to pass Exam P</u>. Exam P is never a piece of cake. Otherwise, everyone with a remote interest in actuarial careers will become an actuary. Set passing Exam P as a priority of your life. Put your heart into it. <u>Finally, don't give up</u>. If you find a concept difficult to understand or you failed Exam P, don't quit. Hang in there. Try a different approach to understand the concept and a different method to study for Exam P. Many people became fine actuaries after failing an exam many times.

Sample Supplemental Lecture Note with Worked-out Problems

Core concept for this week: Bayes' Theorem

This is the supplemental lecture note for this week. Before reading this document, make sure you have read Chapter 7 (Bayes' theorem and posterior probabilities) in "Deeper Understanding, Faster Calc."

Summary of Bayes' Theorem

Bayes' Theorem has an intimidating formula. However, the logic behind it is really simple. As a matter of fact, you use Bayes' Theorem everyday, except you don't know you use it.

Bayes' Theorem says, "**Reevaluate things if you have more evidence**." Everyday, we reevaluate our beliefs after we have new information.

Example 1   From best bargain to lemon

You bought a used car. The car looked great and drove fine. And best of all, it was dirt cheap. You thought you got the best bargain ever in your life time. However, a week later, you ran into problems. The paint of your car started peeling off. The door couldn't close tight. Worst of all, the engine couldn't start. Your car was just dead. Now you realized you got a lemon.

Your original belief: You got a bargain.
New info: Your car started to have problems.
Your new belief: You bought a lemon.

Example 2   From low risk to high risk

A customer just purchased an auto insurance policy. He was classified as a standard risk and would pay a standard premium rate, not too high or too low. Then this customer had many accidents in a year. The insurer decided to classify him as a high risk driver and decided to charge him higher premiums.

The insurer's original belief: This customer was a standard risk.
New info: He had many accidents in a year.
The insurer's new belief: He was a high risk driver and should pay higher premiums.

Example 3   Searching for a missing vessel

How about using Bayes' Theorem to help find a missing vessel? A nuclear submarine was missing. It probably had a wreck and you wanted to search for its remains. You started off your search at certain places. As the search went on and more evidence was gathered, you could eliminate some places where you originally thought the missing ship could be. In addition, you could identify some hotspots where the missing vessel was most likely to be found given the new information you had.

This is a true story. In May 1968 the US nuclear submarine USS Scorpion (SSN-589) didn't arrive at her home port of Norfolk, Virginia. People couldn't find it. A search team used Bayes' Theorem to continuously focus on now hotspots and successfully found the missing ship. For details, click this link http://en.wikipedia.org/wiki/Bayesian_inference.

Formula

$$P(G_i \mid E) = \frac{P(G_i) \times P(E \mid G_i)}{P(E)}$$

Where $P(E) = P(G_1) \times P(E \mid G_1) + P(G_2) \times P(E \mid G_2) + ... + P(G_n) \times P(E \mid G_n)$

Different textbooks may use different notion, but the essence of the formula $P(G_i \mid E) = \dfrac{P(G_i) \times P(E \mid G_i)}{P(E)}$ is the same. In the above formula, E represents the arrival of some new information that an event E has occurred. $\Pr(G_i)$ is our original estimate of the probability that G_i occurs before E's occurrence; it's called the prior probability. $\Pr(G_i \mid E)$ is our updated estimate of the probability for G_i after we know E's occurred; it's called the posterior probability.

$P(E) = P(G_1) \times P(E \mid G_1) + P(G_2) \times P(E \mid G_2) + ... + P(G_n) \times P(E \mid G_n)$ represents the total probability that E occurs. To calculate the total probability of E's occurrence, we break down the sources that cause E's occurrence into segments G_1, G_2, …, G_n. Each segment is a distinct cause. Then we calculate each segment's contribution (or each cause's share)

to E's occurrence. Next, we sum up each segment's contribution and this gives us the total probability of $P(E)$.

For example, $P(G_1) \times P(E \mid G_1)$ represents G_1's contribution to E's occurrence. For G_1 to contribute to E's occurrence, two things must happen: First, G_1 needs to occur, meaning that the cause G_1 must be present. If G_1 doesn't occur, it certainly can't cause E to occur. Second, after G_1 occurs, it must cause E to occur. $P(G_1)$ is the probability that G_1 occurs; $P(E \mid G_1)$ is the probability that G_1 causes E's occurrence given G_1 already occurs. The product of these two terms, $P(G_1) \times P(E \mid G_1)$, represents G_1's contribution to E's occurrence.

Similarly, $P(G_2) \times P(E \mid G_2)$ is G_2's contribution to E's occurrence. $P(G_n) \times P(E \mid G_n)$ is G_n's contribution to E's occurrence. Then the total probability that E occurs is:

$$P(E) = P(G_1) \times P(E \mid G_1) + P(G_2) \times P(E \mid G_2) + \ldots + P(G_n) \times P(E \mid G_n)$$

Bayes' Theorem simply says this: if we know an event E has occurred, we can identify who causes E to occur. We can assign different probabilities to each distinct cause using the following formula:

$$P(G_i \mid E) = \frac{P(G_i) \times P(E \mid G_i)}{P(E)} = \frac{P(G_i) \times P(E \mid G_i)}{P(G_1) \times P(E \mid G_1) + P(G_2) \times P(E \mid G_2) + \ldots + P(G_n) \times P(E \mid G_n)}$$

$P(G_i \mid E)$ is the probability that the event E is caused by G_i

What you don't need to know

You'll never need to learn the proof of Bayes' Theorem. Proving a theorem, though theoretically interesting, is not important in SOA exams. As long you understand the common sense behind Bayes' Theorem and can solve problems, you are fine.

You don't need to memorize the ugly formula

$$P(G_i \mid E) = \frac{P(G_i) \times P(E \mid G_i)}{P(E)} = \frac{P(G_i) \times P(E \mid G_i)}{P(G_1) \times P(E \mid G_1) + P(G_2) \times P(E \mid G_2) + \ldots + P(G_n) \times P(E \mid G_n)}$$

Perhaps the 1st time you solve a Bayes' theorem problem, you want to try this formula out. When you solve the 2nd or 3rd problem, don't ever use this formula. Always use the table driven approach (to be explained next).

3 approaches to solving Bayes' Theorem related problems

I'll use an example to show you how to solve a problem using 3 approaches. Make sure you understand all these approaches. However, use the 3rd method when you solve a problem in the exam. It's simple and fast.

Illustrative Problems
Problem 1
In a factory, Machine A produces 20% of the output, Machine B produces 35% of the output, and Machine C produces the remaining 45% of the output. 1% of output produced by Machine A is defective. 2% of output produced by Machine B is defective. 3% of output produced by Machine C is defective.

One output randomly chosen is found defective. Calculate
- the probability that a randomly chosen output is defective
- this output was produced by Machine A,B, and C respectively.

Solution

Method 1   Formula driven approach – complex and prone to errors

Define the following events:

E=defective output	G_1=output produced by A
G_2=output produced by B	G_3=output produced by C

Then   P(E)= Probability that a randomly chosen output is defective

$P(G_1/E)$= Probability that given a defective output, it is produced by A
$P(G_2/E)$= Probability that given a defective output, it is produced by B
$P(G_3/E)$= Probability that given a defective output, it is produced by C

We are asked to calculate P(E), $P(G_1/E)$, $P(G_2/E)$, and $P(G_3/E)$. We know that Machine A produces 20% of the output, Machine B produces 35% of the output, and Machine C produces the remaining 45% of the output. This means the following:

$P(G_1)$=20%, $P(G_2)$=35%, $P(G_3)$=45%

We know that 1% of output produced by Machine A is defective. 2% of output produced by Machine B is defective. 3% of output produced by Machine C is defective. This means:

$P(E/G_1)$=1%, $P(E/G_2)$=2%, $P(E/G_3)$=3%.

Finally, we are ready to use the following memorized formula:

$$P(E) = P(G_1) \times P(E \mid G_1) + P(G_2) \times P(E \mid G_2) + P(G_3) \times P(E \mid G_3)$$

$$P(G_i \mid E) = \frac{P(G_i) \times P(E \mid G_i)}{P(E)} \text{ for } i = 1, 2, 3$$

$$P(G_1) \times P(E \mid G_1) = 20\%(1\%), \ P(G_2) \times P(E \mid G_2) = 35\%(2\%), \ P(G_3) \times P(E \mid G_3) = 45\%(3\%)$$

$$P(E) = P(G_1) \times P(E \mid G_1) + P(G_2) \times P(E \mid G_2) + P(G_3) \times P(E \mid G_3)$$
$$= 20\%(1\%) + 35\%(2\%) + 45\%(3\%) = 0.0225$$

$$P(G_1 \mid E) = \frac{P(G_1) \times P(E \mid G_1)}{P(E)} = \frac{20\%(1\%)}{0.0225} = 8.89\%$$

$$\text{Similarly,} \quad P(G_2 \mid E) = \frac{P(G_2) \times P(E \mid G_2)}{P(E)} = \frac{35\%(2\%)}{0.0225} = 31.11\%$$

$$P(G_3 \mid E) = \frac{P(G_2) \times P(E \mid G_3)}{P(E)} = \frac{45\%(3\%)}{0.0225} = 60\%$$

Method 2   Tree diagram

Using the tree diagram, you don't need to mess with the formula.
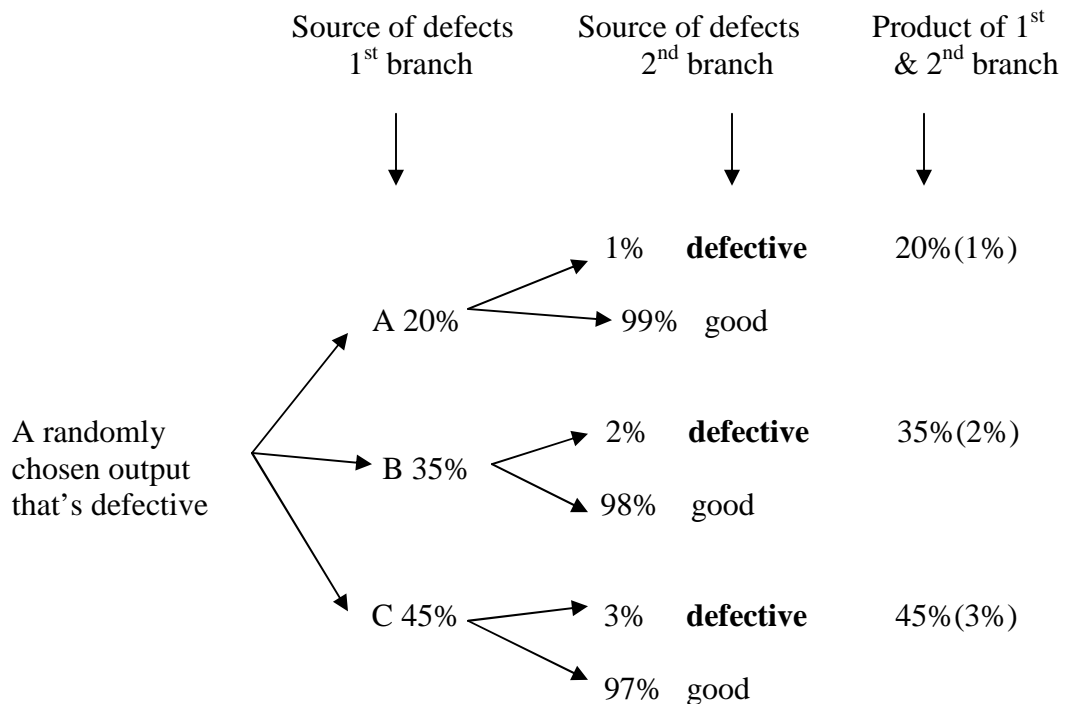
The meaning of this tree diagram is as follows. If a randomly chosen output is defective, we ask "Who causes the defect?" We find that Machine A, B, and C are the possible causes; the output could have been produced by A, or B, or C. Since the defective part is

randomly chosen, we don't know for sure which machine, A or B or C, produces the output and hence is responsible for the defect. So we have 3 suspects for one crime; we have no way of knowing which suspect is guilty and which innocent. As a result, we hold all 3 suspects partially responsible for the crime. A is 20% guilty; B is 35% guilty; and C is 45% guilty.

Next, we analyze each suspect's capability to cause the defect, assuming this suspect alone is the true criminal. If A is really guilty (i.e. 100% guilty), then he can cause 1% harm (i.e. produces 1% defects); if B is really guilty, then he can cause 2% harm; and if C is really guilty, then he can cause 3% harm.

We then calculate each suspect's cumulative guilt. A's guilt amount is 20%(1%)=0.002; B is 35%(2%)=0.007; and C is 45%(3%)=0.0135. So the total guilt is

0.002+0.007+0.135=0.0225.

So the probability that a randomly chosen output is defective is 0.0225.

Finally, we calculate each suspect's % share of the guilt:
A's % share is simply 0.002/0.0225= 8.89%.
B's % share is simply 0.007/0.0225= 31.11%.
C's % share is simply 0.135/0.0225= 60%.

So if a randomly chosen output is defective, there's 8.89% chance that A produced this output, 31.11% chance that B produced this output, and 60% chance that C produced this output.

Let's change the question slightly. What's the probability that a randomly chosen output is non-defective? Given that a randomly chosen part is found non-defective, what's the probability that this output was produced by A, B, and C respectively?

Answer: The probability that a randomly chosen output is non-defective:

$$20\%(99\%)+35\%(98\%)+45\%(97\%)=0.9775$$

The probability that A produced this output is: $\dfrac{20\%(99\%)}{0.9775}=20.26\%$

The probability that B produced this output is: $\dfrac{35\%(98\%)}{0.9775}=35.09\%$

The probability that C produced this output is: $\dfrac{45\%(97\%)}{0.9775}=44.65\%$

Method 3   Table approach

This method is really a simplification of the tree diagram. Instead of drawing a complex tree, we simply set the following table to kept track of prior and post probability.

Event: Have a defective item.

Segment (distinct cause)	Segment's size (i.e. prior probability)	This segment's likelihood to produce the event (a defective item)	this segment's contribution amount	This segment's contribution % (i.e. post probability)
A	20%	1%	20%(1%)=0.0020	0.002/0.0225=8.89%
B	35%	2%	35%(2%)=0.0070	0.007/0.0225=31.11%
C	45%	3%	45%(3%)=0.0135	0.0135/0.0225=60.00%
Total	100%		0.0225	100%

Intuitively, we see that the total pool of defective items is 0.0225. This is the probability that a randomly chosen output is defective.

Machine A contributes 0.002 (or 8.89%) to this pool; B contributes 0.007 (or 3.11%); and C contributes to 0.0135 (60%). So if a randomly chosen output is defective, the probability is 8.89% that A produced this output, 3.11% that B produced this output, and 60% that C produced this output.

Problem 2
1% of the women at age 45 who participate in a study are found to have breast cancer. 80% of women with breast cancer will have a positive mammogram. 10% of women without breast cancer will also have a positive mammogram. One woman aged 45 who participated in the study was found to have a positive mammogram.

Calculate the probability that this woman has breast cancer.

Solution
We can still use any of the 3 approaches to solve this problem. However, I'll use the table method. This problem is tricky and many folks won't be able to solve this problem right. To solve this problem, we need to correctly identify the following 3 items:

- What's the event?

- What are the distinct causes (i.e. segments) that can possibly produce the event? Make sure your causes are mutually exclusive (i.e. no two causes can happen simultaneously) and collectively exhaustive (i.e. there are no other causes).

- What is each cause's probability to produce the event?

Event – a woman (who participated in a study) is found to have a positive mammogram.

Causes of this event – two distinct causes. Women with breast cancer and without breast cancer. These are the two segments. In terms of size of each segment, women with breast cancer account for 1% of the participants; and women without breast cancer account for 99%.

Each cause' probability to produce the event – women with breast cancer have 80% chance of having a positive mammogram. Women without breast cancer have 10% of the chance of having a positive mammogram.

Next, we set up the following table:

Event: a woman in the study is found to have a positive mammogram.

Segment (distinct causes)	Segment's size	Segment's probability to produce the event	Segment's contribution amount to the event	Segment's contribution % to the event (post event probability)
women with breast cancer	1%	80%	1%(80%) =0.008	0.008/0.107=7.48%
women without breast cancer	99%	10%	99%(10%)=0.099	0.009/0.107=92.52%
Total	100%		0.107	100%

So if a woman aged 45 who participated in the study is found to have a positive mammogram, then she has 7.48% chance of actually having breast cancer.

Problem 3 (SOA May 2003, Course 1, #31)
A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers.

Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers.

A randomly selected participant from the study died over the five-year period.

Calculate the probability that the participant was a heavy smoker.

Solution
Let p =the probability that a non-smoker will die during the next 5 years. Then,

The probability that a light smoker will die during the next 5 years is $2p$
The probability that a heavy smoker will die during the next 5 years is $4p$

Please note that we don't need to know the value of p to solve the problem.

Event: A participant died during the 5-year period

Segment	Segment size	Segment's probability to produce the event	Segment's contribution amount	Segment's contribution %
Heavy smoker	20%	$4p$	20%(4p)=0.8p	0.8p/1.9p=42.11%
Light smoker	30%	$2p$	30%(2p)=0.6p	0.6p/1.9p=31.58%
Non smoker	50%	p	50%(p)= 0.5p	0.5p/1.9p=26.32%
Total	100%		1.9p	100%

The probability that the participant was a heavy smoker is 42.11%.
The probability that the participant was a heavy smoker is 31.58%.
The probability that the participant was a heavy smoker is 26.32%.

In problems related to Bayes' Theorem, the absolute size of each segment doesn't matter; only the ratio of each segment size matters. Similarly, the absolute probability for each segment to produce the event doesn't matter; only the ratio of probabilities matters.

If we are to solve this problem quickly, we can set up the following table:

Event: A participant died during the 5-year period

Segment	Segment size	Segment's probability to produce the event	Segment's contribution amount	Segment's contribution %
Heavy smoker	2	4	2(4)=8	8/19=42.11%
Light smoker	3	2	3(2)=6	6/19=31.58%
Non smoker	5	1	5(1)=5	5/19=26.32%
Total	10		19	100%

In the above table, we change the segment sizes from 20%, 30%, and 50% to 2,3, and 5. Similarly, we change the segments' probabilities from $4p$, $2p$, and p to 4, 2, and 1. This speeds up our calculations. You can use this technique when taking the exam.

Sample Offline Problems

Problem 1
You have 2 coins in your pocket, one fair and the other biased. The biased coin, if flipped, is 3 times likely to have heads as tails. You pick out a coin randomly from your pocket and flip it 5 times. You get 3 heads. What's the probability that the coin you picked is a biased coin?

Problem 2
Assume that a family can have 1, 2, or 3 children with probability of 0.5, 0.25, and 0.25 respectively. John is the only boy in the family. What's the probability that John is the only child in the family?

Problem 3
An auto insurer classifies drivers who apply for auto insurance as one of the 3 risk categories: risky, standard, and preferred. The insurer believes that 40% of the applicants for auto insurance policy are risky drivers, 50% of the applicants are standard drivers, and 10% are preferred.

The probability that a risky driver has one or more accidents in a year is 10%.
The probability that a standard driver has one or more accidents in a year is 5%.
The probability that a preferred driver has one or more accidents in a year is 2%.

It's found that a policyholder doesn't have any accident during the next 2 consecutive years. Calculate the probability that this policyholder is a risky driver.

Problem 4
A balanced die is thrown once. If the result is 1 or 2, a ball is randomly drawn without replacement from Urn 1; if the result is 3,4,5,or 6, a ball is randomly drawn from Urn 2. Urn 1 contains 2 red balls, 3 blue balls, and 5 white balls. Urn 2 contains 1 red ball, 2 blue balls, and 2 white balls.
Two balls are drawn from the same urn. It's found that the two balls drawn are both blue. Calculate the probability that the 1st ball drawn is from Urn 1.

Problem 5
A factor receives 10% of its shipments of parts from Company X, 20% from Company Y , and the remainder of its shipments from Company Z. Each shipment contains a very large number of parts.

For Company X's shipments, 8% of the parts are defective;

for Company Y′ s shipments, 5% of the parts are defective;

for Company Z′ s shipments, 4% of the parts are defective.

The factory tests 30 randomly selected parts from a shipment and finds that 5 are

defective. What is the probability that this shipment came from Company X ?

Solutions to Sample Offline Problems

Problem 1

You have 2 coins in your pocket, one fair and the other biased. The biased coin, if flipped, is 3 times likely to have heads as tails. You pick out a coin randomly from your pocket and flip it 5 times. You get 3 heads. What's the probability that the coin you picked is a biased coin?

Solution

The probability getting k heads of n flips is a binomial distribution $C_n^k p^k (1-p)^{n-k}$

Event: A randomly chosen coin lands with 3 a heads if flipped 5 times

segment	segment size	segment's probability to produce the event	segment's contribution amount	segment's contribution percentage
Fair coin	0.5	$C_5^3 0.5^3 0.5^2 = 0.3125$	0.5(0.3125)=0.1563	0.1563/0.2881=54.24%
biased coin	0.5	$C_5^3 0.75^3 0.25^2 = 0.2637$	0.5(0.2637)=0.1318	0.1318/ 0.2881=45.76%
sum	1		0.2881	100.00%

The probability that the coin is the fair one is 52.24%.
The probability that the coin is the biased one is 45.76%.

Problem 2

Assume that a family can have 1, 2, or 3 children with probability of 0.5, 0.25, and 0.25 respectively. John is the only boy in the family. What's the probability that John is the only child in the family?

Solution
Event: The family has only one boy.

segment (# of kids)	segment size	segment's probability to produce the event	segment's contribution amount	segment's contribution %
1	0.5	0.5[(1)]	0.5(0.5) =0.2500	0.25/0.4688=53.33%
2	0.25	0.5[(2)]	0.25(0.5) =0.1250	0.125/0.4688=26.67%
3	0.25	0.375[(3)]	0.25(0.375)=0.0938	0.0938/0.4688=20.00%
Total	1.00		0.4688	100.00%

(1) If the family has only one child, then the possible outcomes are {boy, girl}. The probability of having a boy is 0.5.

(2) If the family has 2 kids, then the 1st child can be either a boy or girl; the 2nd child can be either a boy or girl. So the # of permutations is 2^2 =4. The # of permutations that leads to a boy is 2; the family can have 1st child as a boy and 2nd as a girl or 1st as a girl and 2nd as a boy. So the probability of having only 1 boy is 2/4=0.5.

(3) If the family has 3 kids, then the 1st child can be either a boy or girl; the 2nd child can be either a boy or girl; the 3rd child can be either a boy or girl. So the # of permutations is 2^3 =8. The # of permutations that leads to a boy is 3. The family can have 1st child as a boy, 2nd as a girl, and 3rd as a girl; 1st child as a girl, 2nd as a boy, and 3rd as a girl; or 1st child as a girl, 2nd as a girl, and 3rd as a boy.

The probability that the boy is the only child given he's the only boy is 53.33%.
The probability that the boy has only one sister given he's the only boy is 26.67%.
The probability that the boy has only two sisters given he's the only boy is 20%.

Problem 3

An auto insurer classifies drivers who apply for auto insurance as one of the 3 risk categories: risky, standard, and preferred. The insurer believes that 40% of the applicants for auto insurance policy are risky drivers, 50% of the applicants are standard drivers, and 10% are preferred.

The probability that a risky driver has one or more accidents in a year is 10%.
The probability that a standard driver has one or more accidents in a year is 5%.
The probability that a preferred driver has one or more accidents in a year is 2%.

It's found that a policyholder doesn't have any accident during the next 2 consecutive years. Calculate the probability that this policyholder is a risky driver.

Solution
Event: a driver doesn't have any accidents in 2 years

segment	segment size	segment's probability to produce the event	segment's contribution amount	segment's contribution %
risky	40%	$(1-10\%)^2$=0.81[1]	40%(0.81) =0.32400	0.324/0.87129 =37.19%
standard	50%	$(1-5\%)^2$=0.9025	50%(0.9025)=0.45125	0.45125/0.87129=51.79%
preferred	10%	$(1-2\%)^2$=0.9604	10%(0.9604)=0.09604	0.09604/0.87129=11.02%
Total	100%		0.87129	100.00%

(1) The probability that a risky driver doesn't have any accident in Year 1 is 1-10%=0.9. He doesn't have any accident in Year 2 is also 0.9. As a result, the probability that he doesn't have any accident for two years is 0.9(0.9)=0.81.

So the probability that the policyholder is a risky driver, given that he doesn't have any accidents for two years, is 37.19%.

Problem 4

A balanced die is thrown once. If the result is 1 or 2, a ball is randomly drawn without replacement from Urn 1; if the result is 3,4,5,or 6, a ball is randomly drawn from Urn 2. Urn 1 contains 2 red balls, 3 blue balls, and 5 white balls. Urn 2 contains 1 red ball, 2 blue balls, and 2 white balls.
Two balls are drawn from the same urn. It's found that the two balls drawn are both blue. Calculate the probability that the 1st ball drawn is from Urn 1.

Solution

Event: two balls drawn are both blue.

segment	segment size	segment's probability to produce the event	segment's contribution amount	segment's contribution %
The 1st ball drawn is from Urn I	$\dfrac{2}{6}$	$\dfrac{3}{10} \times \dfrac{2}{9}$	$\dfrac{2}{6} \times \dfrac{3}{10} \times \dfrac{2}{9} = 0.02222$	0.02222/0.08889=25%
The 1st ball drawn is from Urn II	$\dfrac{4}{6}$	$\dfrac{2}{5} \times \dfrac{1}{4}$	$\dfrac{4}{6} \times \dfrac{2}{5} \times \dfrac{1}{4} = 0.06667$	0.06667/0.08889=75%
Total	100%		0.08889	100%

If a die is thrown, the probability of getting 1 or 2 is $\dfrac{2}{6}$. This is the prior probability that Urn I is chosen 1st. Similarly, the probability that Urn II is chosen is $\dfrac{4}{6}$.

If Urn I is chosen first, what's the probability that 2 balls drawn from Urn I are both blue? We can imagine that 2 balls are taken out one at a time from Urn I. When the 1st ball is taken out, the probability of getting a blue is $\dfrac{3}{10}$; remember that Urn I has 3 blue balls out of 10 balls. If the 1st ball taken out is blue, then Urn I has 9 balls left, 2 of which are blue. So the probability that the 2nd ball drawn from Urn I (given the 1st ball drawn from Urn 1 is blue) is $\dfrac{2}{9}$. So the probability that both balls, if taken out from Urn I, are blue is $\dfrac{3}{10} \times \dfrac{2}{9}$.

Alternatively, we can imagine that 2 balls are taken out simultaneously from Urn I. The # of different combinations of getting 2 balls out of 10 is $C_{10}^2 = \dfrac{10 \times 9}{2 \times 1}$. The # of different combinations of getting 2 blue balls from Urn 1 is $C_3^2 = \dfrac{3 \times 2}{2 \times 1}$. As a result, the probability

of getting 2 blue balls from Urn I is $\dfrac{C_{10}^2}{C_3^2} = \dfrac{10 \times 9}{3 \times 2}$. Similarly, the probability of getting 2

blue balls from Urn II is $\dfrac{2}{5} \times \dfrac{1}{4}$. The remaining calculation is self explanatory. The

probability that the 1$^{\text{st}}$ ball drawn is from Urn 1 is 25%.

Problem 5

A factor receives 10% of its shipments of parts from Company X, 20% from Company Y , and the remainder of its shipments from Company Z. Each shipment contains a very large number of parts.

for Company X's shipments, 8% of the parts are defective;

for Company Y′ s shipments, 5% of the parts are defective;

for Company Z′ s shipments, 4% of the parts are defective.

The factory tests 30 randomly selected parts from a shipment and finds that 5 are

defective. What is the probability that this shipment came from Company X ?

Solution

Event: There are 5 defective parts in 30 items

Segment	Segment size	Segment's probability to produce the event (binomial distribution)	Segment's contribution amount	Segment's contribution %
X	10%	$C_{30}^5 \left(0.08^5\right)\left(0.92^{25}\right)$=0.0581	10%(0.0581)=0.0058	0.0058/ 0.012=48.56%
Y	20%	$C_{30}^5 \left(0.05^5\right)\left(0.95^{25}\right)$=0.0124	20%(0.0124)=0.0025	0.0124/ 0.012=20.66%
Z	70%	$C_{30}^5 \left(0.04^5\right)\left(0.96^{25}\right)$=0.0053	70%(0.0053)=0.0037	0.0053/ 0.012=30.78%
Total			0.0120	100.00%

The probability that this shipment came from Company X is 48.56%.
The probability that this shipment came from Company Y is 20.66%.
The probability that this shipment came from Company Z is 30.78%.

End of offline practice problems. Next, you need to do the timed online quiz at
http://actuary88.com.

Your 1st assignment – Do this before the online class starts

This is your 1st assignment. You need to complete this assignment NOW before this online class starts. This helps you review basic math operations and calculus. Feel free to refer to any books.

Problem 1
You are given:

- $\rho_1 = \phi_1 + \phi_2 \rho_1$
- $\rho_2 = \phi_1 \rho_1 + \phi_2$
- $\rho_1 = 0.5$,
- $\rho_2 = -0.2$

Find ϕ_1 and ϕ_2.

Problem 2
You are given:

- $\rho = -\dfrac{\theta}{1+\theta^2}$

- $\rho = -0.35$

Calculate θ.

Problem 3
You are given:

- $F(x) = 1 - e^{-\left(\frac{x}{\theta}\right)^{\tau}}$ where θ and τ are constants
- $F(10) = 0.5$
- $F(100) = 0.9$.

Find τ and θ.

Problem 4
You are given:

- $f(\theta) = k\left(\theta e^{-\theta} - \theta e^{-2\theta}\right)$, where k is a constant.

- $\displaystyle\int_0^{+\infty} f(\theta)\,d\theta = 1$

Calculate $\displaystyle\int_0^{+\infty} f(\theta)\left(1 - e^{-\theta}\right)d\theta$

Problem 5

$f(p) = p^{20}(1-p)^{13}$, where $0 < p < 1$

Find p such that $\dfrac{d}{dp} f(p) = 0$

Solution to your 1st assignment

Problem 1
You are given:

- $\rho_1 = \phi_1 + \phi_2 \rho_1$
- $\rho_2 = \phi_1 \rho_1 + \phi_2$
- $\rho_1 = 0.5$,
- $\rho_2 = -0.2$

Find ϕ_1 and ϕ_2.

Solution
Though the symbols ρ_1, ρ_2, ϕ_1, and ϕ_2 look intimidating, this problem asks you to solve the following equations:

$\phi_1 + 0.5\phi_2 = 0.5$
$0.5\phi_1 + \phi_2 = -0.2$
This gives us $\phi_1 = 0.8$ and $\phi_2 = -0.6$

Problem 2
You are given:

- $\rho = -\dfrac{\theta}{1+\theta^2}$
- $\rho = -0.35$

Calculate θ.

Solution
This tests your knowledge of the following formula:
If $ax^2 + bx + c = 0$ where $a \neq 0$ and $b^2 - 4ac$, then

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Using the above formula, we have: $x_1 = 0.40837$, $x_2 = 2.4488$.

Problem 3

You are given:

- $F(x) = 1 - e^{-\left(\frac{x}{\theta}\right)^{\tau}}$ where θ and τ are constants
- $F(10) = 0.5$
- $F(100) = 0.9$.

Find τ and θ.

Solution

$$F(10) = 0.5, \quad \Rightarrow \quad F(10) = 1 - e^{-\left(\frac{10}{\theta}\right)^{\tau}} = 0.5, \quad e^{-\left(\frac{10}{\theta}\right)^{\tau}} = 0.5, \quad -\left(\frac{10}{\theta}\right)^{\tau} = \ln 0.5$$

$$F(100) = 0.9, \quad \Rightarrow \quad F(100) = 1 - e^{-\left(\frac{100}{\theta}\right)^{\tau}} = 0.9, \quad e^{-\left(\frac{100}{\theta}\right)^{\tau}} = 0.1, \quad -\left(\frac{100}{\theta}\right)^{\tau} = \ln 0.1$$

$$\Rightarrow \quad \frac{-\left(\frac{10}{\theta}\right)^{\tau}}{-\left(\frac{100}{\theta}\right)^{\tau}} = \frac{\ln 0.5}{\ln 0.1}, \qquad \left(\frac{10}{100}\right)^{\tau} = \frac{\ln 0.5}{\ln 0.1}, \qquad \left(\frac{100}{10}\right)^{\tau} = \frac{\ln 0.1}{\ln 0.5}$$

$$\Rightarrow \quad 10^{\tau} = \frac{\ln 0.1}{\ln 0.5} = 3.3219, \quad \ln 10^{\tau} = \ln 3.3219, \quad \tau \ln 10 = \ln 3.3219$$

$$\tau = \frac{\ln 3.3219}{\ln 10} = 0.521.$$

Next, we'll find θ.

$$-\left(\frac{10}{\theta}\right)^{\tau} = \ln 0.5, \quad \Rightarrow \quad -\left(\frac{10}{\theta}\right)^{0.521} = \ln 0.5, \quad \left(\frac{10}{\theta}\right)^{0.521} = -\ln 0.5$$

$$\Rightarrow \quad \frac{10}{\theta} = (-\ln 0.5)^{\frac{1}{0.521}}, \quad \frac{10}{\theta} = 0.4949 \quad \Rightarrow \quad \theta = \frac{10}{0.4949} = 20.208$$

Problem 4

You are given:

- $f(\theta) = k\left(\theta e^{-\theta} - \theta e^{-2\theta}\right)$, where k is a constant.
- $\displaystyle\int_{0}^{+\infty} f(\theta)d\theta = 1$

Calculate $\displaystyle\int_{0}^{+\infty} f(\theta)\left(1 - e^{-\theta}\right)d\theta$

Solution

First, we need to find the constant k.

$$\int_0^{+\infty} f(\theta)d\theta = 1, \qquad \Rightarrow \quad \int_0^{+\infty} k\left(\theta e^{-\theta} - \theta e^{-2\theta}\right)d\theta = 1, \qquad k = \frac{1}{\displaystyle\int_0^{+\infty}\left(\theta e^{-\theta} - \theta e^{-2\theta}\right)d\theta}$$

$$\int_0^{+\infty}\left(\theta e^{-\theta} - \theta e^{-2\theta}\right)d\theta = \int_0^{+\infty}\theta e^{-\theta}d\theta - \int_0^{+\infty}\theta e^{-2\theta}d\theta$$

Next, we need to use integration by parts: $\int u\,dv = uv - \int v\,du$. First, we calculate $\int_0^{+\infty}\theta e^{-2\theta}d\theta$, the more complex integration.

$$\int_0^{+\infty}\theta e^{-2\theta}d\theta = -\frac{1}{2}\int_0^{+\infty}\theta\,de^{-2\theta} = -\frac{1}{2}\left[\theta e^{-2\theta}\Big|_0^{+\infty} - \int_0^{+\infty}e^{-2\theta}d\theta\right]$$

$$\theta e^{-2\theta}\Big|_0^{+\infty} = \lim_{\theta\to+\infty}\theta e^{-2\theta} - \lim_{\theta\to 0}\theta e^{-2\theta}$$

$$\lim_{\theta\to+\infty}\theta e^{-2\theta} = \lim_{\theta\to+\infty}\frac{\theta}{e^{2\theta}} = 0$$

(because $e^{2\theta}$ approaches $+\infty$ much faster than θ approaches $+\infty$)

$$\lim_{\theta\to 0}\theta e^{-2\theta} = 0\times e^{-2\times 0} = 0$$

$$\Rightarrow \quad \theta e^{-2\theta}\Big|_0^{+\infty} = \lim_{\theta\to+\infty}\theta e^{-2\theta} - \lim_{\theta\to 0}\theta e^{-2\theta} = 0 - 0 = 0$$

$$\int_0^{+\infty}e^{-2\theta}d\theta = \frac{1}{2}\int_0^{+\infty}e^{-2\theta}d(2\theta) = \frac{1}{2}e^{-2\theta}\Big|_0^{+\infty} = -\frac{1}{2}$$

$$\Rightarrow \quad \int_0^{+\infty}\theta e^{-2\theta}d\theta = -\frac{1}{2}\left[\theta e^{-2\theta}\Big|_0^{+\infty} - \int_0^{+\infty}e^{-2\theta}d\theta\right] = \frac{1}{4}$$

Similarly,

$$\int_0^{+\infty}\theta e^{-\theta}d\theta = -\int_0^{+\infty}\theta\,de^{-\theta} = -\theta e^{-\theta}\Big|_0^{+\infty} + \int_0^{+\infty}e^{-\theta}d\theta = \int_0^{+\infty}e^{-\theta}d\theta = \left[-e^{-\theta}\right]_0^{+\infty} = 1$$

$$\Rightarrow \quad \int_0^{+\infty}\left(\theta e^{-\theta}-\theta e^{-2\theta}\right)d\theta = \int_0^{+\infty}\theta e^{-\theta}d\theta - \int_0^{+\infty}\theta e^{-2\theta}d\theta = 1-\frac{1}{4}=\frac{3}{4}$$

$$\Rightarrow \quad k=\frac{1}{\displaystyle\int_0^{+\infty}\left(\theta e^{-\theta}-\theta e^{-2\theta}\right)d\theta}=\frac{1}{\dfrac{3}{4}}=\frac{4}{3}$$

$$\int_0^{+\infty}f(\theta)\left(1-e^{-\theta}\right)d\theta = \int_0^{+\infty}\frac{4}{3}\left(\theta e^{-\theta}-\theta e^{-2\theta}\right)\left(1-e^{-\theta}\right)d\theta = \frac{4}{3}\int_0^{+\infty}\left(\theta e^{-\theta}-2\theta e^{-2\theta}+\theta e^{-3\theta}\right)d\theta$$

We already know that $\displaystyle\int_0^{+\infty}\theta e^{-\theta}d\theta = 1$, $\displaystyle\int_0^{+\infty}\theta e^{-2\theta}d\theta=\frac{1}{4}$.

Once again, using integration by parts, we get:

$$\int_0^{+\infty}\theta e^{-3\theta}d\theta = -\frac{1}{3}\int_0^{+\infty}\theta d e^{-3\theta} = -\frac{1}{3}\left[\theta e^{-3\theta}\Big|_0^{+\infty}-\int_0^{+\infty}e^{-3\theta}d\theta\right]=\frac{1}{3}\int_0^{+\infty}e^{-3\theta}d\theta$$

$$=-\frac{1}{3}\times\frac{1}{3}\int_0^{+\infty}e^{-3\theta}d(-3\theta)=-\frac{1}{9}e^{-3\theta}\Big|_0^{+\infty}=\frac{1}{9}$$

$$\Rightarrow \quad \int_0^{+\infty}f(\theta)\left(1-e^{-\theta}\right)d\theta = \frac{4}{3}\int_0^{+\infty}\left(\theta e^{-\theta}-2\theta e^{-2\theta}+\theta e^{-3\theta}\right)d\theta$$

$$=\frac{4}{3}\left(1-2\times\frac{1}{4}+\frac{1}{9}\right)=\frac{4}{3}\times\frac{11}{18}\approx 0.8148$$

Problem 5

$f(p)=p^{20}(1-p)^{13}$, where $0<p<1$

Find p such that $\dfrac{d}{dp}f(p)=0$

Solution

$f(p)=p^{20}(1-p)^{13}$.

Using the general formula: $\dfrac{d}{dx}\left[f(x)g(x)\right]=g(x)\dfrac{d}{dx}\left[f(x)\right]+f(x)\dfrac{d}{dx}\left[g(x)\right]$, we get:

$$\frac{d}{dp}f(p)=\frac{d}{dp}p^{20}(1-p)^{13}=(1-p)^{13}\frac{d}{dp}p^{20}+p^{20}\frac{d}{dp}(1-p)^{13}$$

$$= (1-p)^{13} 20 p^{19} - p^{20} 13 (1-p)^{12} = p^{19} (1-p)^{12} \left[20 (1-p) - 13 p \right]$$

$$= p^{19} (1-p)^{12} (20 - 33 p)$$

$$\frac{d}{dp} f(p) = 0, \ \Rightarrow \ p^{19} (1-p)^{12} (20 - 33 p) = 0$$

$$0 < p < 1, \ \Rightarrow \ p \neq 0, \ 1 - p \neq 0.$$

So the only way to satisfy $p^{19} (1-p)^{12} (20 - 33 p) = 0$ is to have $20 - 33 p = 0$. This gives us: $p = \dfrac{20}{33}$.

About the instructor

Yufeng Guo was born in central China. After receiving his Bachelor's degree in physics at Zhengzhou University, he attended Beijing Law School and received his Masters of law. He was an attorney and law school lecturer in China before immigrating to the United States. He received his Masters of accounting at Indiana University. He has pursued a life actuarial career and passed exams 1, 2, 3, 4, 5, 6, and 7 in rapid succession after discovering a successful study strategy.

Mr. Guo's exam records are as follows:

Fall 2002	Passed Course 1
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Mr. Guo is the author of the "Deeper Understanding, Faster Calc" study manuals for Exam P, FM, M, and C. He's also teaching an online course for FM and M.

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Testimonies of Guo Manual

Guo Manual has been used extensively by many Exam P candidates. For users' reviews of Guo manual at http://www.actuarialoutpost.com, click here Review of the manual by Guo.

Testimonies:

Second time I used the Guo manual and was able to do some of the similar questions in less than 25% of the time because of knowing the shortcut.
Testimony #1 of the manual by Guo

I just took the exam for the second time and feel confident that I passed. I used Guo the second time around. It was very helpful and gives a lot of shortcuts that I found very valuable. I thought the manual was kind of expensive for an e-file, but if it helped me pass it was well worth the cost.
Testimony # 2 of the manual by Guo

I took the last exam in Feb 2006, and I ran out of time and I ended up with a five. I needed to do the questions quicker and more efficiently. The Guo's study guide really did the job.
Testimony #3 of the manual by Guo