



*Three formulas for  ${}_{t|u}q_x$  in terms of  
non-deferred  $p$ 's and  $q$ 's*



$${}_{t|u}q_x = {}_t p_x u q_{x+t}$$

$${}_{t|u}q_x = {}_t p_x - {}_{t+u} p_x$$

$${}_{t|u}q_x = {}_{t+u} q_x - {}_t q_x$$



*$k\rho_x$  in terms of  $l$ 's*

## Survival Distributions



$${}_k p_x = \frac{l_{x+k}}{l_x}$$



*$kq_x$  in terms of  $l$ 's and  $d$ 's*



$${}_kq_x = \frac{{}_k d_x}{l_x} = \frac{l_x - l_{x+k}}{l_x}$$



*$t|uq_x$  in terms of  $l$ 's and  $d$ 's*



$${}_{t|u}q_x = \frac{u d_x}{l_x} = \frac{l_{x+t} - l_{x+t+u}}{l_x}$$



*Definition of  ${}_tq_x$  in terms of probabilities of  $X$ ,  
the random variable for age at death.*



$${}_tq_x = \Pr(x < X \leq x + t \mid X > x)$$



${}_s q_x$  under UDD,  $s \leq 1$



$${}_s q_x = s q_x$$



${}_s p_x$  under UDD,  $s \leq 1$

## Survival Distributions



$${}_s p_x = 1 - {}_s q_x$$



$l_{x+s}$  *under UDD*,  $s \leq 1$



$$l_{x+s} = l_x - s d_x$$



${}_{1-s}q_{x+s}$  *under UDD*,  $s \leq 1$



$${}_{1-s}q_{x+s} = \frac{(1-s)q_x}{1-sq_x}$$



${}_s q_{x+t}$  under UDD,  $s + t \leq 1$



$${}_s q_{x+t} = \frac{{}_s q_x}{1 - {}_t q_x}$$



$\mu_{x+s}$  *under UDD*,  $s \leq 1$



$$\mu_{x+s} = \frac{q_x}{1 - sq_x}$$



${}_s p_x \mu_{x+s}$  *under UDD*,  $s \leq 1$



$${}_s p_x \mu_{x+s} = q_x$$



*$q_x^{(1)}$  in terms of  $q_x'^{(1)}$  and  $q_x'^{(2)}$  when there are only two decrements and both are uniformly distributed in the associated single-decrement tables*

## Multiple Decrements



$$q_x^{(1)} = q_x^{(1)} \left( 1 - \frac{q_x^{(2)}}{2} \right)$$



*$q_x^{(1)}$  in terms of  $q_x'^{(1)}$ ,  $q_x'^{(2)}$ , and  $q_x'^{(3)}$  when there are only three decrements and they are uniformly distributed in the associated single-decrement tables*

## Multiple Decrements



$$q_x^{(1)} = q_x^{(1)} \left( 1 - \frac{q_x^{(2)} + q_x^{(3)}}{2} + \frac{q_x^{(2)} q_x^{(3)}}{3} \right)$$



*${}_{t|s}q_x^{(1)}$ ,  $t + s \leq 1$ , in terms of  $q_x'^{(1)}$ ,  $q_x'^{(2)}$ , and  $q_x'^{(3)}$  when there are only three decrements and they are uniformly distributed in the associated single-decrement tables*

## Multiple Decrements



$${}_{t|s}q_x^{(1)} = q_x^{(1)} \left( s - \frac{(t+s)^2 - t^2}{2} (q_x^{(2)} + q_x^{(3)}) \right. \\ \left. + \frac{(t+s)^3 - t^3}{3} (q_x^{(2)} q_x^{(3)}) \right)$$



*If there are two decrements, with (1) discrete at time  $0 \leq a \leq 1$ , and (2) uniformly distributed over  $(0, 1]$  in the associated single decrement table, what are  $q_x^{(1)}$  and  $q_x^{(2)}$  in terms of  $q_x'^{(1)}$  and  $q_x'^{(2)}$ ?*

## Multiple Decrements



$$q_x^{(1)} = q_x'^{(1)} (1 - a q_x'^{(2)})$$
$$q_x^{(2)} = q_x'^{(2)} (a + (1 - a)(1 - q_x'^{(1)}))$$



*If there are three decrements, with (1) discrete at time  $0 \leq a \leq 1$ , and (2),(3) uniformly distributed in the associated single-decrement tables, what are  $q_x^{(1)}$ ,  $q_x^{(2)}$ , and  $q_x^{(3)}$  in terms of  $q_x'^{(1)}$ ,  $q_x'^{(2)}$ , and  $q_x'^{(3)}$ ?*

## Multiple Decrements



$$q_x^{(1)} = q_x'^{(1)}(1 - aq_x'^{(2)})(1 - aq_x'^{(3)})$$

$$q_x^{(2)} = q_x'^{(2)}\left(a(1 - 0.5aq_x'^{(3)}) + (1 - a)(1 - q_x'^{(1)})(1 - 0.5(1 + a)q_x'^{(3)})\right)$$

$$q_x^{(3)} = q_x'^{(3)}\left(a(1 - 0.5aq_x'^{(2)}) + (1 - a)(1 - q_x'^{(1)})(1 - 0.5(1 + a)q_x'^{(2)})\right)$$