ACTEX Five Practice Exams for P/1

Errata 3rd Printing 9/15/10

Following are corrected problems and solutions for insertion into this manual.

Question 2-9

Suppose $P(A)=0.6$, $P(B)=0.52$, $P(A\cap B)=0.14$ and $P(A^C\cap B^C)=0$. Furthermore, suppose $P\left(C\middle|\left(A\cap B^C\right)\right)=0.35$, $P\left(C\middle|\left(A^C\cap B\right)\right)=0.55$ and $P\left(C\middle|\left(A\cap B\right)\right)=0.85$.

Calculate $P(C)$.

A) 0.37 B) 0.49 C) 0.50 D) 0.62 E) 0.65

Question 3-21

Suppose B and C are mutually exclusive events. Also, assume the following:

- $P(A)=0.30$
- $P(B)=1.25\,P(C)$
- $P(A\cap B)+P(A\cap C)=0.15$
- $P\left(A^C\cap B\right)=0.40$
- $P\left(A^C\cap B^C\cap C^C\right)=0.10$

Calculate $P(A\cap C)$.

A) 0.04 B) 0.08 C) 0.13 D) 0.16 E) 0.25

Question 4-1

Suppose $P(A) = 0.48$, $P(B) = 0.42$ and $P(A \cap B) = 0.17$ and $P(A^C \cap B^C) = 0$. Furthermore, suppose $P\left(C \middle| \left(A \cap B^C\right)\right) = 0.37$, $P\left(C \middle| \left(A^C \cap B\right)\right) = 0.52$ and $P\left(C \middle| \left(A \cap B\right)\right) = 0.81$.

Calculate $P(C)$.

A) 0.24

B) 0.38

C) 0.40

D) 0.44

E) 0.53

Question 4-8

Suppose a scientist has two tests to measure the age of a fossil. The first test measures the age with mean error of $0.001x$ and variance $0.02x^2$, and the second test measures the age with mean error $0.0015x$ and variance $0.015x^2$, where x is the actual age of the fossil, in years. Note this means that both tests, on average, slightly overstate the age of the fossil.

If the error of each test is normally distributed with the parameters given and the error of each is independent of the other, calculate the probability that the average age given by the two tests is within the interval $(0.995x,\ 1.005x)$.

A) 0.001

B) 0.004

C) 0.028

D) 0.044

E) 0.210

Question 4-19

Suppose B and C are mutually exclusive events. Also, assume the following:

$$P(A) = 0.40$$
$$P(B) = 1\,P(C)$$
$$P(A \cap B) + P(A \cap C) = 0.10$$
$$P\left(A^C \cap B\right) = 0.30$$
$$P\left(A^C \cap B^C \cap C^C\right) = 0.20$$

Calculate $P(A \cap C)$.

A) 0.05 B) 0.10 C) 0.15 D) 0.20 E) 0.25

Solution 2-9

To solve, we are going to use

$$P(C) = P\left(C \middle| \left(A \cap B^C\right)\right) P\left(A \cap B^C\right) + P\left(C \middle| \left(A^C \cap B\right)\right) P\left(A^C \cap B\right) + P\left(C \middle| (A \cap B)\right) P(A \cap B)$$

$$+P(C|(A^C \cap B^C))P(A^C \cap B^C)$$

So we only need to find $P\left(A \cap B^C\right)$ and $P\left(A^C \cap B\right)$ to find $P(C)$.

$$P(A) = P(A \cap B) + P\left(A \cap B^C\right)$$
$$P\left(A \cap B^C\right) = P(A) - P(A \cap B) = 0.6 - 0.14 = 0.46$$

$$P(B) = P(A \cap B) + P\left(A^C \cap B\right)$$
$$P\left(A^C \cap B\right) = P(B) - P(A \cap B) = 0.52 - 0.14 = 0.38$$

Now we just plug in our known values and get

$$P(C) = (0.35)(0.46) + (0.55)(0.38) + (0.85)(0.14) + (0) = 0.489$$

Answer: B

Solution 3-11

We could find c explicitly by setting the integral of $f(x)$ equal to 1 and solving for c, but we can tell by the form of the probability density function that this distribution is Exponential with a mean of 100. Now we just need to find p_{50} such that $P(X \le p_{50}) = \frac{1}{2}$.

The cumulative distribution function of an Exponential is 1-e$^{-\frac{x}{\mu}}$, where μ is the mean. So we need to solve $1 - e^{-0.01 p_{50}} = \frac{1}{2}$ for p_{50}.

This gives us that the median benefit paid is 69.31.

Note that if the median of the distribution was higher than 110, then the median benefit paid would be 110.

Answer: B

Solution 3-21

First, we find $P(C)$.

Note that $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$.

B and C are mutually exclusive, $P(B \cap C) = 0$ and $P(A \cap B \cap C) = 0$.

Also, note that $P(A \cup B \cup C) = 1 - P(A^C \cap B^C \cap C^C)$.

So now we plug in our known values, and we get

$$1 - 0.10 = 0.90 = 0.30 + 1.25P(C) + P(C) - 0.15 = 0.15 + 2.25P(C)$$

This gives us that $P(C) = 0.33$.

Next we will use that, since B and C are mutually exclusive,

$$P(A \cup C) = P(A \cup B \cup C) - P(A^C \cap B) = 0.90 - 0.40 = 0.50$$

Finally,

$$P(A \cap C) = P(A) + P(C) - P(A \cup C) = 0.30 + 0.33 - 0.50 = 0.13$$

Answer: C

Solution 4-1

To solve, we are going to use

$$P(C) = P\left(C \middle| \left(A \cap B^C\right)\right) P\left(A \cap B^C\right) + P\left(C \middle| \left(A^C \cap B\right)\right) P\left(A^C \cap B\right) + P\left(C \middle| (A \cap B)\right) P(A \cap B)$$

$$+ P(C|(A^C \cap B^C))P(A^C \cap B^C)$$

So we only need to find $P\left(A \cap B^C\right)$ and $P\left(A^C \cap B\right)$ to find $P(C)$.

$$P(A) = P(A \cap B) + P\left(A \cap B^C\right)$$

$$P\left(A \cap B^C\right) = P(A) - P(A \cap B) = 0.48 - 0.17 = 0.31$$

$$P(B) = P(A \cap B) + P\left(A^C \cap B\right)$$

$$P\left(A^C \cap B\right) = P(B) - P(A \cap B) = 0.42 - 0.17 = 0.25$$

Now we can just plug in our known values and we get

$$P(C) = (0.37)(0.31) + (0.52)(0.25) + (0.81)(0.17) + (0) = 0.382$$

Answer: B

Solution 4-8

Let X_1 and X_2 be random variables that represent the error of tests 1 and 2, respectively. Then the error of the average of tests 1 and 2 is given by the random variable $Y = \dfrac{X_1 + X_2}{2},$ which is normally distributed with mean $0.00125x$ (the average of the means of X_1 and X_2) and variance $0.00875x^2$, where

$$Var\left(\frac{X_1 + X_2}{2}\right) = \left(\frac{1}{2}\right)^2 Var(X_1 + X_2) = \frac{Var(X_1) + Var(X_2)}{4}.$$

The problem is asking for the probability that the average of the two tests gives an age in the interval $(0.995x, 1.005x),$ which means Y must be in the interval $(-0.005x, 0.005x)$. So the probability statement we need to solve is $P(-0.005x \le Y \le 0.005x)$. Notice that this is $P(Y \le 0.005x) - P(Y \le -0.005x)$.

To find these probabilities, we must convert Y to a standard normal random variable. So now we must find

$$P\left(\frac{Y - \mu_Y}{\sigma_Y} \le \frac{0.005x - \mu_Y}{\sigma_Y}\right) - P\left(\frac{Y - \mu_Y}{\sigma_Y} \le \frac{-0.005x - \mu_Y}{\sigma_Y}\right).$$

We have that $\mu_Y = 0.00125x$ and $\sigma_Y = 0.093541x,$ so our probability statement simplifies to $P(Z \le 0.040089) - P(Z \le -0.066815)$. Since our normal tables only go to two decimals, we must simplify this to $P(Z \le 0.04) - P(Z \le -0.07)$.

According to the normal tables, $P(Z \le 0.04) = 0.5159,$ and $P(Z \le -0.07) = 0.4721.$ Thus our answer is $P(Z \le 0.04) - P(Z \le -0.07) = 0.0439.$

Answer: D

Solution 4-19

First, we find $P(C)$.

Note that
$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(A\cap C)-P(B\cap C)+P(A\cap B\cap C)$.

B and C are mutually exclusive, $P(B\cap C)=0$ and $P(A\cap B\cap C)=0$.

Also, note that $P(A\cup B\cup C)=1-P\left(A^C\cap B^C\cap C^C\right)$.

So now we plug in our known values, and we get

$$1-0.20=0.80=0.40+1P(C)+P(C)-0.10=0.30+2P(C)$$

This gives us that $P(C)=0.25$.

Next we will use that, since B and C are mutually exclusive,

$P(A\cup C)=P(A\cup B\cup C)-P\left(A^C\cap B\right)=0.80-0.30=0.50$

Finally,

$P(A\cap C)=P(A)+P(C)-P(A\cup C)=0.40+0.25-0.50=0.15$

Answer: C