SOA Exam C
CAS Exam 4

2009 Edition

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Product Preview

Flashcards

Continuous Random Variable

A continuous random variable X assumes values from an Interval of numbers. X has a probability density function (pdf) denoted $f(x)$ or $f_X(x)$, which is a continuous function (except possibly at a finite or countably infinite number of points). Probabilities are found by integrating the density function over an interval.

$$P\left[X \in (a,b)\right] = P(a < X < b) = \int_a^b f(x)dx.$$

$f(x)$ must satisfy

(i) $f(x) \geq 0$, and

(ii $\int_{-\infty}^{\infty} f(x)\, dx = 1.$

Often, the region of non-zero density is a finite interval, and $f(x) = 0$ outside that interval.

Conditional Distributions

If X and Y have joint pf or pdf $f(x, y)$, then the conditional distribution of Y given $X = x$ has pf/pdf $f_{Y|X}(y \mid X = x) = \dfrac{f(x, y)}{f_X(x)}$.

The same definition applies to the conditional distribution of X given $Y = y$. Conditional distributions must satisfy the same requirements as any distribution. For instance, if X and Y are discrete then $\sum_y f_{Y|X}(y \mid X = x) = 1$, and if X and Y are continuous then

$$\int_{-\infty}^{\infty} f_{Y|X}(y \mid X = x) = 1.$$

If we are given the marginal density of X, $f_X(x)$ and the conditional density of Y given $X = x$, then the joint density of X and Y is

$$f(x, y) = f_{Y|X}(y \mid X = x) \cdot f_X(x).$$

Kaplan-Meier Product-Limit Estimate $S_n(t)$, of the Survival Probability $S(t) = P[T > t]$

$$S_n(t) = 1 \qquad \text{for} \qquad 0 \le t < y_1,$$

$$S_n(t) = 1 - \frac{s_1}{r_1} \qquad \text{for} \qquad y_1 \le t < y_2,$$

$$S_n(t) = \left(1 - \frac{s_1}{r_1}\right)\left(1 - \frac{s_2}{r_2}\right) \qquad \text{for} \qquad y_2 \le t < y_3,$$

$$S_n(t) = \left(1 - \frac{s_1}{r_1}\right)\left(1 - \frac{s_2}{r_2}\right)\left(1 - \frac{s_3}{r_3}\right) \qquad \text{for} \qquad y_3 \le t < y_4,$$

$$\vdots \qquad \vdots \qquad\qquad\qquad \vdots$$

$$S_n(t) = \prod_{i=1}^{j-1}\left(1 - \frac{s_i}{r_i}\right) \qquad \text{for} \qquad y_{j-1} \le t < y_j$$

Maximum Likelihood Estimate of θ
For the Inverse Weibull Distribution with τ Given
Based on Complete Data

The pdf of the Inverse Weibull is $f(x) = \dfrac{\tau\theta^{\tau} e^{-(\theta/x)^{\tau}}}{x^{\tau+1}}, \quad x > 0.$

If τ is given and $x_1, x_2, ..., x_n$ is a random sample of observations, then the mle of θ is $\hat{\theta} = \left[\dfrac{1}{n}\sum_{i=1}^{n} x_i^{-\tau}\right]^{-1/\tau}$.

Another way to think of this is that given the sample, if $z_i = x_i^{-\tau}$ for each *i*, then the values $z_1, z_2, ..., z_n$ are like a random sample from the exponential distribution with mean $\theta^{-\tau}$, so the mle of $\theta^{-\tau}$ is $\dfrac{1}{n}\sum z_i$ and then the mle of θ is $\hat{\theta} = \left[\dfrac{1}{n}\sum_{i=1}^{n} x_i^{-\tau}\right]^{-1/\tau}$.

Bayesian Credibility for the Gamma Prior Distribution and Inverse Exponential Model Distribution

Prior parameter λ: Gamma $\alpha, \theta,\ \pi(\lambda) = \dfrac{\lambda^{\alpha-1} e^{-\lambda/\theta}}{\theta^{\alpha}\Gamma(\alpha)}$

Model dist. $X \mid \lambda$: Inverse Exponential $\lambda,\ f(x \mid \lambda) = \dfrac{\lambda}{x^2} e^{-\lambda/x}$

With observations $x_1, x_2, ..., x_n,$ the posterior distribution of λ is gamma with $\alpha' = \alpha + n$ and $\dfrac{1}{\theta'} = \dfrac{1}{\theta} + \sum \dfrac{1}{x_i}$.

The predictive distribution is inverse Pareto with the same α' and θ' as the posterior distribution. The mean of the inverse Pareto is infinite, so the Bayesian premium is infinite.

Conditional Tail Expectation for a Continuous Loss

If L is a continuous loss random variable, then the Conditional Tail Expectation at confidence level α is

$$CTE_{\alpha} = E[L \mid L > Q_{\alpha}] = Q_{\alpha} + \frac{E[L] - E[L \wedge Q_{\alpha}]}{1-\alpha}.$$