Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance

Third Edition

Product Preview

Robert L. Brown
Ph.D., FSA, FCIA, ACAS
Leon R. Gottlieb
FCAS, MAAA

Example 3.7

Given the following information, and assuming the revised rates take effect July 1, 2007 for one year on one-year policies, determine new rates for each of Class 1 and Class 2, for each of Territory 1 and Territory 2. (Class 1/2 differentials will not change.) Use the loss ratio and loss cost methods, and base the overall average rate change on 2005 policy year data, assuming they are fully credible for that purpose. The permissible loss ratio is .600.

Policy Year 2004 Losses

As of March 31, 2006		As of March 31, 2007	
Paid	Outstanding	Paid	Outstanding
400,000	100,000	625,000	0

Trend Factors

July 1, 2006 to July 1, 2007	July 1, 2006 to July 1, 2008	January 1, 2006 to July 1, 2007	January 1, 2006 to July 1, 2008
1.18	1.30	1.24	1.36

	Territory 1	Territory 2
Present Rates		
Class 1 (Differential)	100 (1.00)	200 (2.00)
Class 2 (Differential)	300 (3.00)	600 (6.00)
Collected Earned Premium	700,000	600,000
Policy Year 2005 Incurred Losses as of March 31, 2007	360,000	240,000
Earned Exposure Units		
Class 1	5,000	2,000
Class 2	1,000	500

Solution

For each of the two methods, loss ratio and loss cost, the rate change involves the three stages of (i) overall or average rate change, (ii) change in differentials, and (iii) balance back.

Basic Ingredients

We must first calculate the expected losses to be incurred in the future exposure period, trended and developed. We begin by calculating a loss-development factor to apply to the 2005 policy year losses. From the policy year 2004 data, we can see that, at the 2006 reporting (i.e., as of 27 months), the total incurred losses were 500,000 (400,000 + 100,000). One year later (as of 39 months), the total incurred losses were 625,000. These losses are considered fully mature because there are no reserves. Thus, policy year 2004 data exhibited loss development of +25% from first report to fully mature.

Assuming the same loss development for 2005 data, we would use a loss-development factor of 1.25 to develop the 2005 policy year losses to their expected ultimate value.

Next we calculate the trend factor, making the use of the following diagram.

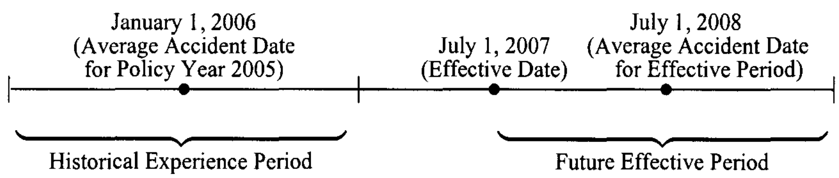

Figure 3.5

The experience data come from policy year 2005 with an average accident date of January 1, 2006. The effective date is July 1, 2007 so the future effective period is from July 1, 2007 through June 30, 2009, with an average accident date of July 1, 2008. Thus the required trend factor is the one that goes from January 1, 2006 to July 1, 2008, which is given as 1.36. Then in total we find

$$E\text{[Dollars of Future Losses (Developed and Trended)]}$$

$$= (360{,}000+240{,}000)(1.25)(1.36)$$

$$= 1{,}020{,}000.$$

Loss Ratio Method

(i) Overall Rate Change

For the loss ratio method the actuary must calculate the earned premium at current rates. The accounting entry for collected earned premium is not the correct denominator, because it could contain earned premiums based on the rates in rate manuals older than the current manual.

Earned Premium at Current Rates

$$= \sum_{i,j} CR_{ij} \cdot e_{ij}$$

$$= (100)(5{,}000) + (300)(1{,}000) + (200)(2{,}000) + (600)(500)$$

$$= 1{,}500{,}000.$$

This produces

Expected Effective Period Loss Ratio at Current Rates

$$= \frac{1{,}020{,}000}{1{,}500{,}000} = .680,$$

which, based on a permissible loss ratio of .600, leads to

$$\text{Indicated Rate Change} = \frac{.680}{.600} - 1 = +13.3\%.$$

(ii) Change in Differentials

The given data allow for a territorial differential change analysis but not a class differential change analysis, because loss data by class were not given. We are told that class differentials will remain the same, and are asked to determine the indicated new differentials for Territories 1 and 2. (We must assume the data are 100% credible.)

Territory 1 Earned Premium at Current Rates:

$$(100)(5000) + (300)(1000) = 800{,}000$$

Territory 2 Earned Premium at Current Rates:

$$(200)(2000) + (600)(500) = 700{,}000$$

Territory	Existing Differential	Loss Ratio at Current Rates	Indicated Differential
1	1.00	$\frac{360,000}{800,000} = .450$	1.00
2	2.00	$\frac{240,000}{700,000} = .3429$	$\frac{.3429}{.45}(2.00) = 1.5238$

It is irrelevant to this territorial differential analysis whether losses are trended and developed or not, since we are analyzing ratios of loss ratios. Note that as presented, the Territory 1 differential has been left at 1.00, whereas the Territory 2 differential has been reduced from 2.00 to 1.5238. This suggests that the actuary could define the new rates as follows:

	Territory 1	Territory 2
Class 1	113.33	172.70
Class 2	340.00	518.09

If this were done, however, the resulting rate increase would be less than the required +13.3%, due to the off-balance created by the method used to change differentials. This is adjusted in the balance-back step.

(iii) Balance Back

Old Average Differential:

$$\frac{(5000)(1) + (1000)(3) + (2000)(2) + (500)(6)}{8500} = 1.7647$$

New Average Differential:

$$\frac{(5000)(1) + (1000)(3) + (2000)(1.5238) + (500)(4.5714)}{8500} = 1.5686$$

The balance-back factor is

$$\frac{\text{Old Average Differential}}{\text{New Average Differential}} = \frac{1.7647}{1.5686} = 1.1250,$$

leading to the following proposed rates:

	Territory 1	Territory 2
Class 1	127.50	194.28
Class 2	382.50	582.85

These proposed rates will result in a 13.3% increase in premium income, as required.

Loss Cost Method

(i) Average Rate Change

We have already calculated:

E[Dollars of Future Losses (Developed and Trended)] = 1,020,000,

from which we find

$$\text{Indicated Loss Cost} = \frac{1{,}020{,}000}{8{,}500} = 120.00$$

and

$$\text{Average Gross Rate} = \frac{120}{PLR} = \frac{120}{.600} = 200.00.$$

(Note that this is the indicated *average* gross rate, but not the indicated rate for any particular territory or class, which can be determined only when we know the new average differential for our expected book of business.)

(ii) Change in Differentials

To set the new territorial differentials, the actuary normally calculates the average loss costs for Territory 1 and Territory 2, and compares them as follows:

Territory	Existing Differential	Loss Cost	Indicated Differential
1	1.00	$\frac{360{,}000}{6{,}000} = 60$	1.00
2	2.00	$\frac{240{,}000}{2{,}500} = 96$	1.60

This is not the same answer as we got from the loss ratio method. As seen in Example 3.6, the loss cost method normally leads to the correct answer only if all cross-variable distributions are homogeneous, which is not the case here. Recall the following earned exposure unit data:

	Territory 1	Territory 2
Class 1	5,000	2,000
Class 2	1,000	500

In Territory 1, $\frac{5}{6}$ of drivers are Class 1 and $\frac{1}{6}$ are Class 2. In Territory 2, $\frac{4}{5}$ of drivers are Class 1 and $\frac{1}{5}$ are Class 2. To arrive at the correct answer, this heterogeneity of cross-variable distributions must be reflected. One way to accomplish this is to use exposure units that are weighted by their cross-parameter differentials. That is, Class 1 will count as an exposure unit with weight 1.00, but Class 2 will count as an exposure unit with weight 3.00, because of its class differential of 3.00. This leads to the following results:

Territory	Existing Differential	Weighted[5] Units of Exposure	Loss Cost per Weighted Unit of Exposure	Indicated Differential
1	1.00	8000	$\frac{360{,}000}{8{,}000} = 45.00$	1.00
2	2.00	3500	$\frac{240{,}000}{3{,}500} = 68.57$	1.5238

(iii) Balance Back

For any risk cell:

$$\text{Rate } (i, j) = (\text{Base Rate})(D_i)(D_j).$$

Thus, the actuary determines the gross rate for the base class: Class 1, Territory 1 that will produce the correct manual rates by balancing back for the average indicated differential. Thus

$$\text{Base Gross Rate} = \frac{\text{Average Gross Rate}}{\text{Average Differential}},$$

where the average gross rate is 200 and the average differential is

[5]These are normally called "Base Exposures." For further discussion see: Risk Classification by Robert J. Finger in the textbbook *Foundations of Casualty Actuarial Science*, Casualty Actuarial Society, 2001.

$$\frac{(5000)(1) + (1000)(3) + (2000)(1.5238) + (500)(4.5714)}{8500} = 1.5686.$$

This leads to

$$\text{Base Gross Rate} = \frac{200}{1.5686} = 127.50.$$

The resulting manual rates are the same as with the loss ratio method, as expected. A general proof of the equivalence of the manual rates under the loss cost and the loss ratio methods is given in Appendix A. □

About The Text

This text provides a basic foundation of knowledge concerning two fundamental building blocks of property/casualty actuarial work: ratemaking and loss reserving. Although the material is of property/casualty origins, the methods presented have potential application in other insurance areas.

The text contains a number of worked examples and end-of-chapter exercises.

This third edition includes overviews of individual risk rating, increased limits factors, and reinsurance.

About The Authors

Robert L. Brown, Ph.D., FSA, FCIA, ACAS

Rob Brown graduated from the University of Waterloo in 1971 with a BMath degree. He added a MA in Gerontology in 1994 (Waterloo) and a Ph.D. (Gerontology) from Simon Fraser University in 1997. Rob is a Fellow of the Canadian Institute of Actuaries, a Fellow of the Society of Actuaries and an Associate of the Casualty Actuarial Society.

Professor Brown was President of the Canadian Institute of Actuaries in 1990/91 and served on the Board of Governors of the Society of Actuaries from 1992 to 1996. From 1997-99, he was SOA Vice-President and, for 2000/01, Rob was President of the Society of Actuaries.

Rob Brown is presently Professor of Actuarial Science and Director of the Institute of Insurance and Pension Research at the University of Waterloo.

Leon R. Gottlieb, FCAS, MAAA

Leon R. Gottlieb is a Principal of Mercer Oliver Wyman Actuarial Consulting, in Chicago. His actuarial career spans 35 years and includes consulting, insurance, and reinsurance company employment.

Mr. Gottlieb is a Fellow of the Casualty Actuarial Society and a Member of the American Academy of Actuaries. He has served on numerous professional and industry committees and is a former President of the Casualty Actuaries of the Southeast.

ACTEX Publications
Winsted, CT

ISBN: 978-1-56698-611-3