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ARCONES' MANUAL FOR THE SOA EXAM FM/CAS EXAM 2, FINANCIAL MATHEMATICS.

SPRING 2010 EDITION.

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Title

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Contents

Preface . . . i
Contents . . . iii

1 Basic interest theory **1**
1.1 Amount and accumulation functions. . . . 1
1.2 Simple interest . . . 10
1.3 Compound interest . . . 16
1.4 Present value and discount. . . . 24
1.5 Nominal rate of interest . . . 26
1.6 Force of interest . . . 32
1.7 Problems . . . 37
1.8 Questions from actuarial exams . . . 42
1.9 Solutions to problems . . . 44
1.10 Solutions to questions from actuarial exams . . . 54

2 Cash flows and yield rates **57**
2.1 Net present value of cash flows . . . 57
2.2 Method of equated time . . . 62
2.3 Yield of return . . . 64
2.4 Dollar–weighted and time–weighted rates of return . . . 72
2.5 Investment and portfolio methods . . . 75
2.6 Continuous payments . . . 77
2.7 Problems . . . 79
2.8 Questions from actuarial exams . . . 85
2.9 Solutions to problems . . . 89
2.10 Solutions to questions from actuarial exams . . . 99

3 Annuities **103**
3.1 Geometric and arithmetic series . . . 103
3.2 Level payments annuities . . . 106
3.3 Level payments perpetuities . . . 123
3.4 Non level payments annuities . . . 126
3.5 Continuous annuities . . . 141
3.6 Remarks . . . 145
3.7 Problems . . . 146

3.8 Questions from actuarial exams 155
3.9 Solutions to problems 160
3.10 Solutions to questions from actuarial exams 175

4 Amortization, sinking funds and reinvestment 181
4.1 Amortization schedules 181
4.2 Sinking funds 196
4.3 Reinvestment 199
4.4 Problems 201
4.5 Questions from actuarial exams 207
4.6 Solutions to the problems 211
4.7 Solutions to the questions from actuarial exams 221

5 Bonds and other securities 227
5.1 Securities 227
5.2 Price of a bond 227
5.3 Book value and amortization schedules 231
5.4 Book value between coupon payments dates 238
5.5 Callable bonds 241
5.6 More securities 243
5.7 Problems 247
5.8 Questions from actuarial exams 253
5.9 Solutions to problems 256
5.10 Solutions to questions from actuarial exams 267

6 Variable interest rates and portfolio insurance 271
6.1 Inflation 271
6.2 Arbitrage 272
6.3 Term structure of the interest rates 272
6.4 Duration, convexity 281
6.5 Asset–liability management 304
6.6 Problems 310
6.7 Questions from actuarial exams 318
6.8 Solutions to problems 320
6.9 Solutions to questions from actuarial exams 337

7 Derivatives **339**

7.1 Derivatives . . . 339

7.1.1 Risk sharing . . . 339

7.1.2 Derivatives. . . . 339

7.2 Forwards . . . 345

7.2.1 Forwards . . . 345

7.2.2 Alternative ways to buy an asset. . . . 348

7.2.3 Pricing a forward contract. . . . 349

7.2.4 Pricing a prepaid forward contract. . . . 349

7.2.5 Pricing a forward contract. . . . 352

7.2.6 Hedging a forward contract. . . . 357

7.2.7 Pricing currency forwards. . . . 359

7.3 Futures . . . 359

7.4 Call options . . . 362

7.5 Put options . . . 379

7.6 Put–call parity . . . 391

7.6.1 Put–call parity formula. . . . 391

7.6.2 Synthetic forward. . . . 395

7.6.3 Floor. . . . 396

7.6.4 Cap. . . . 401

7.7 Equity linked certificates of deposit . . . 404

7.8 Spreads . . . 407

7.8.1 Speculating on increase/decrease of an asset price. . . . 407

7.8.2 Speculating on volatility. . . . 418

7.8.3 Box spreads. . . . 424

7.9 Risk Management . . . 425

7.9.1 A producer perspective on risk management. . . . 430

7.9.2 The buyer's perspective on risk management. . . . 436

7.10 Swaps . . . 439

7.10.1 LIBOR. . . . 439

7.10.2 Structure of interest rates. . . . 440

7.10.3 Forward rate agreement. . . . 442

7.10.4 Interest rate swaps. . . . 445

7.10.5 Commodity swaps. . . . 451

7.11 Problems . . . 456

7.12 Questions from actuarial exams . . . 473

7.13 Solutions to problems 477
7.14 Solutions to questions from actuarial exams 497

8 Practice Exams **501**
8.1 Practice Exam 1 501
8.2 Practice Exam 2 506
8.3 Practice Exam 3 510
8.4 Practice Exam 4 514
8.5 Exam FM, May 2005. 518
8.6 Exam FM, November 2005. 522
8.7 Answer key 527
8.8 Solutions to Practice Exam Number 1 528
8.9 Solutions to Practice Exam Number 2 532
8.10 Solutions to Practice Exam Number 3 536
8.11 Solutions to Practice Exam Number 4. 541
8.12 Solutions to Exam FM, May 2005 546
8.13 Solutions to Exam FM, November 2005 549

9 Appendix **553**
9.1 How to use the calculator TI–BA–II–Plus 553
9.2 How to set–up the calculator TI–BA–II–Plus during the actuarial exam 556
9.3 Cheat sheet 557
Subject Index 568

About the Author **573**

CHAPTER 7
Derivatives

7.1 Derivatives

7.1.1 Risk sharing Real life is full of potential risks. A risk is a contingent financial loss. For example, the burning of house, an accident, a death, etc, are (contingent) uncertain events which have financial consequences. These financial losses can be catastrophic for a particular individual (or family). But, societies have created ways to deal with financial risks. A few centuries ago in some communities, if a community member's house burned down, the community would pitch together to rebuild the house. Although, a particular individual cannot sustain a particular risk, a community can pool together to overcome risks. In modern societies, the main way to transfer the individual uncertainty into a pool of individuals is done through the insurance industry. There is a business opportunity in managing the risks which individuals can face. The motto of the Society of Actuaries is **Risk is Opportunity**. Social Security also offers a safety net against risk. These methods of facing risks have in common that they use **risk sharing**. Everybody (the lucky and unlucky) contribute to cover the cost of a catastrophe. Insurance companies combine the insurance payments into a pool of money and use this money to reimburse the insurees who submit a claim.

In the business world changes in commodity prices, currency exchange rates and interest rates are potential risks. Their prices can either increase dramatically or collapse. A farmer faces the possible fall of the price of his/her crop. Surging oil prices can wipe out airlines' profits. Manufacturing companies face high rising prices of commodities. An importer of goods from Canada faces financial insecurity due to possible changes in exchange rates. The loan payments taken by a company can vary according with the flexible interest rate taken on the loan. These changes in prices could hurt the viability of a business. The change of price of assets over time is risk called the **price volatility risk**. A measure of the price volatility risk is the standard deviation of the asset's price over time.

Many of the risks faced by business are **diversifiable**. A risk is diversifiable if it is unrelated to another risk. Markets permit diversifiable risks to be widely shared. A financial institution can offer the service of protecting parties with combinable risks. Again there is a business opportunity in managing these combinable risks (**Risk is Opportunity**). Risk is **nondiversifiable** when it does vanish when spread across many investors. A way to do risk sharing for companies is to do contracts to avoid risks.

7.1.2 Derivatives.

Definition 7.1. *A* **derivative** *is a contract which specifies the right or obligation to receive or deliver certain asset for a certain price. The value of a derivative contract depends on the value of another asset.*

Example 7.1. *Suppose that a farmer grows wheat and a baker makes bread using wheat and other ingredients. If the price of the wheat decreases, the farmer loses money. If the price of the wheat increases, the baker loses money. Both the farmer and the baker faces price volatility risk. In order to avoid possible financial losses which may jeopardy their businesss profitability, the farmer and the baker can agree to sell/buy wheat one year from now at a certain price. The contract they enter is a derivative. It is a win–win contract for both of them. The two risks are diversifiable. Usually, the contract is not made directly between them. A* **market–maker** *or* **scalper** *makes a contract with the farmer and another with the baker. The farmer and the baker enter into this contract to do* **hedging**. *Hedging means to minimize risk.*

Example 7.2. *An airline company faces rising oil prices, which mean increasing costs and the risk of reduced profits, even losses. If oil prices rise above $80 per barrel the airline company will lose money. An oil company plans to build an oil platform. For the project to be financially viable it must be able to sell oil in one year at least for $40 per barrel. Both companies face risk when oil prices change in opposite directions. They can make a contract so that the airline company buys one million of barrels from the oil company one year from now at $60 per barrel. This contact will be a win–win situation for both parties. As before a financial institution will make separate contracts with each party and make a profit for its service.*

Derivatives are traded because the parties entering the contract believe that they will benefit from the contract. In the case of a scalper, a scalper runs derivatives to make a living. He can do that because there are enough people interested in opposite needs, i.e. the risk associated with these derivatives are diversifiable. A market–maker is a person or firm that stands ready to buy and sell a particular asset or financial instrument on a regular and continuous basis at a quoted price.

They are several possible reasons to enter into a derivative market:

- **Risk management.** Parties enter derivatives to avoid risks, like the farmer and the baker in Example 7.1, or a cattle producer and a hamburger chain.
- **Speculation.** Someone having more information on the possible evolution of the price commodity can bet that the price of a commodity will go in a certain way and make money. Suppose that Mr. Smith works for the National Weather Service. He believes that next year there will be a bad crop of wheat. He could speculate by buying the farmer's wheat at the price set–up for next year. A market–maker enters into derivatives to make money in this transaction.
- **Reduce transaction costs.** A business can enter into a derivative contract instead of buying in the regular market and pay different costs for delivery. A firm can use derivatives to borrow money at a rate lower than the current rate. This can happen specially if the firm is not very solvent.
- **Arbitrage.** When derivatives are miss priced, investors can make a profit from trading into derivatives.
- **Regulatory arbitrage.** Sometimes business enter into derivatives to get around regulatory limitations, accounting regulations and taxes.

in a year. To this price, insurance costs has to be added. The call holder is hedging against high prices of an asset. He has to pay for that. Hence, $\text{Call}(K,T) > E[\max(0, S_T - K)](1+i)^{-T}$.

Table 7.4 shows the premium of a call option for different strike prices. We have used $S_0 = 75$, $T = 1$, $\sigma = 0.20$, $\delta = 0$, $r = \ln(1.05)$.

Table 7.4:

K	65	70	75	80	85
$\text{Call}(K,T)$	14.31722	10.75552	7.78971	5.444947	3.680736

Example 7.49. *The current price of XYZ stock is \$75 per share. The annual effective rate of interest is 5%. The redemption time is one year from now. The price of stock one year from now is \$73.5. Calculate the profit per share at expiration for the holder of each one of the call options in Table 7.4.*

Solution: The profit is

$$\max(S_T - K, 0) - (1.05)\text{Call}(K,T) = \max(73.5 - K, 0) - (1.05)\text{Call}(K,T).$$

The corresponding profits are:

$$\begin{aligned}
&\text{if } K = 65, \text{ profit} = \max(73.5 - 65, 0) - (1.05)(14.31722) = -6.533081, \\
&\text{if } K = 70, \text{ profit} = \max(73.5 - 70, 0) - (1.05)(10.75552) = -7.793296, \\
&\text{if } K = 75, \text{ profit} = \max(73.5 - 75, 0) - (1.05)(7.78971) = -8.1791955, \\
&\text{if } K = 80, \text{ profit} = \max(73.5 - 80, 0) - (1.05)(5.444947) = -5.71719435, \\
&\text{if } K = 85, \text{ profit} = \max(73.5 - 85, 0) - (1.05)(3.680736) = -3.8647728.
\end{aligned}$$

If K is very small, the call option will almost certainly be executed. Hence, if K is very small, $\text{Call}(K,T) = F_{0,T}$, i.e. $\lim_{K\to 0+} \text{Call}(K,T) = F_{0,T}$. If K is very large, the call option will almost certainly not be executed. Hence, $\lim_{K\to\infty} \text{Call}(K,T) = 0$. As a function on K, $\text{Call}(K,T)$ is a decreasing function with $\lim_{K\to 0+} \text{Call}(K,T) = F_{0,T}$ and $\lim_{K\to\infty} \text{Call}(K,T) = 0$. Figure 7.9 shows the graph of $\text{Call}(K,T)$ as a function of T.

Example 7.50. *Using the Black–Scholes formula with $T = 1$, $S_0 = 100$, $T = 1$, $\sigma = 0.25$, $r = \ln(1.06)$ and $\delta = 0.0$, the following table of call option premiums was obtained:*

$\text{Call}(K,T)$	76.4150	52.8366	30.0399	12.7562	4.1341	0.8417	0.2672	0.0605
K	25	50	75	100	125	150	175	200

Figure 7.9 shows the graph of this function.

When we consider $\text{Call}(K,T)$ as function of T. If T is small enough, then the option will be exercised if $S_0 > K$ with a profit of $S_0 - K$. Hence, if $S_0 > K$, $\lim_{T\to 0+} \text{Call}(K,T) = S_0 - K$. Notice

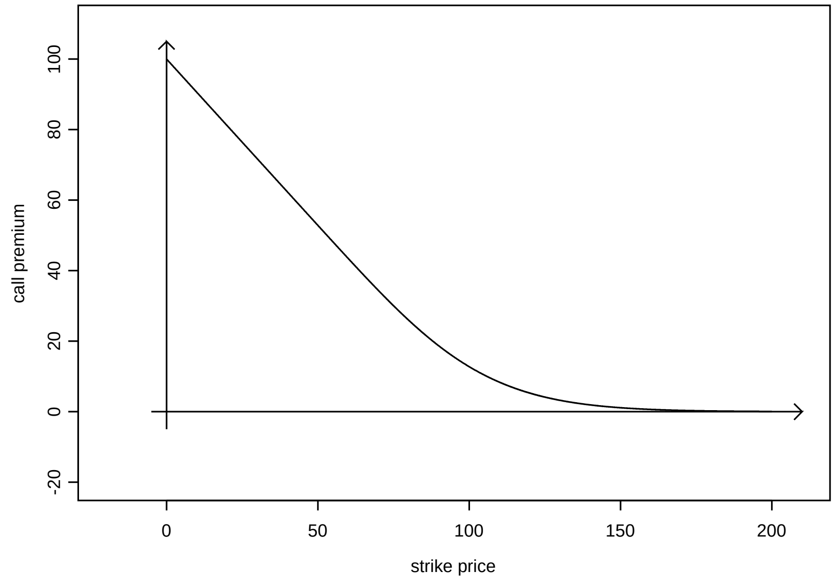


Figure 7.9: Example 7.50. Graph of Call(K,T) as a function of K.

that by buying the call option for Call(K,T), we buy an asset worth S_0 for K. If T is small enough and $S_0 < K$, the option is not exercised and his value is zero, i.e. $\lim_{T\to 0+} \text{Call}(K,T) = 0$.

An option is **in–the–money option** if it would have a positive payoff if exercised immediately. An option is **out–the–money** option if it would have a negative payoff if exercised immediately. An option is **at–the–money** option if it would have a zero payoff if exercised immediately. The previous definition hold for both call and put options. Put options will considered shortly. For a purchased call option, we have

- The purchased call option is in–the–money, if $S_0 > K$.
- The purchased call option is out–the–money, if $S_0 < K$.
- The purchased call option is at–the–money, if $S_0 = K$.

7.5 Put options

Definition 7.7. *A* **put option** *is a financial contract which gives the (***holder***)* **owner** *the right, but not the obligation, to sell a specified amount of a given security at a specified price at a specified time.*

The put option owner exercises the option by selling the asset at the specified call price to the put writer. A put option is executed only if the put owner decides to do so. A put option owner executes a put option only when it benefits him, i.e. when the specified call price is bigger than the current (market value) spot price. Since the owner of a put option can make money if the option is exercised, put options are sold. The **owner of the put option** must pay to its counterpart for holding a put option. The price of a put option is called its **premium**.

$\mathrm{Put}(K,T)$ denotes the premium of a K–strike option for one unit with expiration time T. Notice that $\mathrm{Put}(K,T) > 0$. The premium of a put option for N units is $N\mathrm{Put}(K,T)$.

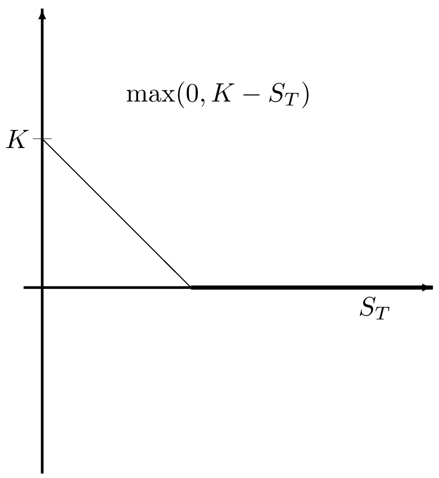
The (owner) buyer of a put option is called the **put option holder**. The holder of a put is said to have a **long put position**. The seller of a put option is called the **put option writer**. The writer of a put is said to have a **short put position**.

The dates when a put option can be exercised are determined by the type of the option. There are European, American and Bermudan types of put options. The exercise time for these types of put options are defined similarly to the exercise time of a call option. Unless said otherwise, we assume that all put options are European.

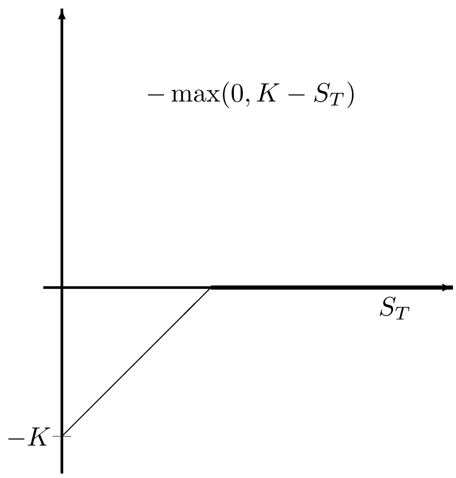
Example 7.51. *John buys a six–month put option for 150 shares with a strike price of \$45 per share.*

(i) If the price per share six months from now is \$40, John sells 150 shares to the put option writer for $(150)(45) = 6750$*. Since the market value of these 150 shares is* $(150)(40) = 6000$*. John makes (before expenses)* $6750 - 6000 = 750$ *on this contract.*

(ii) If the price per share six months from now is \$50, John does not exercise the put option.



Payoff for the put option holder



Payoff for the put option writer

Figure 7.10: Payoff of a put option

The put option buyer's payoff per share is

$$\max(K - S_T, 0) = \begin{cases} K - S_T & \text{if } S_T < K, \\ 0 & \text{if } S_T \geq K, \end{cases}$$

where K is the strike price and S_T is the spot price at redemption. The put option writer's payoff per share is

$$-\max(K - S_T, 0) = \begin{cases} -(K - S_T) & \text{if } S_T < K, \\ 0 & \text{if } S_T \geq K, \end{cases}$$

Figure 7.10 shows the graph of the payoff of a put option. A put option is a zero–sum game.

	payoff	minimum payoff	maximum payoff
put option holder	$\max(K - S_T, 0)$	0	K
put option writer	$-\max(K - S_T, 0)$	$-K$	0

Example 7.52. *Daniel buys a 55–strike put option on XYZ stock with a nominal amount of 5000 shares. The expiration date is 6 months from now. The nominal amount of the put option is 5000 shares of XYZ stock.*

(i) Calculate Daniel's payoff for the following spot prices per share at expiration: 40, 45, 55, 60, 60.

(ii) Calculate Daniel's minimum and maximum payoffs.

Solution: (i) Daniel's payoff is $5000 \max(55 - S_T, 0)$. The corresponding payoffs are:

$$\begin{aligned}
&\text{if } S_T = 40, \text{ payoff} = (5000) \max(55 - 40, 0) = 75000, \\
&\text{if } S_T = 45, \text{ payoff} = (5000) \max(55 - 45, 0) = 50000, \\
&\text{if } S_T = 50, \text{ payoff} = (5000) \max(55 - 50, 0) = 25000, \\
&\text{if } S_T = 55, \text{ payoff} = (5000) \max(55 - 55, 0) = 0, \\
&\text{if } S_T = 60, \text{ payoff} = (5000) \max(55 - 60, 0) = 0.
\end{aligned}$$

(ii) Daniel's minimum payoff is zero. Daniel's maximum payoff is $(5000)(55) = 275000$.

Example 7.53. *Isabella sells a 55–strike put option on XYZ stock. The expiration date is 18 months from now. The nominal amount of the put option is 10000 shares of XYZ stock.*

(i) Calculate Isabella's payoff for the following spot prices per share at expiration: 40, 45, 55, 60, 60.

(ii) Calculate Isabella's minimum and maximum payoffs.

Solution: (i) Isabella's payoff is $-10000 \max(55 - S_T, 0)$. The corresponding payoffs are:

$$\begin{aligned}
&\text{if } S_T = 40, \text{ payoff} = -(10000) \max(55 - 40, 0) = -150000, \\
&\text{if } S_T = 45, \text{ payoff} = -(10000) \max(55 - 45, 0) = -100000, \\
&\text{if } S_T = 50, \text{ payoff} = -(10000) \max(55 - 50, 0) = -50000, \\
&\text{if } S_T = 55, \text{ payoff} = -(10000) \max(55 - 55, 0) = 0, \\
&\text{if } S_T = 60, \text{ payoff} = -(10000) \max(55 - 60, 0) = 0.
\end{aligned}$$

(ii) Isabella's minimum payoff is $-(10000)(55) = -550000$. Isabella's maximum payoff is zero.

Let Put(K,T) be the premium per unit paid of a put option with strike price K and expiration time T years. Notice that Put$(K,T) > 0$. Let i be the risk free annual effective rate of interest. The put option holder's profit is

$$\max(K - S_T, 0) - \mathrm{Put}(K,T)(1+i)^T = \begin{cases} K - S_T - \mathrm{Put}(K,T)(1+i)^T & \text{if } S_T < K, \\ -\mathrm{Put}(K,T)(1+i)^T & \text{if } S_T \geq K. \end{cases}$$

Put$(K,T)(1+i)^T$ is the future value at time T of the purchase price. The put option writer's profit is

$$-\max(K - S_T, 0) + \mathrm{Put}(K,T)(1+i)^T = \begin{cases} -K + S_T + \mathrm{Put}(K,T)(1+i)^T & \text{if } S_T < K, \\ \mathrm{Put}(K,T)(1+i)^T & \text{if } S_T \geq K. \end{cases}$$

Figure 7.11 shows a graph of the put profit.

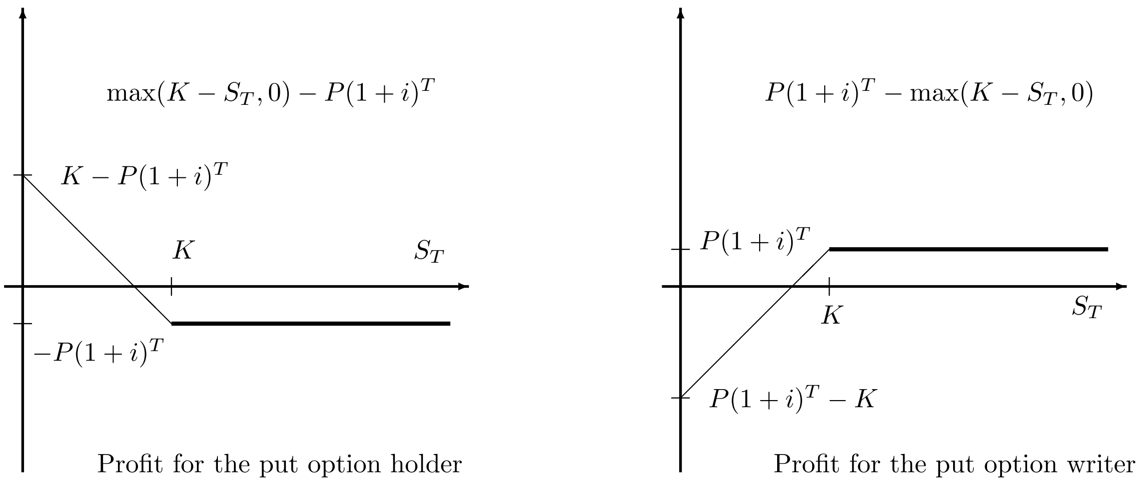


Figure 7.11: Profit of a put option

	profit	minimum profit	maximum profit
put holder	$\max(K - S_T, 0) - \mathrm{Put}(K,T)e^{rT}$	$-\mathrm{Put}(K,T)e^{rT}$	$K - \mathrm{Put}(K,T)e^{rT}$
put writer	$-\max(K - S_T, 0) + \mathrm{Put}(K,T)e^{rT}$	$-K + \mathrm{Put}(K,T)e^{rT}$	$\mathrm{Put}(K,T)e^{rT}$

Notice that the put option holder's profit as a function of S_T is nonincreasing. The put option holder benefits from a decrease on the spot price. The minimum of the put option holder's profit is

$$-\mathrm{Put}(K,T)(1+i)^T.$$

The maximum of the put option's holder profit is $K - \mathrm{Put}(K,T)(1+i)^T$. If there exists no arbitrage

$$-\mathrm{Put}(K,T)(1+i)^T < 0 < K - \mathrm{Put}(K,T)(1+i)^T,$$

which is equivalent to

$$0 < \text{Put}(K, T) < K(1+i)^{-T}.$$

Theorem 7.1. *If there exists no arbitrage, then*

$$\max((1+i)^{-T}K - S_0, 0) < \text{Put}(K, T) < K(1+i)^{-T}.$$

Proof. Consider the portfolio consisting of buying an asset and a put option on this asset, both for the same notional amount. The profit at expiration is

$$\begin{aligned} &S_T + \max(K - S_T, 0) - S_0(1+i)^T - \text{Put}(K, T)(1+i)^T \\ =&\max(K, S_T) - (\text{Put}(K, T) + S_0)(1+i)^T. \end{aligned}$$

The maximum profit is ∞. The minimum profit is $K - (\text{Put}(K, T) + S_0)(1+i)^T$. If there exists no arbitrage $K - (\text{Put}(K, T) + S_0)(1+i)^T < 0$, which is equivalent to $(1+i)^{-T}K - S_0 < \text{Put}(K, T)$. From this bound and the bounds before the theorem, the claim follows. □

Example 7.54. *The current price of XYZ stock is 160 per share. The annual effective interest rate is* 7%. *The price of a one–year European* 200*–strike put option for XYZ stock is* \$20 *per share. Find an arbitrage strategy and the minimum profit per share.*

Solution: We have that

$$\text{Put}(K, T) + S_0 - (1+i)^{-T}K = 20 + 160 - (200)(1.07)^{-1} = -6.91588785 < 0.$$

The put premium is too low. Consider the portfolio consisting of buying the put and the stock, both for the same nominal amount. The profit per share is

$$\max(200 - S_T, 0) - 20(1.07) + S_T - 160(1.07) = \max(200, S_T) - 192.6.$$

The minimum profit per share is $200 - 192.6 = 7.4$.

Example 7.55. *The current price of XYZ stock is* 160 *per share. The annual effective interest rate is* 7%. *The price of a one–year European* 200*–strike put option for XYZ stock is* \$190 *per share. Find an arbitrage strategy and the minimum profit per share.*

Solution: We have that

$$(1+i)^{-T}K - \text{Put}(K, T) = (200)(1.07)^{-1} - 190 = -3.08411215 < 0.$$

The put is overpriced. Consider the portfolio consisting of selling the put. The profit per share is $190(1.07) - \max(200 - S_T, 0)$. The minimum profit per share is $190(1.07) - 200 = 3.3$.

The profit of a put option holder is positive if

$$\max(K - S_T, 0) - \text{Put}(K, T)(1+i)^T > 0,$$

which is equivalent to

$$K - \text{Put}(K, T)(1+i)^T > S_T.$$

If $K - \text{Put}(K, T)(1+i)^T < S_T$, the put option holder's profit is negative.

Example 7.56. *Ashley buys a 85–strike put option for 4.3185816 per share. The nominal amount of the put option is 2500 shares of XYZ stock. The expiration date of this option is one year. The annual interest rate compounded continuously is* 5%.

(i) Calculate Ashley's profit function.

(ii) Calculate Ashley's profit for the following spot prices at expiration: 75, 80, 85, 90, 95.

(iii) Calculate Ashley's minimum and maximum profits.

(iv) Calculate the spot prices at which Ashley's profit is positive.

(v) Calculate the spot price at expiration at which Ashley breaks even.

(vi) Calculate Ashley's annual yield in her investment for the spot prices at expiration in (ii).

Solution:(i) Ashley's profit is

$$(2500)(\max(85 - S_T, 0) - 4.3185816e^{0.05}) = (2500)\max(85 - S_T, 0) - 11350.$$

(ii) The profits corresponding to the considered spot prices are:

$$\begin{aligned}
&\text{if } S_T = 75,\ \text{profit} = (2500)\max(85 - 75, 0) - 11350 = 13650,\\
&\text{if } S_T = 80,\ \text{profit} = (2500)\max(85 - 80, 0) - 11350 = 1150,\\
&\text{if } S_T = 85,\ \text{profit} = (2500)\max(85 - 85, 0) - 11350 = -11350,\\
&\text{if } S_T = 90,\ \text{profit} = (2500)\max(85 - 90, 0) - 11350 = -11350,\\
&\text{if } S_T = 95,\ \text{profit} = (2500)\max(85 - 95, 0) - 11350 = -11350.
\end{aligned}$$

(iii) Ashley's minimum profit is -11350. Ashley's maximum profit is $(2500)(85) - 11350 = 201150$.

(iv) Ashley's profit is positive if $(2500)\max(85 - S_T, 0) - 11350 > 0$, which is equivalent to $S_T < 85 - \frac{11350}{2500} = 80.46$.

(v) Ashley breaks even if $(2500)\max(85 - S_T, 0) - 11350 = 0$, i.e. if $S_T = 85 - \frac{11350}{2500} = 80.46$.

(vi) Ashley invests $(2500)(4.3185816) = 10796.454$. Ashley's return in her investment is $(2500)\max(85-S_T, 0)$. Let j be Ashley's annual yield. Then, $10796.454(1+j)^{0.5} = (2500)\max(85-S_T, 0)$. Hence, $j = \left(\frac{(2500)\max(85-S_T,0)}{10796.454}\right)^2 - 1$ and

$$\begin{aligned}
&\text{if } S_T = 75, j = \left(\frac{(2500)\max(85 - 75, 0)}{10796.454}\right)^2 - 1 = 436.1888022\%,\\
&\text{if } S_T = 80, j = \left(\frac{(2500)\max(85 - 80, 0)}{10796.454}\right)^2 - 1 = 34.04720055\%,\\
&\text{if } S_T = 85, j = \left(\frac{(2500)\max(85 - 85, 0)}{10796.454}\right)^2 - 1 = -100\%,\\
&\text{if } S_T = 90, j = \left(\frac{(2500)\max(85 - 90, 0)}{10796.454}\right)^2 - 1 = -100\%,\\
&\text{if } S_T = 95, j = \left(\frac{(2500)\max(85 - 95, 0)}{10796.454}\right)^2 - 1 = -100\%.
\end{aligned}$$