

ACTEX MLC Study Manual

Spring 2012 Edition

Errata

18 April 2012

- P-4 Delete “Exercise 1” and “Solutions to Exercise 1”
- C1-3 In equations (1.2) and (1.3), replace the dummy variable by u , i.e.:
- $$S_0(t) = \int_t^{\infty} f_0(u)du = 1 - \int_0^t f_0(u)du = 1 - F_0(t)$$
- $$\Pr(a < T_0 \leq b) = \int_a^b f_0(u)du = F_0(b) - F_0(a) = S_0(a) - S_0(b)$$
- C1-6 The answer to part (b) of Example 1.2 should be
“ $\Pr(T_{10} > 15) = S_{10}(15) = 1 - \frac{15}{90} = \frac{5}{6}$.”
- The answer to part (c) should be “ $\Pr(T_{10} \leq 15) = F_{10}(15) = 1 - S_{10}(15) = \frac{1}{6}$ ”.
- C1-7 Line 6: add “when we describe survival distributions” after “Note that”
- C1-8 Last line: Delete the second ${}_t q_x$.
- C1-15 The line after the function μ_x in Makeham’s law:
Replace “ $0 < B < 1$ ” with “ $A \geq -B, B > 0$ ”
- C1-17 Delete the first line of the solution. Replace $\mu_x(t)$ with μ_{x+t} in the third line.
- C1-18 Question 6: Statement (i) replace x with t , Statement (ii) should read $\mu_{40} = 2\mu_{20}$.
- C1-23 First line of Solution to #2: replace the x in $\frac{(30-x)^3}{27000}$ with t
- C1-25 The equation in the first line should read: ${}_{15}p_{10} = S_{10}(15) = \frac{S_0(25)}{S_0(10)} = 0.9277$.
- C2-2 The equation for ${}_t d_x$ should read ${}_t d_x = l_x - l_{x+t} = d_x + d_{x+1} + \dots + d_{x+t-1}$.
- C2-3 Change the equation for ${}_t d_x$ as above.
- C2-5 The paragraph starting with “For a nonsmoker at age 40, the remaining lifetime for...”: delete the extra for in “lifetime for is uniformly”
- C2-13 The paragraph immediately after the end of the previous example: replace factional by fractional.
- C2-14 The paragraph starting with “ $q_{[x]+t}$ is the probability”: the sentence should end with “was selected at age x ”
- C2-16 Line 6: $I(41, 1) = 1 - q_{[41]+1} / q_{42}$
- C2-17 Example 2.17(c): The question asks for the value of ${}_{1|2}q_{[31]+1}$.

C2-20 Line 3: delete “should”.

C2-38 Solution to #10: delete the extra 0.7 in $0.7 \int_0^{\infty} 0.7e^{-0.02u} du$

C3-6 Line –5: We denote $E(Z)$ for a unit-benefit...

C3-9 Example 3.2 (b). The final answer is 0.9072, not 0.8068.

C3-13 Example 3.3 is 2002 Fall #32.

C3-15 The solution to (b), last line: $0.4877 + \underline{0.9278} = 1.4155$.

C3-23 Line –7: replace $A_{x:\overline{m}|}^{1(m)}$ with $A_{x:\overline{n}|}^{1(m)}$.

C3-28 Part (c) First, we have $A_{x:\overline{15}|}^1 = v^{15} p_x \dots$

C3-31 Line 7: replace the equation with the following:

$$\Rightarrow 0.00433 = {}^2A_{41} - \left(\frac{0.0028}{1.05^2} + \frac{0.9972}{1.05^2} \times {}^2A_{41} \right)$$

C3-31 Figure: change “Pay \$ n if death occurs in year 1” to “Pay \$5 if death occurs in year 1”

C3-32 Line 1: A death benefit of \$5 would be ...

Line –4: A death benefit of \$1 would be ...

Line –1: insured would have an annually increasing ...

C3-33 Line 1: plus a unit-benefit ...

C3-49 Last line of the solution for #1: replace \bar{A}_x with \bar{A}_{30} .

C4-5 Next to last line: and the fact that

C4-8 Line 2: The first second equation for Y should read $\bar{a}_{\overline{n}|} = \frac{1-v^n}{\delta}$, $T_x > n$, but not $T_x < n$.

C4-9 Line 1: $\bar{a}_{x:\overline{n}|} = \frac{1-\bar{A}_{x:\overline{n}|}}{\delta}$, Line 5: which allows us to calculate $\text{Var}(Y)$...

C4-10 Last line: The payments from an n -year guaranteed ...

C4-13 The equation for “ n -year temporary”: $\bar{a}_{x:\overline{n}|} = \frac{1-\bar{A}_{x:\overline{n}|}}{\delta}$.

C4-23 Last line: change “made at the beginning of year n ” to “made at time n .”

C4-25 Lines 6 to 8: change all minus signs to plus signs, i.e.:

$$\begin{aligned} A_x &= A_{x:\overline{20}|}^1 + {}_{20|}A_x \\ &= A_{x:\overline{20}|}^1 + v^{20} p_x A_{x+20} \\ &= A_{x:\overline{20}|}^1 + A_{x:\overline{20}|}^1 A_{x+20} \end{aligned}$$

C4-26 The paragraph starting with “We can define”, first line: change “death probabilities” to “survival probabilities”

C4-26 Last equation: an ${}_nE_x$ is missing. It should read ${}_n|a_x^{(m)} = {}_n|\ddot{a}_x^{(m)} - \frac{{}_nE_x}{m}$

C4-28 The last two equations in the column labeled “Equation 2” should read

$${}_n|\ddot{a}_x^{(m)} = v^n {}_n p_x \ddot{a}_{x+n}^{(m)}, \quad {}_n|a_x^{(m)} = v^n {}_n p_x a_{x+n}^{(m)}$$

C4-31 Line -7: The APV at time 0 in this case is ...

C4-39 Question 2: Statement (i) should read $\ddot{a}_{20:\overline{30}|} = 12.13$

C4-40 Question 10: Statement (iv) should read ${}_5p_{25} = 0.9^5$

C4-43 Question 22 statement (iv) should read $\ddot{a}_{x+1} = 6.951$

C4-50 Solution to Exercise 8: the final answer is 9.498.

C4-51 Solution to Exercise 12: Replace first set of three equations with

$$\ddot{a}_{25:\overline{20}|} = \ddot{a}_{25} - {}_{20}|\ddot{a}_{25} = \ddot{a}_{25} - v^{20} {}_{20}p_{25} \ddot{a}_{45} = \ddot{a}_{25} - {}_{20}E_{25} \ddot{a}_{45}$$

C4-52 Replace the Solution to Exercise 15 with the following:

Using Woolhouse’s formula with three terms, we have

$$\begin{aligned} \ddot{a}_{25}^{(2)} &\approx \ddot{a}_{25} - \frac{2-1}{2 \times 2} - \frac{2^2-1}{12 \times 2^2} (\mu_{25} + \delta) \\ &= \ddot{a}_{25} - \frac{1}{2} - \frac{3}{48} (\mu_{25} + \delta) = \ddot{a}_{25} - \frac{1}{2} - \frac{3 \times 0.07}{48} \\ \ddot{a}_{45}^{(2)} &\approx \ddot{a}_{45} - \frac{1}{2} - \frac{3}{48} (\mu_{45} + \delta) = \ddot{a}_{45} - \frac{1}{2} - \frac{3 \times 0.08}{48}. \end{aligned}$$

Hence,

$$\begin{aligned} \ddot{a}_{25:\overline{20}|}^{(2)} &= \ddot{a}_{25}^{(2)} - {}_{20}E_{25} \ddot{a}_{45}^{(2)} \\ &\approx \ddot{a}_{25} - \frac{1}{2} - \frac{3 \times 0.07}{48} - {}_{20}E_{25} \left(\ddot{a}_{45} - \frac{1}{2} - \frac{3 \times 0.08}{48} \right) \\ &= (\ddot{a}_{25} - {}_{20}E_{25} \ddot{a}_{45}) + {}_{20}E_{25} \left(\frac{1}{2} + \frac{3 \times 0.08}{48} \right) - \frac{1}{2} - \frac{3 \times 0.07}{48} \\ &= \ddot{a}_{25:\overline{20}|} + 0.8 \times e^{-0.05 \times 20} \times \left(\frac{1}{2} + \frac{3 \times 0.08}{48} \right) - \frac{1}{2} - \frac{3 \times 0.07}{48} \\ &= 16.6442 \end{aligned}$$

C5 There is no Example 5.10 in this chapter.

C5-4 The paragraph preceding Example 5.1, replace “short” with “shorter”

C5-8 Table: m -payment whole life: The notation is ${}_m\bar{P}(\bar{A}_x)$.

C5-11 Replace the last equation with

$$d^{(4)} = 4[1 - (1 - d)^{1/4}] = 4[1 - v^{1/4}] = 4(1 - 1.06^{-0.25}) = 0.057846553$$

$$\ddot{a}_{40:10}^{(4)} = \frac{1 - A_{40:10}^{(4)}}{d^{(4)}} = \frac{1 - 0.5649473}{0.057846553} = 7.520806$$

C5-12 Line 3: $5000(0.3360531) / 7.520806 = 223.416$
 Line 4: each quarterly premium is **55.854**

C5-14 Next to last line: change $k + 1$ in “the death benefit payable at time $k + 1$ ” to k .

C5-19 First equation: replace $T_x < -\frac{1}{\delta} \ln \frac{l\delta + \pi}{S\delta + \pi}$ with $T_x < -\frac{1}{\delta} \ln \frac{\pi}{S\delta + \pi}$

Figure: replace $t = F_x \left(-\frac{1}{\delta} \ln \frac{\pi}{S\delta + \pi} \right)$ with $t = -\frac{1}{\delta} \ln \frac{\pi}{S\delta + \pi}$

$$\text{C5-20 (a) } {}_0L = \begin{cases} Sv^{T_x} - \pi \bar{a}_{\overline{T_x}|} & T_x \leq n \\ -\pi \bar{a}_{\overline{n}|} & T_x > n \end{cases} = \begin{cases} \left(S + \frac{\pi}{\delta} \right) v^{T_x} - \frac{\pi}{\delta} & T_x \leq n \\ \left(0 + \frac{\pi}{\delta} \right) v^n - \frac{\pi}{\delta} & T_x > n \end{cases}$$

C5-28 Example 5.16: For (ii), also add $l_{63} = 7,833,904$, $l_{64} = 7,683,980$

(a) Calculate the median and **15th percentile** of T_{45} .

C5-29 (a) Add the following:

Then we need to solve $F_{45}(t) = 0.15$. Notice that ${}_{18}q_{45} = 0.145150$ and ${}_{18}q_{45} = 0.161510$. So t is in between 18 and 19. We set ${}_{32}p_{45} {}_{t-32}p_{77} = S_{45}(t) = 0.85$:

$$(1 - 0.145150) \left[1 - (t - 18) \left(1 - \frac{7683980}{7833904} \right) \right] = 0.85$$

and this gives $t = 18.296455$.

(b) ${}_{20}q_{45} = 0.17788 < 0.5$. From (i), we get $\delta = 0.04879$. So the 50th percentile premium is $\frac{S}{\ddot{s}_{\overline{20}|}} = \frac{1000 \times 0.04879}{e^{20 \times 0.04879} - 1} = 29.5109$.

For the 15th percentile premium, since ${}_{18}q_{45} \leq 0.15 < {}_{19}q_{45}$,

the 15th percentile premium is $\frac{S}{\ddot{s}_{\overline{18.296455}|}} = \frac{1000 \times 0.04879}{e^{18.296455 \times 0.04879} - 1} = 33.8426$.

(c) The 50th percentile premium is $\frac{S}{\ddot{s}_{\overline{20}|}} = \frac{1000 \times 0.05 / 1.05}{1.05^{20} - 1} = 28.8025$.

The 15th percentile premium is $\frac{S}{\ddot{s}_{\overline{19}|}} = \frac{1000 \times 0.05 / 1.05}{1.05^{19} - 1} = 31.1857$.

C5-50 #23(b) next-to-last line: $Z_1 Z_2 = \begin{cases} 0 & K_x \leq n - 1 \\ v^{2n} & K_x \geq n \end{cases}$

C6-4 Part (a), line 6: $1 + \frac{0.92}{1.06} + \frac{0.8464}{1.06^2} + \frac{0.76176}{1.06^3} = 3.2608059$

line 8: $\pi = 895.84034 / 3.2608059 = 274.7297$.

Part(b), line -2: $\pi \ddot{a}_{42:\overline{2}|} = 274.7297 \left(1 + \frac{1}{1.06} \times 0.9 \right) = 507.99083$

line -1: ${}_2V = 1039.51584 - 507.99083 = 531.5250$.

C6-5 Solution to Example 6.2:

If $K_{40} = 2$ (with conditional probability $q_{42} = 0.1$), then

$${}_2L = \frac{1000}{1.06} - \pi = 668.6668.$$

If $K_{40} = 3$ (with conditional probability ${}_1q_{42} = 0.9 \times 0.1 = 0.09$), then

$${}_2L = \frac{1000}{1.06^2} - \pi \left(1 + \frac{1}{1.06} \right) = 356.0878.$$

If $K_{40} \geq 4$ (with conditional probability ${}_2p_{42} = 0.9 \times 0.9 = 0.81$), then

$${}_2L = \frac{1200}{1.06^2} - \pi \left(1 + \frac{1}{1.06} \right) = 534.0871.$$

We get $E({}_2L | K_{40} \geq 2) = 531.5251$ (the slight difference is due to rounding).

Also,

$$E({}_2L^2 | K_{40} \geq 2) = 668.6668^2 \times 0.1 + 356.0878^2 \times 0.09 + 534.0871^2 \times 0.81 = 287175.1105.$$

So, $\text{Var}({}_2L | K_{40} \geq 2) = 287175.1105 - 531.5251^2 = 4656.18$.

C6-24 Solution to Example 6.10: The first equation should read

$$({}_3V + 274.7297)(1.06) = 1200 \times 0.9 + 1000 \times 0.1 \Rightarrow {}_3V = 838.47815.$$

The third equation should read

$$\text{Var}({}_2L | K_{40} \geq 42) = \frac{1}{1.06^2} \times (1000 - 838.47815)^2 \times 0.9 \times 0.1 + \frac{0.9}{1.06^2} \times 3203.9872 = 4656.13.$$

C6-27 The next-to-last equation should read $\pi \ddot{a}_{k|} = 100 \sum_{j=0}^{k-1} (j+1)v^{j+1} q_{x+j} + v^k {}_kV$. Do

the same for the first equation on C6-28.

C6-30 In the figure, change “ $\pi = 319.5034$ ” to “ $\pi = 274.7297$ ”

C6-31 1st line: $274.7297 \times \frac{0.9375}{1.06^{0.6}} = 248.71007$

2nd line: ${}_{2.4}V = 1068.128411 - 248.71007 = 819.4183$

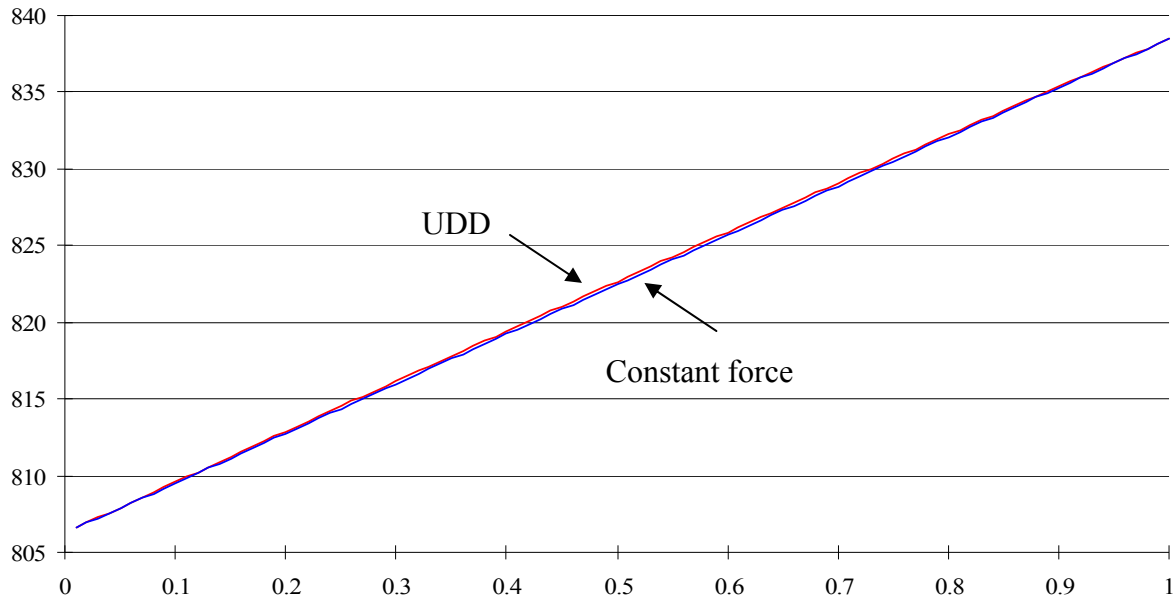
5th line: $274.7297 \times \frac{0.9387404}{1.06^{0.6}} = 249.03914$

6th line: ${}_{2.4}V = 1068.264008 - 249.03914 = 819.2249$

Next-to-last line: ${}_2V = 531.5250$

Last line: $\frac{(531.5250 + 274.7297)(1.06)^{0.4} - 1000(0.04)/1.06^{0.6}}{1 - 0.04} = 819.4183$

C6-32 The graph should be:



C6-33 Line -6: ${}_{2.4}V \approx 0.6 \times (531.5250 + 274.7297) + 0.4 \times 838.47815 = 819.1441$

C6-34 Part (a) for the solution of Example 6.15

$$({}_5V + 3.2117516)(1.05)^{1/6} = \frac{1000}{1.05^{1/2-1/6}} \times 0.002293406 + 0.997706593 {}_{5\frac{1}{6}}V$$

$${}_{5\frac{1}{6}}V = 15.2569755$$

C6-35 The last equation in part (c) of the solution to Example 6.15:

$${}_{4\frac{5}{6}}V(1.05)^{1/6} = 1000 \times 0.002125369 + 0.997874631 {}_5V, \quad {}_{4\frac{5}{6}}V = 16.08907$$

Line -4: change ${}_{10\frac{1}{6}}V$ to ${}_{5\frac{1}{6}}V$. Same for line -3 on C6-34.

C6-77 Solution to #42: $0.285 \approx \frac{4(0.2) + 5(0.3) + 7\pi}{9} \Rightarrow \pi \approx 0.0379$

C6-83 Solution to #56: line 7, 9 and 11, swap (B) and (D).

C7-10 Line 4: In $\frac{d_t V}{dt} = \begin{cases} P + 0.1_t V - 60 & 0 < t < 2 \\ 0.1_t V - 60 & 2 \leq t \leq 5 \end{cases}$, P should be replaced by π .

C7-20 Line -3: The benefit premium for a benefit of 1 dollar is P_{35}

C7-46 Solution to #9: Replace the two 1158.85 with 1155.85.

C8-5 The solution to Example 8.2 should be as follows:

Suppose that there are 100 students at the beginning (it does not matter if you use 100 or 1000). Let (1) be academic failure and (2) be withdrawal for all other reasons. Then information (i) can be converted into the following double decrement table:

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
0	100	$100 \times 0.4 = 40$	$100 \times 0.2 = 20$
1	$100 - 40 - 20 = 40$	$d_1^{(1)}$	$40 \times 0.3 = 12$
2	$40 - 12 - d_1^{(1)} = 28 - d_1^{(1)}$	$d_2^{(1)}$	$d_2^{(2)}$
3	$[28 - d_1^{(1)}] \times 0.6$	–	–

The number of students who survive year two is $28 - d_1^{(1)}$. Information (iii) gives $d_1^{(1)} = 0.4 \times (28 - d_1^{(1)})$, and hence $d_1^{(1)} = 8$. Information (ii) gives $28 - d_1^{(1)} = 10d_2^{(1)}$, and hence $d_2^{(1)} = 2$. As a result, we now have:

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
0	100	40	20
1	40	8	12
2	20	2	$d_2^{(2)}$
3	12	–	–

As a result, $d_2^{(2)} = 20 - 2 - 16 = 6$. The probability required is $\frac{20 + 12 + 6}{100} = 0.38$.

Thus the correct answer is given by (B).

C8-17 3 lines preceding (b): $\int_0^{0.5} {}_t p_{50}^{(\tau)} \mu_{50}^{(2)}(t) dt$ should read $\int_0^{0.5} {}_t p_{50}^{(\tau)} \mu_{50+t}^{(2)} dt$

C8-25 Line –1: replace 0.004415 with 0.04415.

C8-26 The last line for Example 8.12: replace 0.004415 with 0.04415.

C8-49 Solution to #28: When $t \in [0.2, 0.6)$, ${}_t q_x^{(2)} = 0.12 \times 3/4 = 0.09$

The sizes of the jumps are **0.09** and $0.12 - 0.09 = 0.03$. So,

$$q_{25}^{(2)} = {}_{0.2}p_{25}^{(1)} \times 0.09 + {}_{0.6}p_{25}^{(1)} \times 0.03$$

$$= (1 - 0.1 \times 0.2) \times 0.09 + (1 - 0.1 \times 0.6) \times 0.03 = 0.1164$$

C9-5 The chart in the middle and the bottom, 1st row, 2nd column, replace 1 with 0

C9-7 Example 9.4: In the solution we illustrate Euler's method, backward recursion, version I as specified in chapter C0.

C9-9 The equation in the middle should read

$$({}_h V + \pi_h)(1 + i) = p_{x+h}^{(\tau)} {}_{h+1} V + \sum_{j=1}^n b_{h+1}^{(j)} q_{x+h}^{(j)}$$

C9-10 Line –6: delete the extra “in the”

C9-11 6th line in the solution of (a): “The APV of death and survival benefit is”.
1st line in the solution of (b): “no one would withdraw”

C9-12 The equation at the middle should read ${}_k V - {}_k CV$

C9-23 Question 4: replace the second (iv) with (v).

C10-11 Example 10.3(a): in state 0 now and the beginning ...

C10-14 Line 6: change “state 2” to “states 0 and 1”.

C10-27 Line 5 of the solution to part (a), and Line 1 of the solution to part (b):
in the integral, replace ${}_s P_{60}^{01}$ with ${}_s P_{60}^{00}$.

C10-28 First line: replace ${}_s P_{60}^{01} \mu_{60+s}^{01} {}_{10-s} P_{60+s}^{11}$ with ${}_s P_{60}^{00} \mu_{60+s}^{01} {}_{10-s} P_{60+s}^{11}$
The equation for ${}_{10} P_{60}^{01}$ should read

$${}_{10} P_{60}^{01} \approx \frac{2.5}{3} [0.016518 + 4(0.021935 + 0.03391) + 2(0.028047) + 0.037762] \\ = 0.27813$$

The final answer for Example 10.9 is 0.42571.

C10-36 The table should read

s	${}_s P_{60}^{11}$	s	${}_s P_{60}^{11}$
0.5	0.983589	3	0.893227
1	0.966626	3.5	0.873497
1.5	0.949110	4	0.853224
2	0.931037	4.5	0.832419
2.5	0.912409	5	0.811095

Solution: Integral in the first line: $1000 \int_0^5 e^{-0.05t} {}_t P_{60}^{11} \mu_{60+t}^{12} dt$

Second line: $f(t) = e^{-0.05t} {}_t P_{60}^{11} \mu_{60+t}^{12}$

t	${}_t P_{60}^{11}$	μ_{60+t}^{12}	$f(t)$
0	1	0.032274	0.032274
0.5	0.983589	0.033929	0.032548
1	0.966626	0.035669	0.032797
1.5	0.949110	0.037497	0.033018
2	0.931037	0.03942	0.033209
2.5	0.912409	0.041441	0.033368
3	0.893227	0.043566	0.033494
3.5	0.873497	0.045799	0.033583
4	0.853224	0.048148	0.033634
4.5	0.832419	0.050616	0.033645
5	0.811095	0.053211	0.033613

The value of the integral is approximately

$$\frac{0.5}{2} [0.032274 + 0.033613 + 2(0.032548 + 0.032797 \\ + \dots + 0.033634 + 0.033645)] = 0.16612$$

So, the APV is approximately equal to 166.12.

C10-46 Question 23: Replace “Kolmogorv” with “Kolmogorov”

C10-53 Question 5: The first row of the matrix should read 844/888, 3/888, 41/888.

The sum of the probabilities is $\frac{844}{888} + \frac{3}{888} + 1 = 1.95383$.

C10-61 (c) The probability is $\int_0^{\infty} {}_t p_x^{\overline{00}} \mu_{x+t}^{02} dt = \frac{1}{7} \int_0^{\infty} e^{-0.06t} \times 0.02 dt = \frac{1}{3}$.

C11-9 The “warning” at the top should read:

While ${}_{m|n} q_{xy} = {}_m p_{xy} n q_{x+m:y+m}$, ${}_{m|n} q_{\overline{xy}} \neq {}_m p_{\overline{xy}} n q_{\overline{x+m:y+m}}$!

C11-10 Line 3: the 2nd formula should read $E(T_{xy}^-) = \int_0^{\infty} {}_t p_{xy}^- dt$.

C11-12 Line -5: $\text{Var}(T_{xy}) = \frac{1}{0.05^2} = 400$

C11-15 Example 11.8 statement (ii) ${}_{\infty} q_{80:98}^1$ should read ${}_{\infty} q_{80:98}^1$.

C11-16 The first line of the equation: ${}_{\infty} q_{80:98}^1$ should read ${}_{\infty} q_{80:98}^2$.

C11-42 Question 21, statement (iii) should read “ $\ddot{a}_{30:40} = 14.2068$, $\ddot{a}_{40:50} = 12.4784$ ”

C11-49 Second line: Replace “Questions 37 to 39” with “Questions 38 to 40”.

C11-59 Solution to #21:

$$\begin{aligned} a_{\overline{30:40:\overline{10}}} &= a_{\overline{30:40}} - {}_{10} E_{\overline{30:40}} a_{\overline{40:50}} \\ &= (\ddot{a}_{\overline{30:40}} - 1) - v^{10} \frac{l_{40}}{l_{30}} \frac{l_{50}}{l_{40}} (\ddot{a}_{\overline{40:50}} - 1) \\ &= 13.2068 - 1.06^{-10} \frac{89509.00}{95013.79} \times 11.4784 = 7.1687 \end{aligned}$$

C11-67 and 68 Question #46(c) Starting from “As a result”:

As a result, ${}_t p_y = {}_t p_{xy}^{00} + {}_t p_{xy}^{02} = e^{-0.18t} + \frac{5}{3}(e^{-0.12t} - e^{-0.18t}) \times 0.8 = \frac{4e^{-0.12t} - e^{-0.18t}}{3}$.

$$\bar{a}_{\overline{y:\overline{10}}} = \int_0^{10} e^{-0.05t} \times \frac{1}{3} (4e^{-0.12t} - e^{-0.18t}) dt = \frac{1}{3} \left(\frac{4(1 - e^{-1.7})}{0.17} - \frac{(1 - e^{-2.3})}{0.23} \right) = 5.106353$$

By symmetric relation, $\bar{a}_{\overline{xy:\overline{10}}} = \bar{a}_{\overline{x:\overline{10}}} + \bar{a}_{\overline{y:\overline{10}}} - \bar{a}_{\overline{xy:\overline{10}}} = 6.599396$.

C13-2 Line -4 and -3: the surplus emerging at time $h + 1$ should be

$$[G_h(1 - c_h) - e_h](1 + i) - (b_{h+1} + E_{h+1})q_{x+h}$$

C13-4 Last line of solution to Example 13.1: replace 7.0866635 with 7.1694635.

C13-6 and 7 Example 13.2: 4th line: ${}_1 V^{(1)} = 590.9576$

$$\text{Pr}_0^{(0)} = -130$$

$$\text{Pr}_1^{(0)} = 260 \times 1.08 - [0 \times 0.7 + 590.9576 \times 0.25 + 1005 \times 0.05] = 82.8106$$

$$\text{Pr}_2^{(0)} = (260 - 26) \times 1.08 - [0 \times 0.7 + 377.358 \times 0.2 + 1005 \times 0.1] = 76.7484$$

$$\text{Pr}_3^{(0)} = (260 - 13) \times 1.08 - 1005 \times 0.1 = 166.26$$

$$\text{Pr}_2^{(1)} = (590.9576 - 26) \times 1.08 - (377.358 \times 0.75 + 1005 \times 0.25) = 75.8857$$

$$\text{Pr}_3^{(1)} = (377.358 - 13) \times 1.08 - 1005 \times 0.25 = 142.2566$$

(The profits under state 2 are all 0s.)

$$\text{So, } \Pi_0 = -130, \Pi_1 = 82.8106, \Pi_2 = 0.7 \times 76.7484 + 0.25 \times 75.8857 = 72.6953$$

$$\text{Pr}(Y_2 = 0 \mid Y_0 = 0) = 0.7^2 = 0.49,$$

$$\text{Pr}(Y_2 = 1 \mid Y_0 = 0) = 0.7 \times 0.2 + 0.25 \times 0.75 = 0.3275$$

(delete the extra) after 0.75)

$$\Pi_3 = 0.49 \times 166.26 + 0.3275 \times 142.2566 = 128.0564.$$

The profit signature is $\Pi = (-130, 82.8106, 72.6953, 128.0564)'$.

C13-8 Line -3: delete the second “is”.

C13-10 Line 2 in the paragraph starting with “For life insurance policies”: Replace R_k by G_k .

C13-11 Solution to Example 13.3:

Line 1 and 3: Replace 7.0866635 with 7.1694635

Line 3: The final value for NPV(0.05) in line 3 should be changed to 0.728477.

Line 3: the IRR is found to be 6.44%

Table: for $m = 2$, the summation should be -4.94096

for $m = 3$, the summation should be 0.728477

Line -3 to line -1:

$$\sum_{k=0}^2 \frac{G_{k,k} P_x}{(1+r)^k} = G \left(1 + \frac{0.92}{1.05} + \frac{0.92 \times 0.91}{1.05^2} \right) = 269.95497.$$

The profit margin is $0.728477 / 269.95497 = 0.27\%$.

The NPV as a percentage of acquisition costs is $0.728477 / 35.6070275 = 2.05\%$.

C13-12 Question 1: Consider the set up in Example 13.2.

C13-15 The profit signature is $(-130, 82.8106, 72.6953, 128.0564)'$.

$$\begin{aligned} \text{(a)} \quad & -130 + \frac{82.8106}{1.1} + \frac{72.6953}{1.1^2} + \frac{128.0564}{1.1^3} \\ & = -130 + 75.2824 + 60.0788 + 96.2107 = 101.5719 \end{aligned}$$

(b) Replace 89.084% with 47.2%.

(c) Since $-130 + 75.2824 < 0$ and $-130 + 75.2824 + 60.0788 > 0$,

(d) So the profit margin is $101.5719 / 530.7438 = 19.1\%$.

(e) $101.5719 / 130 = 78.1\%$.

C14-25 Solution to Example 11

Part (a)

For “At time I ”, change the following two items:

$$\text{ESB}_1: (1 - 1.2 \times 0.00278) \times 0.05 \times (2000 - 600 + 100) = 74.7498.$$

$$\text{Pr}_1: (0 + 2500 - 0)(1.1) - 333.9336 - 74.7498 - 1893.662 = 447.6546$$

For “*At time 2*”, change the following two items:

$$ESB_2: (1 - 1.2 \times 0.00298) \times 0.1 \times (4900 - 600 + 100) = 438.4266$$

$$Pr_2: (2000 + 2500 - 62.5)(1.1) - 357.9576 - 438.4266 - 4394.2298 = -309.3640$$

For “*At time 3*”, change the following two items:

$$ESB_3: (1 - 1.2 \times 0.00320) \times 1 \times (8000 - 600 + 100) = 7471.2$$

$$Pr_3: (4900 + 2500 - 62.5)(1.1) - 384.384 - 7471.2 - 0 = 215.666$$

Final sentence: the profit vector is given by $(-200, 447.65, -309.36, 215.67)'$.

Part (b) final sentence: the profit signature is $(-200, 447.65, -292.92, 183.12)'$.

C14-35 Question 17: Replace the choices with

(A) $(-300, 100, -81, 749)'$

(B) $(-300, 100, 81, 749)'$

(C) $(-300, 100, -162, 668)'$

(D) $(-300, 100, 162, 668)'$

(E) $(-300, 100, 271, 877)'$

C14-36 Question 18: Replace the choices with

(A) $(-200, 36, 103, 717)'$

(B) $(-200, 53, 61, 445)'$

(C) $(-200, 36, 61, 717)'$

(D) $(-200, 53, 103, 445)'$

(E) $(-200, 53, 19, 445)'$

C14-44 Solution to Question 17

For “*At time 1*”, change the following two items:

$$ESB_1: (1 - 1.2 \times 0.00278) \times 0.1 \times (2900 - 1000 + 150) = 204.3161.$$

$$Pr_1: (0 + 3000 - 0)(1.08) - 334.1004 - 204.3161 - 2601.2930 = 100.2905$$

For “*At time 2*”, change the following two items:

$$ESB_2: (1 - 1.2 \times 0.00298) \times 0.2 \times (6200 - 1000 + 150) = 1066.1737$$

$$Pr_2: (2900 + 3000 - 80)(1.08) - 358.1364 - 1066.1737 - 4942.2630 = -80.9731$$

For “*At time 3*”, change the following two items:

$$ESB_3: (1 - 1.2 \times 0.00320) \times 1 \times (9600 - 1000 + 150) = 8716.4$$

$$Pr_3: (6200 + 3000 - 80)(1.08) - 384.576 - 8716.4 - 0 = 748.624$$

Final sentence: the profit vector is given by $(-300, 100, -81, 749)'$.

C14-45 Solution to Question 18

For “*At time 1*”, change the following two items:

$$ESB_1: (1 - 1.2 \times 0.00278) \times 0.1 \times (1950 - 800 + 100) = 124.583.$$

$$Pr_1: (0 + 2000 - 0)(1.1) - 273.7188 - 124.583 - 1749.1453 = 52.5529$$

For “*At time 2*”, change the following two items:

$$ESB_2: (1 - 1.2 \times 0.00298) \times 0.2 \times (4100 - 800 + 100) = 677.5683$$

$$Pr_2: (1950 + 2000 - 70)(1.1) - 301.0992 - 677.5683 - 3268.2707 = 21.0618$$

For “*At time 3*”, change the following two items:

$$ESB_3: (1 - 1.2 \times 0.00320) \times 1 \times (6400 - 800 + 100) = 5678.112$$

$$Pr_3: (4100 + 2000 - 70)(1.1) - 332.16 - 5678.112 - 0 = 622.726$$

Line -6: the profit vector is given by $(-200, 52.5529, 21.0618, 622.726)'$.

Line -1: the profit signature is $(-200, 52.5529, 18.8924, 445.2690)'$.

C15-5 The equation at the middle: $\frac{\text{Salary earned in } (t - 0.5h, t + 0.5h)}{h} = 1.05$

for any $h \leq 1$. In this case, the salary rate is exactly 1.05.

C15-6 Last -1: $100,000 \frac{s_{55}}{s_{34.5}} = 246,538.0$

C15-7 Line 4: during $(55.5, 56)$ is $100,000(1.045)^{21}/2$

C15-8 Line 1 of Example 15.4 (c) Change 40 to 35.

C15-11 Line 3 of Step 1 of the solution: Change 1.000375 to 1.00375.

C15-15 Last paragraph: delete "As you shall see shortly,"

C15-16 Line 2 of #3: change September to October.

C15-19 Line 1 of #3: change September to October.

#5: The monthly salary over year of age $(49\frac{5}{6}, 50\frac{5}{6})$ is 6000. The expected salary in the final year of work is

$$6000 \times 12 \frac{s_{64}}{s_{49\frac{5}{6}}} = 72000 \times \frac{7.70}{\frac{1}{6} \times 3.21 + \frac{5}{6} \times 3.41} = 164,185.587$$

The expected target pension benefit per year is $164,185.587 \times 0.65 = 106,721$.

T1-16 Replace the choices with

- (A) $(-200, 850, 779, 1346)'$ (B) $(-200, 819, 870, 1346)'$
(C) $(-200, 928, 1089, 1255)'$ (D) $(-200, 1089, 928, 1255)'$
(E) $(-200, 928, 1089, 1346)'$

T1-30 and T1-31

For "At time 1", change the following two items:

ESB₁: $(1 - 1.2 \times 0.00592) \times 0.1 \times (1950 - 800 + 100) = 124.112$.

Pr₁: $(0 + 3000 - 0)(1.1) - 582.8832 - 124.112 - 1742.5325 = 850.4723$

For "At time 2", change the following two items:

ESB₂: $(1 - 1.2 \times 0.00642) \times 0.2 \times (4100 - 800 + 100) = 674.7613$

Pr₂: $(1950 + 3000 - 80)(1.1) - 648.6768 - 674.7613 - 3254.7309 = 778.831$

For "At time 3", change the following two items:

ESB₃: $(1 - 1.2 \times 0.00697) \times 1 \times (6400 - 800 + 100) = 5652.3252$

Pr₃: $(4100 + 3000 - 80)(1.1) - 723.486 - 5652.3252 - 0 = 1346.1888$

Final sentence: Therefore, the profit vector is given by $(-200, 850, 779, 1346)'$.

T2-2 #4: The question should read "Find $1.8p_{[66]}$ ".

T2-6 #12 (v)(1) Change 200 to 320

T2-12 #22 change the choices for (C) and (D) to 10555 and 10778.

T2-18 Key: #4: Change (D) to (E).

T2-19 #4: Change the answer to (E). Last two line of the solution:

$$\text{Finally, } {}_0.4p_{[66]+0.6} = p_{[66]} / {}_0.6p_{[66]} = 0.88 / (1 - 0.6 \times 0.12) = 0.948276.$$

So, the answer is 0.76776.

T2-22 #12 Replace “200” in $200\ddot{a}_{30:25} + 120(\ddot{a}_{\overline{30:25}} - \ddot{a}_{30:25}) = 80\ddot{a}_{30:25} + 120(\ddot{a}_{30} + \ddot{a}_{25})$ with 320.

T2-28 Second line: change 0.64^3 to 0.94^3 . Third line: change 37.20572 to 56.11430618.

$$5^{\text{th}} \text{ line: } \pi = 56.11430618 / 2.5458112 = 22.04181762.$$

$$8^{\text{th}} \text{ line: } {}_2V = 0.94(0 + 0.14 \times 300) - \pi = 17.4381824.$$

13th and 14th line:

$$= 0.94^2 \times (200 - 17.4381824)^2 \times 0.88 \times 0.12 + 0.94^2 \times 0.88 \times 9574.6896$$

$$= 10554.823$$

$$16^{\text{th}} \text{ line: } {}_1L = \begin{cases} 200v - \pi = 165.9581824 & K_{62} = 0 \\ 300v^2 - \pi(1+v) = 222.318874 & K_{62} = 1 \\ -\pi(1+v) = -42.76112618 & K_{62} \geq 2 \end{cases}$$

19th and 20th line:

$$E({}_1L | K_{61} \geq 1) = 14.94304684, E({}_1L^2 | K_{61} \geq 1) = 10778.1175$$

$$\text{This gives } \text{Var}({}_1L | K_{61} \geq 1) = 10554.823.$$