TABLE OF CONTENTS - VOLUME 1

INTRODUCTORY COMMENTS

MODELING

MODELING

MODEL ESTIMATION

MODEL ESTIMATION

INTRODUCTORY COMMENTS

This study guide is designed to help in the preparation for the Society of Actuaries Exam C and Casualty Actuarial Society Exam 4. The exam covers the topics of modeling, model estimation, construction and selection, credibility, simulation and risk measures.

The study manual is divided into two volumes. The first volume consists of a summary of notes, illustrative examples and problem sets with detailed solutions on the modeling and model estimation topics. The second volume consists of notes examples and problem sets on the credibility, simulation and risk measures topics, as well as 14 practice exams.

The practice exams all have 35 questions. The level of difficulty of the practice exams has been designed to be similar to that of the past 4-hour exams. Some of the questions in the problem sets are taken from the relevant topics on SOA/CAS exams that have been released prior to 2009 but the practice exam questions are not from old SOA exams.

I have attempted to be thorough in the coverage of the topics upon which the exam is based. I have been, perhaps, more thorough than necessary on a couple of topics, such as maximum likelihood estimation, Bayesian credibility and applying simulation to hypothesis testing.

Because of the time constraint on the exam, a crucial aspect of exam taking is the ability to work quickly. I believe that working through many problems and examples is a good way to build up the speed at which you work. It can also be worthwhile to work through problems that have been done before, as this helps to reinforce familiarity, understanding and confidence. Working many problems will also help in being able to more quickly identify topic and question types. I have attempted, wherever possible, to emphasize shortcuts and efficient and systematic ways of setting up solutions. There are also occasional comments on interpretation of the language used in some exam questions. While the focus of the study guide is on exam preparation, from time to time there will be comments on underlying theory in places that I feel those comments may provide useful insight into a topic.

The notes and examples are divided into sections anywhere from 4 to 14 pages, with suggested time frames for covering the material. There are over 330 examples in the notes and over 800 exercises in the problem sets, all with detailed solutions. The 14 practice exams have 35 questions each, also with detailed solutions. Some of the examples and exercises are taken from previous SOA/CAS exams. Questions in the problem sets that have come from previous SOA/CAS exams are identified as such. Some of the problem set exercises are more in depth than actual exam questions, but the practice exam questions have been created in an attempt to replicate the level of depth and difficulty of actual exam questions. In total there are aver 1600 examples/problems/sample exam questions with detailed solutions. ACTEX gratefully acknowledges the SOA and CAS for allowing the use of their exam problems in this study guide.

I suggest that you work through the study guide by studying a section of notes and then attempting the exercises in the problem set that follows that section. My suggested order for covering topics is
(1) modeling (includes risk measures), (2) model estimation , (Volume 1) ,
(3) credibility theory , and (4) simulation , (Volume 2).

It has been my intention to make this study guide self-contained and comprehensive for all Exam C topics, but there are occasional references to the Loss Models reference book (3rd edition) listed in the SOA/CAS catalog. While the ability to derive formulas used on the exam is usually not the focus of an exam question, it is useful in enhancing the understanding of the material and may be helpful in memorizing formulas. There may be an occasional reference in the review notes to a derivation, but you are encouraged to review the official reference material for more detail on formula derivations. In order for the review notes in this study guide to be most effective, you should have some background at the junior or senior college level in probability and statistics. It will be assumed that you are reasonably familiar with differential and integral calculus. The prerequisite concepts to modeling and model estimation are reviewed in this study guide. The study guide begins with a detailed review of probability distribution concepts such as distribution function, hazard rate, expectation and variance.

Of the various calculators that are allowed for use on the exam, I am most familiar with the BA II PLUS. It has several easily accessible memories. The TI-30X IIS has the advantage of a multi-line display. Both have the functionality needed for the exam.

There is a set of tables that has been provided with the exam in past sittings. These tables consist of some detailed description of a number of probability distributions along with tables for the standard normal and chi-squared distributions. The tables can be downloaded from the SOA website www.soa.org .

If you have any questions, comments, criticisms or compliments regarding this study guide, please contact the publisher ACTEX, or you may contact me directly at the address below. I apologize in advance for any errors, typographical or otherwise, that you might find, and it would be greatly appreciated if you would bring them to my attention. ACTEX will be maintaining a website for errata that can be accessed from www.actexmadriver.com .

It is my sincere hope that you find this study guide helpful and useful in your preparation for the exam. I wish you the best of luck on the exam.

Samuel A. Broverman November, 2009

Department of Statistics www.sambroverman.com
University of Toronto E-mail: sam@utstat.toronto.edu or 2brove@rogers.com

MODELING SECTION 20 - STOP LOSS INSURANCE

The material in this section relates to Loss Models, Sections 9.3.
The suggested time for this section is less than 2 hours.

When a deductible is applied to aggregate losses (not individual losses), the insurance payment will be the aggregate loss in excess of the deductible. If aggregate losses are S for a period, and the deductible for the period is d, then the **stop-loss insurance payment** is

$$Max\{S-d,0\}=(S-d)_+=S-(S\wedge d)=\begin{cases}0 & \text{if } S\le d\\ S-d & \text{if } S>d\end{cases}. \tag{20.1}$$

This is algebraically identical to ordinary deductible covered earlier in Section 11 of this study guide.

The expected value of stop-loss insurance paid is the **net stop-loss premium**, which is equal to $E[(S-d)_+]$. This can be formulated various ways.

$E[(S-d)_+]=\int_d^\infty (y-d)\cdot f_S(y)\,dy$ if S is continuous, (20.2)

or $\sum_{k=d+1}^{\infty}(k-d)\cdot f_S(k)$ if S is discrete and integer valued. (20.3)

We also have the formulations $E[(S-d)_+]=E[S]-E[S\wedge d]=\int_d^\infty [1-F_S(x)]\,dx$. (20.4)
Equation 20.4 is valid for any non-negative distribution of S, continuous or discrete. For most exam questions, the formulation $E[(S-d)_+]=E[S]-E[S\wedge d]$ is usually quite efficient to use.

Insurance with a deductible was considered earlier in Section 11 of this study guide, and we are considering the same idea here, except that the random variable to which the deductible is applied is the aggregate loss S.

The following illustration will provide some insight into the mechanics of applying a deductible to a loss. Suppose that S has the following discrete distribution:

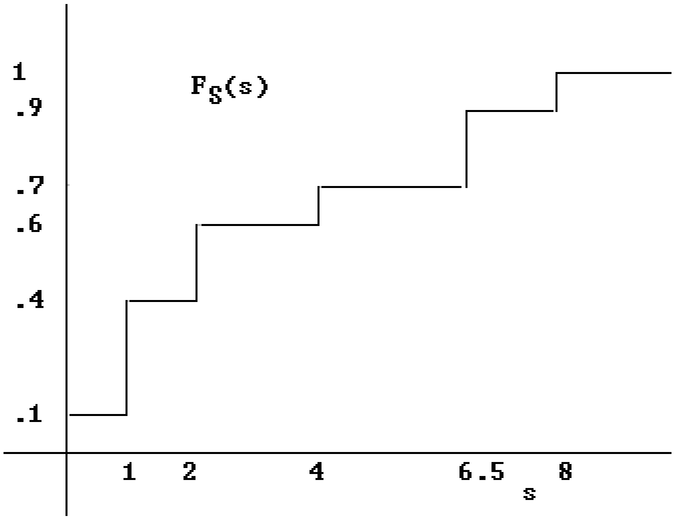
y :	0	1	2	4	6.5	8
$f_S(y)$:	.1	.3	.2	.1	.2	.1
$F_S(y)$:	.1	.4	.6	.7	.9	1.0
$1-F_S(y)$:	.9	.6	.4	.3	.1	0

The mean can be found from

$$E[S]=\sum_{\text{all } y} y\cdot f_S(y)=(0)(.1)+(1)(.3)+(2)(.2)+(4)(.1)+(6.5)(.2)+(8)(.1)=3.2 .$$

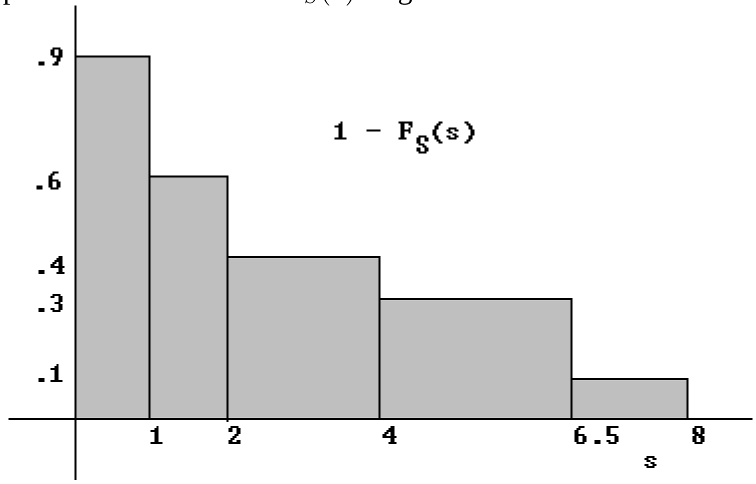
The mean is also equal to $E[S]=\int_0^\infty [1-F_S(s)]\,ds$. In the case that S has a discrete distribution, the distribution function and its complement are step-functions.

The graph of $F_S(s)$ is on the next page.



Note that $F_S(8) = 1$ and $F_S(y) = 1$ for $y \geq 8$.

The graph of the function $1 - F_S(s)$ is given below.



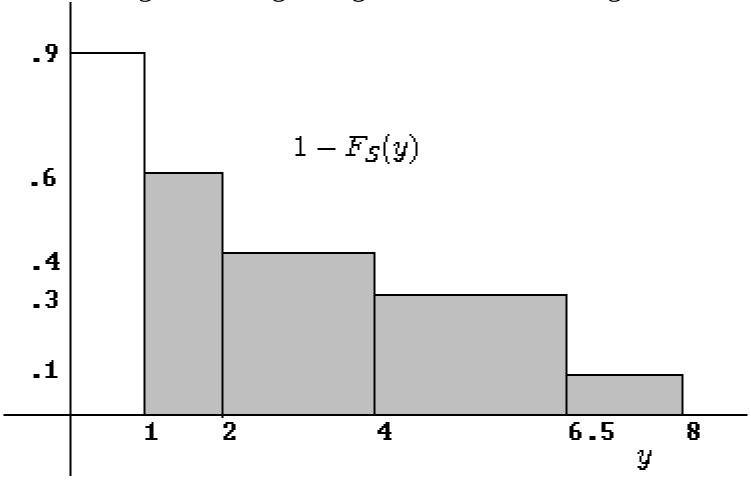
Note that $1 - F_S(8) = 0$.
Therefore, in this case $E[S] = \int_0^\infty [1 - F_S(s)]\, ds = \int_0^8 [1 - F_S(s)]\, ds$.
Since $1 - F_S(s)$ is a step function, $\int_0^8 [1 - F_S(s)]\, ds$ is the area under the curve, which becomes the area of a series of rectangles, the area of the shaded region above:
$\int_0^8 [1 - F_S(s)]\, ds = (1)(.9) + (1)(.6) + (2)(.4) + (2.5)(.3) + (1.5)(.1) = 3.2$.

Suppose that we wish to apply a deductible of $d = 1$ to create the stop loss insurance $(S-1)_+$ for this example. We can find $E[(S-1)_+]$ from $E[(S-1)_+] = E[S] - E[S \wedge 1]$, where

$$S \wedge 1 = \begin{cases} S & \text{if } S \leq 1 \\ 1 & \text{if } S > 1 \end{cases} = \begin{cases} 0 & \text{if } S = 0 \text{ , } f_S(0) = .1 \\ 1 & \text{if } S \geq 1 \text{ , } P(S > 1) = .9 \end{cases} .$$

Then, $E[S \wedge 1] = (0) \cdot f_S(0) + (1) \cdot P(S > 1) = .9$, and
$E[(S-1)_+] = E[S] - E[S \wedge 1] = 3.2 - .9 = 2.3$.

We can also use the relation $E[(S-d)_+]=\int_d^\infty[1-F_S(y)]\,dy$.
With a deductible of 1, this becomes $E[(S-1)_+]=\int_1^8[1-F_S(y)]\,dy$. The integral is the area of the following shaded region, again a series of rectangles.



$E[(S-1)_+]=\int_1^8[1-F_S(y)]\,dy=(1)(.6)+(2)(.4)+(2.5)(.3)+(1.5)(.1)=2.3$.

Notice that $E[(S-1)_+]=\int_1^8[1-F_S(y)]\,dy$
$=\int_0^8[1-F_S(y)]\,dy-\int_0^1[1-F_S(y)]\,dy=E[S]-[1-F_S(0)]$.
This is true because $1-F_S(y)=1-F_S(0)=.9$ is constant for $0\le y<1$.
From the graphical point of view, $E[(S-1)_+]$ is found by subtracting the area of the first rectangle on the left of the graph of $1-F_S(y)$

Now suppose that we wish to apply a deductible of $d=2$ to create the stop loss insurance $(S-2)_+$ for this example. Again, to find $E[(S-2)_+]$ we can use the relationship $E[(S-2)_+]=E[S]-E[S\wedge 2]$,

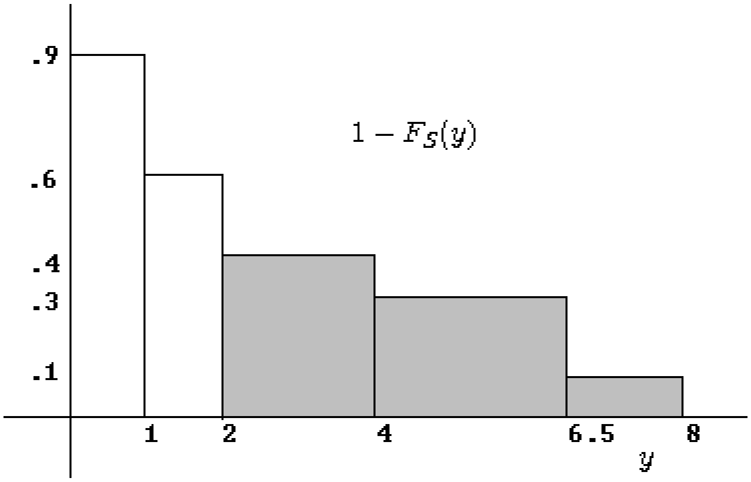
where $S\wedge 2=\begin{cases}0 & S=0\,,\,f_S(0)=.1\\ 1 & S=1\,,\,f_S(1)=.3\\ 2 & S\ge 2\,,\,P(S>2)=.6\end{cases}$.

Then $E[S\wedge 2]=(0)(.1)+(1)(.3)+(2)(.6)=1.5$, and $E[(S-2)_+]=3.2-1.5=1.7$.

We can also use the relation $E[(S-2)_+]=\int_2^\infty[1-F_S(x)]\,dx$. In this case, this becomes $E[(S-2)_+]=\int_2^8[1-F_S(x)]\,dx$. The integral is the area of the shaded region on the next page, again a series of rectangles.

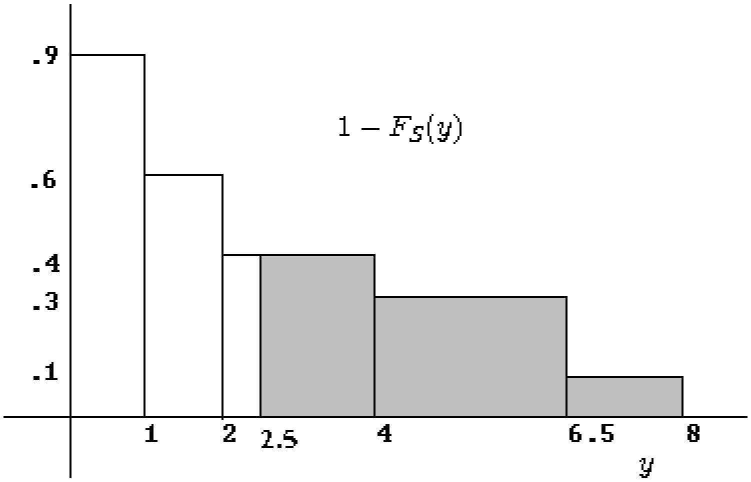
$E[(S-2)_+]=\int_2^8[1-F_S(x)]\,dx=(2)(.4)+(2.5)(.3)+(1.5)(.1)=1.7$.

Notice that $E[(S-2)_+]=\int_2^8[1-F_S(x)]\,dx$
$=\int_1^8[1-F_S(x)]\,dx-\int_1^2[1-F_S(x)]\,dx=E[(S-1)_+]-[1-F_S(1)]$.
This is true because $1-F_S(x)=1-F_S(1)=.6$ is constant for $1\le x<2$.



Now suppose that we wish to apply a deductible of $d = 2.5$ to create the stop loss insurance $(S-2.5)_+$ for this example. We can use $E[(S-2.5)_+] = E[S] - E[S \wedge 2.5]$, and $E[S \wedge 2.5] = (0)(.1) + (1)(.3) + (2)(.2) + (2.5)(.4) = 1.7$, so that $E[(S-2.5)_+] = 3.2 - 1.7 = 1.5$.

We can also use the relation $E[(S-2.5)_+] = \int_{2.5}^{\infty}[1-F_S(x)]\,dx$. In this case, this becomes $E[(S-2.5)_+] = \int_{2.5}^{8}[1-F_S(x)]\,dx$. The integral is the area of the following shaded region, again a series of rectangles. One of the rectangles is a part of the rectangle whose base runs from $x = 2$ to $x = 4$.



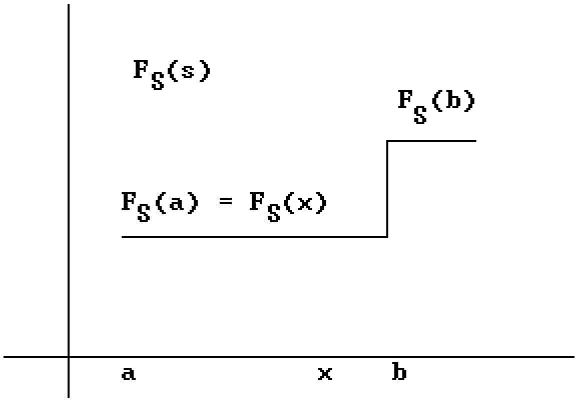
$E[(S-2.5)_+] = \int_{2.5}^{8}[1-F_S(x)]\,dx = (1.5)(.4) + (2.5)(.3) + (1.5)(.1) = 1.5$.

Notice that $E[(S-2.5)_+] = \int_{2.5}^{8}[1-F_S(x)]\,dx$

$$= \int_{2}^{8}[1-F_S(x)]\,dx - \int_{2}^{2.5}[1-F_S(x)]\,dx = E[(S-2)_+] - (.5)(1-F_S(2)) \ .$$

This is true because $1-F_S(x) = 1-F_S(2) = .4$ is constant for $2 \le x < 4$.

The general point being made by the example above using the graphical approach can be described in the following way. Suppose that S has a discrete distribution, and suppose that a and b are two successive values of S (so that $a < b$, and there are no values of S between a and b). Since S has a discrete distribution, $F_S(x)$ is a step function which steps up at each successive point of probability of S. Then for any number x between a and b (actually, for $a \leq x < b$), we have $F_S(x) = F_S(a)$ since the next step after a is at b and there is no probability between a and b. This can be seen in the following diagram.



Therefore, it is also true that if $a \leq x < b$ then $1 - F_S(x) = 1 - F_S(a)$.

Now suppose that d is a deductible such that $a \leq d \leq b$, and suppose we know the expected stop loss payment when the deductible is a (that is, we know $E[(S-a)_+]$).
We wish to find $E[(S-d)_+]$. We use the relationship $E[(S-d)_+] = \int_d^\infty [1 - F_S(x)]\, dx$.

This can be written in the form
$$E[(S-d)_+] = \int_d^\infty [1 - F_S(x)]\, dx = \int_a^\infty [1 - F_S(x)]\, dx - \int_a^d [1 - F_S(x)]\, dx \,. \tag{20.5}$$

We know that $E[(S-a)_+] = \int_a^\infty [1 - F_S(x)]\, dx$.
Also, since $1 - F_S(x) = 1 - F_S(a)$ for $a \leq x < b$, we have
$\int_a^d [1 - F_S(x)]\, dx = \int_a^d [1 - F_S(a)]\, dx = (d-a)[1 - F_S(a)]$
(this is the integral of a constant over the interval from a to d) .

Therefore, $E[(S-d)_+] = E[(S-a)_+] - (d-a)[1 - F_S(a)]$. (20.6)
This relationship is valid for any d between the successive probability points a and b.
In particular, $E[(S-b)_+] = E[(S-a)_+] - (b-a)[1 - F_S(a)]$ if a and b are successive points of probability for S.

The most typical illustration of this relationship occurs when S is integer-valued. If that is so, then for successive integer deductibles we have
$$E[(S-1)_+] = E[S] - [1 - F_S(0)] \,, \tag{20.7}$$
$$E[(S-2)_+] = E[(S-1)_+] - [1 - F_S(1)] \,, \ldots \tag{20.8}$$
$$\boldsymbol{E\big[[S-(d+1)]_+\big] = E[(S-d)_+] - [1 - F_S(d)]} \,, \text{ for any integer } d \geq 0 \,. \tag{20.9}$$
This has come up regularly on the exam.

Some additional properties of stop-loss insurance are as follows.

(i) If S is continuous for $S > 0$, then $E[(S-d)_+] = \int_d^\infty (x-d) \cdot f_S(x)\,dx$.

(ii) If S is discrete, then $E[(S-d)_+] = \sum_{x>d} (x-d) \cdot f_S(x)$.

(iii) If $P[a < S < b] = 0$ and $a \le d \le b$, then
$E[(S-d)_+] = E[(S-a)_+] - (d-a)[1-F_S(a)] = \frac{b-d}{b-a} \cdot E[(S-a)_+] + \frac{d-a}{b-a} \cdot E[(S-b)_+]$.
This is linear interpolation between the deductibles of a and b, and it is valid if S cannot assume values (has no density) between a and b.

(iv) If $a < b$, then $(S-a)_+ - (S-b)_+ = \begin{cases} 0 & \text{if } S \le a \\ S-a & \text{if } a < S \le b \\ b-a & \text{if } S > b \end{cases}$.
The maximum paid under this modified stop-loss arrangement is $b-a$.
This is a combination of maximum covered loss b and policy deductible a that was considered in Section 14 of the study guide.

Example LM20-1: Under a stop-loss insurance arrangement, the insurance amount paid is 80% of the excess of aggregate claims above 20, subject to a maximum payment of 5. All claim amounts are non-negative integers. You are given:

Deductible:	16	20	24	25	26	27
Net stop-loss premium:	3.89	3.33	2.84	2.75	2.69	2.65

Determine the expected amount paid by the insurer.
Solution: The maximum payment is triggered if 80% of claims above 20 is 5, or equivalently, if aggregate claims is above 26.25 (for then the excess over 20 is 6.25 or more, and 80% of 6.25 is 5). The payment made by the insurer can be expressed as
$(.8)[(S-20)_+ - (S-26.25)_+]$, and the expected amount paid by the insurer will be
$(.8)\Big(E[(S-20)_+] - E[(S-26.25)_+]\Big)$. We are given $E[(S-20)_+] = 3.33$.
Since all claim amounts are integer-valued, $P[26 < S < 27] = 0$ (S must be an integer).
Using the linear interpolation relationship (iii) above,
$E[(S-26.25)_+] = \frac{27-26.25}{27-26} \cdot E[(S-26)_+] + \frac{26.25-26}{27-26} \cdot E[(S-27)_+]$
$= (.75)(2.69) + (.25)(2.65) = 2.68$.
The expected amount paid by the insurer is $(.8)(3.33-2.68) = .52$. ☐

Example LM20-2: For aggregate claims, S, you are given:
(i) S can assume values that are multiples of 10
(ii) $E[(S-10)_+] = 0.6$ (iii) $E[(S-20)_+] = 0.2$
Find $F_S(10)$.
Solution: Since $S = 10$ and $S = 20$ are successive points of probability,
$E\Big([S-(20)]_+\Big) = E[(S-10)_+] - 10 \cdot [1-F_S(10)]$
$\rightarrow .2 = .6 - (10)[1-F_S(10)] \rightarrow F_S(10) = .96$. ☐

MODELING - PROBLEM SET 20
Stop-Loss Insurance - Section 20

1. Annual aggregate losses S follow a compound distribution with annual frequency N and severity X (the usual assumption of independence of N and the X's applies).
The probability function of N is uniform on the integers from 0 to 4.
X has a uniform distribution on the integers from 1 to 5 .
Annual stop loss insurance on aggregate losses has a deductible of 2. The insurer collects a premium equal to the sum of the mean and standard deviation of the stop loss. Find the stop loss premium.

2. (SOA) An aggregate claim distribution has the following characteristics:
$P[S=i]=\frac{1}{6}$ for $i=1,2,...,6$. A stop-loss insurance with deductible amount d has an expected insurance payment of 1.5 . Find d.
A) 1.75 B) 2.00 C) 2.25 D) 2.50 E) 2.75

3. (SOA): For aggregate claims, S. you are given :
(i) S takes on only positive integer values;
(ii) $E[S]=\frac{5}{3}$ (iii) $E[(S-2)_+]=\frac{1}{6}$ (iv) $E[(S-3)_+]=0$
Determine $f_S(1)$.

4. (SOA) Aggregate claims have a compound Poisson distribution with
$\lambda=4$, $f_X(1)=\frac{3}{4}$, $f_X(2)=\frac{1}{4}$. Determine $E[(S-2)_+]$.
A) 3.05 B) 3.07 C) 3.09 D) 3.11 E) 3.13

5. (SOA) For an aggregate claim distribution S, the amount paid by a reinsurance policy is the following random variable:

$$I=\begin{cases} 0 & \text{if } S<100 \\ S-100 & \text{if } 100\le S<200 \\ .5S & \text{if } 200\le S<500 \\ 250 & \text{if } S\ge 500 \end{cases}$$

Which of the following correctly expresses $E[I]$ in terms of stop-loss expectations?
A) $E[(S-100)_+]-E[(S-200)_+]-E[(S-500)_+]$
B) $E[(S-100)_+]-.5E[(S-200)_+]-E[(S-500)_+]$
C) $E[(S-100)_+]-E[(S-200)_+]-.5E[(S-500)_+]$
D) $E[(S-100)_+]-.5E[(S-200)_+]-.5E[(S-500)_+]$
E) $E[(S-100)_+]-.25E[(S-200)_+]-.25E[(S-500)_+]$

6. (SOA) For a certain company, losses follow a Poisson frequency distribution with mean 2 per year, and the amount of a loss is 1, 2, or 3, each with probability 1/3. Loss amounts are independent of the number of losses, and of each other. An insurance policy covers all losses in a year, subject to an annual aggregate deductible of 2. Calculate the expected claim payments for this insurance policy.

A) 2.00 B) 2.36 C) 2.45 D) 2.81 E) 2.96

7. (SOA) Prescription drug losses, S, are modeled assuming the number of claims has a geometric distribution with mean 4, and the amount of each prescription is 40.

Calculate $E[(S-100)_+]$.

A) 60 B) 82 C) 92 D) 114 E) 146

8. (SOA) WidgetsRUs owns two factories. It buys insurance to protect itself against major repair costs. Profit equals revenues, less the sum of insurance premiums, retained major repair costs, and all other expenses. WidgetsRUs will pay a dividend equal to the profit, if it is positive.

You are given:

(i) Combined revenue for the two factories is 3.

(ii) Major repair costs at the factories are independent.

(iii) The distribution of major repair costs for each factory is

k	Prob(k)
0	0.4
1	0.3
2	0.2
3	0.1

(iv) At each factory, the insurance policy pays the major repair costs in excess of that factory's ordinary deductible of 1. The insurance premium is 110% of the expected claims.

(v) All other expenses are 15% of revenues.

Calculate the expected dividend.

A) 0.43 B) 0.47 C) 0.51 D) 0.55 E) 0.59

9. (SOA) For a stop-loss insurance on a three person group:

(i) Loss amounts are independent.

(ii) The distribution of loss amount for each person is:

Loss Amount	Probability
0	0.4
1	0.3
2	0.2
3	0.1

(iii) The stop-loss insurance has a deductible of 1 for the group.

Calculate the net stop-loss premium.

A) 2.00 B) 2.03 C) 2.06 D) 2.09 E) 2.12

10. (SOA) For a collective risk model:
(i) The number of losses has a Poisson distribution with $\lambda = 2$.
(ii) The common distribution of the individual losses is:

x	$f_X(x)$
1	0.6
2	0.4

An insurance covers aggregate losses subject to a deductible of 3. Calculate the expected aggregate payments of the insurance.

A) 0.74 B) 0.79 C) 0.84 D) 0.89 E) 0.94

11. (SOA) In a given week, the number of projects that require you to work overtime has a geometric distribution with $\beta = 2$. For each project, the distribution of the number of overtime hours in the week is the following:

x	$f(x)$
5	0.2
10	0.3
20	0.5

The number of projects and the number of overtime hours are independent. You will get paid for overtime hours in excess of 15 hours in the week. Calculate the expected number of overtime hours for which you will get paid in the week.

A) 18.5 B) 18.8 C) 22.1 D) 26.2 E) 28.0

12. (SOA) A compound Poisson claim distribution has $\lambda = 5$ and individual claims amounts distributed as follows:

x	$f_X(x)$	
5	0.6	
k	0.4	where $k > 5$

The expected cost of an aggregate stop-loss insurance subject to a deductible of 5 is 28.03. Calculate k.

A) 6 B) 7 C) 8 D) 9 E) 10

MODELING - PROBLEM SET 20 SOLUTIONS

1. $E[N]=2$, $Var[N]=2$, $E[X]=3$, $Var[X]=2$.
$E[S]=E[N]\cdot E[X]=6$.
The stop loss insurance pays $(S-2)_+=S-(S\wedge 2)$.
$E[S\wedge 2]=P(S=1)+2P(S>1)$.
$P(S=0)=P(N=0)=.2$, $P(S=1)=P(N=1)\cdot P(X=1)=(.2)(.2)=.04$.
$P(S>1)=1-.2-.04=.76$.
$E[S\wedge 2]=.04+2(.76)=1.56$.
$E[(S-2)_+]=6-1.56=4.44$.
$E[(S-2)_+^2]=E[S^2]-E[(S\wedge 2)^2]-2(2)[E(S)-E(S\wedge 2)]$.
$Var[S]=E[N]\cdot Var[X]+Var[N]\cdot(E[X])^2=22=E[S^2]-(E[S])^2$.
$E[S^2]=22+36=58$.
$E[(S\wedge 2)^2]=P(S=1)+4P(S>1)=.04+4(.76)=3.08$.
$E[(S-2)_+^2]=58-3.08-4(6-1.56)=37.16$.
$Var[(S-2)_+]=37.16-(4.44)^2=17.45$.
The premium for the stop loss insurance is $6+\sqrt{17.45}=10.18$.

2. If $d<2$ then
$E[(S-d)_+]=[(2-d)+(3-d)+(4-d)+(5-d)+(6-d)]\cdot\frac{1}{6}=1.5\rightarrow d=2.2$,
which contradicts the assumption that $d<2$. If $d\le 2<3$, then
$E[(S-d)_+]=[(3-d)+(4-d)+(5-d)+(6-d)]\cdot\frac{1}{6}=1.5\rightarrow d=2.25$.

3. From (iv) it follows that $P[S\ge 4]=0$, so that S has probability only at $x=1,2,3$, so that $f_S(1)+f_S(2)+f_S(3)=1$.
From (ii) we get $E[S]=\sum_{k=1}^{3}k\cdot f_S(k)=f_S(1)+2f_S(2)+3f_S(3)=\frac{5}{3}$.
From (iii) we get $E[(S-2)_+]=\sum_{k=3}^{3}(k-2)\cdot f_S(k)=f_S(3)=\frac{1}{6}$.
From the three equations, we can solve for $f_S(1)$, which is $\frac{1}{2}$.
Actually, we do not need (iv). From (i) we have $S\ge 1$, so that $f_S(0)=0$.
Then $E[(S-1)_+=E[S]-[1-F_S(0)]=\frac{5}{3}-1=\frac{2}{3}$.
Then $\frac{1}{6}=E[(S-2)_+=E[(S-1)_+]-[1-F_S(1)]=\frac{2}{3}-[1-F_S(1)]\rightarrow F_S(1)=\frac{1}{2}$.
But $\frac{1}{2}=F_S(1)=f_S(0)+f_S(1)=0+f_S(1)$.

4. When S is integer valued, then for any integer $d\ge 0$ we have
$E[(S-d-1)_+]=E[(S-d)_+]-[1-F_S(d)]$.
$(S-0)_+=S\rightarrow E[(S-0)_+]=E[S]=\lambda E[X]=(4)(\frac{5}{4})=5$.
$F_S(0)=f_S(0)=P[N=0]=e^{-\lambda}=e^{-4}=.0183$
$\rightarrow E[(S-1)_+]=E[(S-0)_+]-[1-F_S(0)]=5-(1-.0183)=4.0183$.
$F_S(1)=F_S(0)+f_S(1)$. $f_S(1)=f_X(1)\cdot P[N=1]=(\frac{3}{4})(e^{-4}\cdot\frac{4}{1})=.0549$
$\rightarrow F_S(1)=.0183+.0549=.0732$.
Then $E[(S-2)_+]=E[(S-1)_+]-[1-F_S(1)]=4.0183-(1-.0732)=3.0915$. C

5. I is the same as $(S-100)_+$ for $0 \leq S < 200$.
Since $(S-200)_+$ pays $S-200$ for $S > 200$, we see that $.5(S-200)_+$ pays $.5S-100$ for $S > 200$. Thus, $(S-100)_+ - .5(S-200)_+$ pays $S-100$ for $100 \leq S \leq 200$, and pays $S-100-(.5S-100) = .5S$ for $S > 200$. To limit the reinsurance coverage to a maximum of $250 = (.5)(500)$, note that $.5(S-500)_+$ pays $.5S-250$ for $S > 500$.
Thus, $(S-100)_+ - .5(S-200)_+ - .5(S-500)_+$ pays $.5S-(.5S-250) = 250$ for $S > 500$. Answer: D

6. We wish to find $E[(S-2)_+] = E[S] - E[S \wedge 2] = E[S] - [f(1) + 2(1-F(1))]$.
Since the claim amounts are integers, S is also an integer.
$P(X=1) = P(X=2) = P(X=3) = \frac{1}{3}$. $E[S] = E[N] \cdot E[X] = (2)(2) = 4$.
$f_S(0) = P(S=0) = P(N=0) = e^{-2} = .1353$.
$f_S(1) = P(S=1) = P(N=1) \cdot P(X=1) = (e^{-2} \cdot 2)\left(\frac{1}{3}\right) = .0902$.
$F_S(1) = f_S(0) + f_S(1) = .2255$
$\rightarrow E[(S-2)_+] = 4 - [.0902 + 2(1 - .1353 - .0902)] = 2.36$. Answer: B

7. $E[(S-100)_+] = E[S] - E[S \wedge 100] = E[S] - 40f(40) - 80f(80) - 100[1-F(80)]$
$= 160 - 40\left(\frac{4}{25}\right) - 80\left(\frac{16}{125}\right) - 100[1 - \frac{1}{5} - \frac{4}{25} - \frac{16}{125}] = 92.16$. Answer: C

8. Expected dividend $=$ Revenue $-$ Insurance Payments $-$ Expected retained costs .
Revenue $= 3$.
Expected major repair cost per factory $= (1)(.3) + (2)(.2) + (3)(.1) = 1$.
Expected retained major repair cost per factory $= .3 + .2 + .1 = .6$.
Expected reinsurance payment per factory $= 1 - .6 = .4$.
Insurance premium per factory $= (1.1)(.4) = .44$.
Insurance payments for both factories $= 2(.44) = .88$.
Revenue $-$ Insurance Payments $-$ Other Costs $= 3 - .88 - .45 = 1.67$
Dividend $=$ Revenue $-$ Insurance Payments $-$ Retained Costs (if ≥ 0).

Fac. 1 Claim	Fac. 2 Claim	Retained Claim	Dividend	Prob.
0	0	0	1.67	.16
0	≥ 1	1	.67	.24
≥ 1	0	1	.67	.24
≥ 1	≥ 1	2	0	.36

Expected dividend $= (1.67)(.16) + (.67)(.24 + .24) = .5888$. Answer: E

9. We use the formulation $E[(S-1)_+] = E[S] - E[S \wedge 1]$,
where $S \wedge 1 = \begin{cases} 0 & P(S=0) \\ 1 & P(S>0) \end{cases}$.
$E[S] = E[X_1] + E[X_2] + E[X_3] = 3 \cdot E[X] = 3 \cdot [(1)(.3) + (2)(.2) + (3)(.1)] = 3$.
$P[S=0] = P[(X_1=0) \cap (X_2=0) \cap (X_3=0)]$,
and by independence of the X_i's this becomes
$P[X_1=0] \cdot P[X_2=0] \cdot P[X_3=0] = (.4)^3 = .064$. Then,
$E[(S-1)_+] = E[S] - E[S \wedge 1] = E[S] - P(S>0) = 3 - [1 - .064] = 2.064$. Answer: C

10. The amount paid is a stop-loss insurance with a deductible of 3 applied to aggregate losses. The aggregate loss has a compound Poisson distribution with an integer-valued severity. Therefore, S is integer-valued. $E[S] = E[N] \cdot E[X] = 2[(1)(.6) + (2)(.4)] = 2.8$.
$E[(S{-}3)_+] = E[S] - E[S \wedge 3]$.
S is integer valued with probability values $P(S=0) = P(N=0) = e^{-2} = .1353$,
$P(S=1) = P(N=1) \cdot P(X=1) = \frac{e^{-2} \cdot 2^1}{1!} \cdot (.6) = .1624$,
$P(S=2) = P(N=1) \cdot P(X=2) + P(N=2) \cdot (P(X=1))^2 = .2057$,
and $P(S \geq 3) = 1 - P(S=0,1,2) = .4966$.

$$\text{Then,} \quad S \wedge 3 = \begin{cases} 0 & \text{prob. } .1353 \\ 1 & \text{prob. } .1624 \\ 2 & \text{prob. } .2057 \\ 3 & \text{prob. } .4966 \end{cases}$$

so that $E[S \wedge 3] = (1)(.1624) + (2)(.2057) + (3)(.4966) = 2.0636$.
$E[(S-3)_+] = 2.8 - 2.0636 = .7364$. Answer: A

11. This is a stop-loss problem where S is the aggregate number of overtime hours worked in the week and the deductible is 15. S has a compound distribution with frequency N that is geometric with mean 2 and severity X that is 5 (prob. .2), 10 (prob. .3) or 20 (prob. .5). We wish to find $E[(S-15)_+] = E[S] - E[S \wedge 15]$. The mean of S is
$E[S] = E[N] \cdot E[X] = 2[5(.2) + 10(.3) + 20(.5)] = 28.$
Note that S must be a multiple of 5, with
$P(S=0) = P[N=0] = \frac{1}{1+\beta} = .3333$ (the only way that $S=0$ is if $N=0$),
$P[S=5] = P[N=1] \cdot P[X=5] = \left(\frac{2}{9}\right)(.2) = .0444$, and
$P[S=10] = (P[N=1] \cdot P[X=10] + P[N=2] \cdot (P[X=5])^2)$
$= \left(\frac{2}{9}\right)(.3) + \left(\frac{4}{27}\right)(.2)^2 = .0726$.
Then, $P(S \geq 15) = 1 - P(S=0,5,10) = .5497$.

$$S \wedge 15 = \begin{cases} 0 & S=0, \text{ prob. } .3333 \\ 5 & S=5, \text{ prob. } .0444 \\ 10 & S=10, \text{ prob. } .0726 \\ 15 & S \geq 14, \text{ prob. } .5497 \end{cases},$$

so $E[S \wedge 15] = 5(.0444) + 10(.0726) + 15(.5497) = 9.194$.
Then, $E[(S-15)_+] = 28 - 9.2 = 18.8$. Answer: B

12. The minimum claim amount is 5 if a claim occurs. The minimum value of S is 0, which occurs if there are no claims,. The next possible value of S is 5.
The stop-loss insurance with deductible 5 pays $(S-5)_+ = S - (S \wedge 5)$,

$$\text{where} \quad S \wedge 5 = \begin{cases} 0 & S=0 \\ 5 & S \geq 5 \end{cases}.$$

$E[S] = E[N] \cdot E[X] = (5)[(5)(.6) + k(.4)] = 15 + 2k$.
$E[S \wedge 5] = 5[1 - P(S=0)] = 5[1 - P(N=0)] = 5[1 - e^{-5}] = 4.9663$.
We are given that $28.03 = E[(S-5)_+] = E[S] - E[S \wedge 5] = 15 + 2k - 4.9663$.
Solving for k results in $k = 9$. Answer: D

TABLE OF CONTENTS - VOLUME 2

CREDIBILITY

SIMULATION

PRACTICE EXAMS AND SOLUTIONS

ACTEX EXAM C/4 - PRACTICE EXAM 9

1. You are given:
- X has density $f(x)$, where $f(x) = 500,000/x^3$, for $x > 500$ (single parameter Pareto with $\alpha = 2$)
- Y has density $g(y)$, where $g(y) = ye^{-y/500}/500,000$ (gamma with $\alpha = 2$ and $\theta = 500$)

Which of the following are true?

    1. X has an increasing mean residual life function.
    2. Y has an increasing hazard rate.
    3. X has a heavier tail than Y based on the hazard rate test.

A) 1 only B) 2 only C) 3 only D) 2 and 3 only E) All of 1, 2 and 3

2. A population of auto insurance policies consists of three types of policies. Low risk policies make up 60% of the population, medium risk policies make up 30% of the population, and the other 10% are high risk. The number of claims per year for a low risk policy has a Poisson distribution with a mean of .2. The number of claims per year for a medium risk policy has a Poisson distribution with a mean of 1. The number of claims per year for a high risk policy has a Poisson distribution with a mean of λ. For a randomly chosen policy from the population, the variance of the number of claims in a year is 1.1701. Find the expected number of claims per year for a high risk policy.
A) 1.5 B) 2.0 C) 2.5 D) 3.0 E) 3.5

Use the following information for Questions 3 and 4.
- losses follow a distribution with density function

$$f(x) = \frac{1}{1000}e^{-x/1000} \ , \ 0 < x < \infty$$

- there is a deductible of 500
- 10 losses are expected to exceed the deductible each year

3. Determine the amount to which the deductible would have to be raised to double the loss elimination ratio.
A) Less than 550 B) At least 550, but less than 850
C) At least 850, but less than 1150 D) At least 1150, but less than 1450
E) At least 1450

4. Determine the expected number of losses that would exceed the deductible each year if all loss amounts doubled, but the deductible remained at 500.
A) Less than 10 B) At least 10, but less than 12
C) At least 12, but less than 14 D) At least 14, but less than 16
E) At least 16

5. A company has 1,000 employees who are partly covered under a disability insurance plan. The plan pays full salary for up to 4 weeks of disability (after which a government plan takes over). Occurrences of disability among employees are independent of one another, and an employee is covered for only one occurrence per year. The following information is known:

Employee Category	Weekly Salary	Number of Employees	Disability Probability
1	400	300	.025
2	600	500	.02
3	800	200	.01

Length of Disability	Probability
1 week	.8
2 weeks	.1
3 weeks	.05
$\geq$ 4 weeks	.05

The company actuary calculates the annual premium required to ensure (using the normal approximation) with 95% probability that the premium will exceed disability claims. What percentage of total weekly payroll is that premium (nearest $\frac{1}{2}$%)?

A) 2% B) 2.5% C) 3% D) 3.5% E) 4%

6. Your are given the following.

- a sample of 2000 claims contains 1700 that are no greater than \$6000, 30 that are greater than \$6000 but no greater than \$7000, and 270 that are greater than \$7000
- the total amount of the 30 claims that are greater than \$6000 but no greater than \$7000 is \$200,000
- the empirical limited expected value for this sample with a policy limit of \$6000 is \$1810

Determine the empirical limited expected value for this sample with a policy limit of \$7000.

A) Less than \$1900 B) At least \$1900, but less than \$1925

C) At least \$1925, but less than \$1950 D) At least \$1950, but less than \$1975

E) At least \$1975

7. A mortality study without any censored observations begins with n individuals under observation. There are d_1 deaths at the first death point t_1 and d_2 deaths at the second death point t_2. You are given the following:

(i) the product limit estimate of $S(t_2)$ is .903614

(ii) the Nelson-Aalen estimate of $S(t_2)$ is .906065 (both values rounded to 6 decimal places)

(iii) $d_1 < d_2$

Find the Nelson-Aalen estimate of $S(t_1)$.

A) $.935$ B) $.945$ C) $.955$ D) $.965$ E) $.975$

8. 40 observed losses have been recorded in thousands of dollars and are grouped as follows:

Interval ($000)	Number of Losses	Total Losses ($000)
$(1\,,\,\frac{4}{3}]$	16	20
$(\frac{4}{3}\,,\,2]$	10	15
$(2,4]$	10	35
$(4,\infty)$	4	20

A deductible of 1 is applied to the loss data. The empirical distribution function is constructed from the numbers of losses in each interval of the resulting data set after the deductible is applied. The method of percentile matching is applied using the after-deductible empirical distribution function to estimate the parameters θ and τ of the Weibull distribution (the distribution function is $F(y) = 1 - exp[-(\frac{y}{\theta})^\tau]$). The percentiles to be matched are the 40-th and the 90-th percentiles. Determine the estimated value of τ.

A) .4 B) .5 C) .6 D) .7 E) .8

Use the following information for Questions 9 and 10.
You are given the following:
- the random variable X has the density function
$$f(x) = \alpha(x+1)^{-\alpha-1}\ ,\ 0 < x < \infty\ ,\ \alpha > 0$$
- a random sample of size n is taken of the random variable X

9. Assuming $\alpha > 1$, determine $\tilde{\alpha}$, the method of moments estimator of α.

A) $\overline{X}$ B) $\frac{\overline{X}}{\overline{X}-1}$ C) $\frac{\overline{X}}{\overline{X}+1}$ D) $\frac{\overline{X}-1}{\overline{X}}$ E) $\frac{\overline{X}+1}{\overline{X}}$

10. Determine the limit of $\hat{\alpha}$ as the sample mean goes to infinity, where $\hat{\alpha}$ is the maximum likelihood estimator of α.

A) 0 B) 1/2 C) 1 D) 2 E) ∞

11. You are given the following claims settlement activity for a book of automobile claims as of the end of 1999:

Number of Claims Settled			
Year	Year Settled		
Reported	1997	1998	1999
1997	Unknown	3	1
1998		5	2
1999			4

$L =$ (Year Settled$-$Year Reported) is a random variable describing the time lag in settling a claim. The probability function of L is $f_L(l) = (1-p)p^l$, for $l = 0, 1, 2, \ldots$.
Determine the maximum likelihood estimate of the parameter p.

A) 3/11 B) 7/22 C) 1/3 D) 3/8 E) 7/15

Use the following information for Questions 12 and 13.

A portfolio of independent risks is divided into two classes. Each class contains the same number of risks.

For each risk in Class 1, the number of claims for a single exposure period follows a Poisson distribution with mean 1.

For each risk in Class 2, the number of claims for a single exposure period follows a Poisson distribution with mean 2.

A risk is selected at random from the portfolio. During the first exposure period, 2 claims are observed for this risk. During the second exposure period, 0 claims are observed for this same risk.

12. Determine the posterior probability that the risk selected came from Class 1.
A) Less than .53 B) At least .53, but less than .58 C) At least .58, but less than .63
D) At least .63, but less than .68 E) At least .68

13. Determine the Buhlmann credibility estimate of the expected number of claims for this same risk for the third exposure.
A) Less than 1.32 B) At least 1.32, but less than 1.34
C) At least 1.34, but less than 1.36 D) At least 1.36, but less than 1.38
E) At least 1.38

14. The prior distribution of the parameter λ is exponential with a mean of 1. The conditional distribution of X, the number of claims for an insured in one year, given λ, is a mixture of two Poisson random variables with probability function

$$p(x|\lambda) = (.5)\left[\frac{e^{-\lambda}\lambda^x}{x!}\right] + (.5)\left[\frac{e^{-2\lambda}(2\lambda)^x}{x!}\right],\ x = 0, 1, 2, ...$$

An insured is chosen at random and observed to have no claims in the first year.
Find the Bayesian estimate of the expected number of claims next year for the same insured.
A) 0.5 B) 0.55 C) 0.60 D) 0.65 E) 0.70

15. The number of claims per month for a given risk is assumed to be Poisson distributed with an unknown mean that varies by risk. It is found that for a risk that has reported no claims for the past month, the semiparametric empirical Bayes estimate of the expected number of claims next month is $\frac{1}{30}$, and it is found that for a risk that has reported no claims for the past two months, the semiparametric empirical Bayes estimate of the expected number of claims next month is $\frac{1}{55}$. Find the semiparametric empirical Bayes estimate of the expected number of claims next month for a risk that has reported no claims for the past three months.
A) $\frac{1}{70}$ B) $\frac{1}{75}$ C) $\frac{1}{80}$ D) $\frac{1}{85}$ E) $\frac{1}{90}$

Use the following information for Questions 16 and 17.
Your are given the following:

- a large portfolio of automobile risks consists solely of youthful drivers
- the number of claims for one driver during one exposure period follows a Poisson distribution with mean $4 - g$, where g is the grade point average of the driver
- the distribution of g within the portfolio is uniform on the interval $[0, 4]$

A driver is selected at random form the portfolio. During one exposure period, no claims are observed for this driver.

16. Determine the posterior probability that the selected driver has a grade point average greater than 3.
A) Less than .15 B) At least .15, but less than .35
C) At least .35, but less than .55 D) At least .55, but less than .75
E) At least .75

17. A second driver is selected at random from the portfolio. During five exposure periods, no claims are observed for this second selected driver. Determine the Buhlmann credibility estimate of the expected number of claims for this second driver during the next exposure period.
A) Less than .375 B) At least .375, but less than .425
C) At least .425, but less than .475 D) At least .475, but less than .525
E) At least .525

18. A sample of ten observations comes from a parametric family $f(x, y; \theta_1, \theta_2)$ with loglikelihood function $ln\, L(\theta_1, \theta_2) = \sum_{i=1}^{10} ln f(x_i, y_i; \theta_1, \theta_2) = -2.5\theta_1^2 - 3\theta_1\theta_2 - \theta_2^2 + 5\theta_1 + 2\theta_2 + k$,
where k is a constant.
Determine the estimated covariance matrix of the maximum likelihood estimator, $\begin{bmatrix} \widehat{\theta}_1 \\ \widehat{\theta}_2 \end{bmatrix}$.

A) $\begin{bmatrix} .5 & .3 \\ .3 & .2 \end{bmatrix}$ B) $\begin{bmatrix} 20 & -30 \\ -30 & 50 \end{bmatrix}$ C) $\begin{bmatrix} .2 & .3 \\ .3 & .5 \end{bmatrix}$ D) $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ E) $\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$

19. Semi-parametric empirical Bayesian credibility is being applied in the following situation. The distribution of annual losses on an insurance policy is uniform on the interval $(0, \theta)$, where θ has an unknown distribution. A sample of annual losses for 100 separate insurance policies is available. It is found that $\sum_{i=1}^{100} X_i = 200$ and $\sum_{i=1}^{100} X_i^2 = 600$.
For a particular insurance policy, it is found that the total losses over a 3 year period is 3.
Find the semi-parametric estimate of the losses in the 4-th year for this policy.
A) Less than 1.5 B) At least 1.5, but less than 1.7 C) At least 1.7, but less than 1.9
D) At least 1.9, but less than 2.1 E) At least 2.1

Questions 20 and 21 relate to the following situation. Claims arrive for processing according to a Poisson process with mean rate $\lambda = 2$ per hour. Claim processing
takes either $\frac{1}{4}$ or $\frac{1}{2}$ hour, with any given claim having a .5 probability of taking $\frac{1}{4}$ hour.

20. Use the inverse transform method to simulate the number of claims in each of the first two hours. The uniform random numbers to be used in sequence to simulate the numbers of claims in hours 1 and 2 are: .7 , .5 . Find the number of claims simulated in each hour.
A) 1 in the first hour and 3 in the second hour
B) 2 in the first hour and 2 in the second hour
C) 3 in the first hour and 2 in the second hour
D) 3 in the first hour and 1 in the second hour
E) None of A, B, C or D is correct

21. The simulated arrival times (in hours) of the claims during the first 2 hours are
.2 , .8 , 1.1 , 1.3 , 1.7
There is only one claims processor. The claim processing times of the successive claims are simulated using the inversion method, using the following uniform random numbers, where small random numbers correspond to small processing times:
.8 , .6 , .1 , .1 , .7 .
Determine the state of the claims processing system at the end of 2 hours.
A) No claims are being processed, no claims are waiting
B) A claim is being processed and no claims are waiting to be processed.
C) A claim is being processed and one claim are waiting to be processed.
D) A claim is being processed and two claims are waiting to be processed.
E) A claim is being processed and three claims are waiting to be processed.

22. X has a uniform distribution on the interval $[0, \sqrt{\Lambda}]$, and Λ has a uniform distribution on the interval $[0, 1]$. Find the mean of the unconditional distribution of X.
A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$ E) $\frac{1}{6}$

23. A loss random variable has a continuous uniform distribution between 0 and \$100 .
An insurer will insure the loss amount above a deductible c. The variance of the amount that the insurer will pay is 69.75 . Find c.
A) 65 B) 70 C) 75 D) 80 E) 85

24. The times of death in a mortality study are t_1 , t_2 , t_3 , ... The following information is given. There was one death at time t_3, two deaths at time t_4 and one death at time t_5 .
The Product-Limit estimate of survival probability for those times are
$S_n(t_3) = .72$, $S_n(t_4) = .60$ and $S_n(t_5) = .50$.
Determine the number of right-censorings that took place in the interval $[t_4, t_5)$.
A) 0 B) 1 C) 2 D) 3 E) 4

25. On Time Shuttle Service has one plane that travels from Appleton to Zebrashire and back and each day. Flights are delayed at a Poisson rate of two per month. Each passenger on a delayed flight is compensated $100. The numbers of passengers on each flight are independent and distributed with mean 30 and standard deviation 50. (You may assume that all months are 30 days long and that years are 360 days long). Calculate the standard deviation of the annual compensation for the delayed flights.
A) Less than $25,000 B) At least $25,000, but less than $50,000
C) At least $50,000, but less than $75,000 D) At least $75,000, but less than $100,000
E) At least $100,000

26. A farmer develops a model for his seeding season for a particular crop. The number of days in which crops can be seeded during the season has a Poisson distribution with a mean of 20. On a day suitable for seeding, the number of acres than can be seeded is either 1 or 2, each with probability .5. The farmer wishes to insure against a poor seeding season. The farmer purchases insurance which will pay if the number of acres seeded during the season is under 20.
For each acre under 20 that is not seeded the insurance will 5000. S represents that number of acres that will be seeded in the season. The farmer has determined $E[(S-20)_+] = 11.2$.
Find the expected insurance payment.
A) 4000 B) 5000 C) 6000 D) 7000 E) 8000

27. You are given the following random sample of 6 observations from the distribution of the random variable X:
2 , 4 , 4 , 5 , 7 , 10

Kernel smoothing is applied to estimate the density function of X. The kernel function used for the data point y is the pdf of the normal distribution with mean y and variance 1. Use kernel smoothing to estimate the distribution function of X at the point $x = 3$, $\widehat{F}(3)$.
A) Less then .06 B) At least .06, but less than .12 C) At least .12 but less than .18
D) At least .18, but less than .24 E) At least .24

28. You are given:
(i) A sample of losses is: 600 700 ≥ 900
(the third loss is known only to be at least 900)
(ii) No information is available about losses of 500 or less.
(iii) Losses are assumed to follow an exponential distribution with mean θ.
Determine the maximum likelihood estimate of θ.
A) Less than 500 B) At least 500 but less than 600 C) At least 600 but less than 700
D) At least 700 but less than 800 E) At least 800

29. If the proposed model is appropriate, which of the following tends to zero as the sample size goes to infinity?
A) Kolmogorov-Smirnov test statistic B) Anderson-Darling test statistic
C) Chi-square goodness-of-fit test statistic D) Schwarz Bayesian adjustment
E) None of A), B), C), or D)

30. When applying the method of limited fluctuation credibility to a certain compound Poisson distribution of total claims cost so that total claims cost will be within r% of expected total claims cost p% of the time, the full credibility standard is 1200 expected claims. We also know that the coefficient of variation of the severity distribution is 2.
Suppose the following changes are made in our assumptions:
(i) the coefficient of variation of the severity distribution is doubled to 4, and
(ii) the standard for full credibility is based on total claims cost being within $2r$% of expected claims cost p% of the time $(p$ is unchanged).
Find the new standard for full credibility.
A) 500 B) 1000 C) 1020 D) 1200 E) 2040

31. In a portfolio of insureds, each insured will have either 0 or 1 claim in a year, with independence from one year to another. The probability that an individual insured will have a claim in a given year is x. The portfolio of insureds is such that for a randomly chosen individual from the portfolio, the probability x is uniformly distributed on $(0,1)$. A randomly chosen individual is found to have no claims in n consecutive years, where $n \geq 1$. Determine the expected number of claims that the individual will have in the $n+1$-st year.
A) $\frac{1}{n-2}$ B) $\frac{1}{n-1}$ C) $\frac{1}{n}$ D) $\frac{1}{n+1}$ E) $\frac{1}{n+2}$

32. Four machines are in a shop. The number needing repair in each week has a binomial distribution with p = 0.5. For each machine, the repair time, in hours, is uniformly distributed on [0,10]. You are to estimate X, the total repair time (in hours) for a three-week period, using the inverse transformation method of simulation. Use the following numbers from the uniform distribution on $[0,1]$ to simulate the number of machines needing repair during each of three weeks: 0.3, 0.6, 0.7. Use the following numbers from the uniform distribution on $[0,1]$ to simulate repair times: 0.3 0.1 0.7 0.6 0.5 0.8 0.1 0.3
Determine X.
A) 17 B) 22 C) 23 D) 30 E) 34

33. X has the following distribution : $Pr[X=0]=.4$, $Pr[X=1]=.6$.
The distribution of Y is conditional on the value of X:
if $X=0$ then the distribution of Y is $Pr[Y=0]=.6$, $Pr[Y=1]=.2$, $Pr[Y=2]=.2$, and
if $X=1$ then the distribution of Y is $Pr[Y=0]=.2, Pr[Y=1]=.3$, $Pr[Y=2]=.5$.
Z is the sum of Y independent normal random variables, each with mean and variance 2.
What is $Var[Z]$?
A) 5.0 B) 6.0 C) 7.0 D) 8.0 E) 9.0

34. Random sampling from the distribution of X results in the three sample values 1, 2 and 4.
A uniform distribution on the interval $[0,\theta]$ is fitted to the data set by finding the value of θ that minimizes the Kolmogorov-Smirnov goodness-of-fit statistic. Find θ.
A) 4.4 B) 4.5 C) 4.6 D) 4.7 E) 4.8

35. It is known that there are two groups of drivers in an insured population. One group has a 20 percent accident probability per year and the other group has a 40 percent accident probability per year. Two or more accidents per year per insured are not possible. The two groups comprise equal proportions of the population and each has the following accident severity distribution:

Probability	Size of Loss
.80	100
.10	200
.10	400

A merit rating plan is based on the pure premium experience of individual insureds for the prior year. Calculate the credibility of an insured's experience.
A) Less than .01 B) At least .01, but less than .02 C) At least .02, but less than .03
D) At least .03, but less than .04 E) At least .04

ACTEX EXAM C/4 - PRACTICE EXAM 9 SOLUTIONS

1. I. The mean residual lifetime with deductible d is $\frac{\int_d^\infty S(t)\,dt}{S(d)}$.
For the single parameter Pareto, $S(t) = \int_t^\infty f(x)\,dx = \int_t^\infty \frac{500{,}000}{x^3}\,dx = \frac{250{,}000}{t^2}$.
Then $\int_d^\infty S(t)\,dt = \int_d^\infty \frac{250{,}000}{t^2}\,dt = \frac{250{,}000}{d}$.
The mean residual lifetime is $\frac{250{,}000/d}{250{,}000/d^2} = d$, which is increasing. I is true.
Alternatively, if X has an decreasing hazard rate, then it has an increasing mean residual lifetime.
The hazard rate for X is $\frac{-S'(x)}{S(x)} = \frac{f(x)}{S(x)} = \frac{500{,}000/x^3}{250{,}000/x^2} = \frac{2}{x}$, which is a decreasing function of x.

II. It can be shown that the gamma distribution with $\alpha > 1$ has an increasing hazard rate.
We would have to find $S(t) = \int_t^\infty g(y)\,dy = \int_t^\infty \frac{ye^{-y/500}}{500{,}000}\,dy$.
Using integration by parts, this is $S(t) = \frac{te^{-t/500}}{500{,}000\times 500} + \frac{e^{-t/500}}{500{,}000\times 500^2}$.
The hazard rate is $\frac{g(t)}{S(t)} = \frac{te^{-t/500}/500{,}000}{\frac{te^{-t/500}}{500{,}000\times 500}+\frac{e^{-t/500}}{500{,}000\times 500^2}} = \frac{t}{\frac{t}{500}+\frac{1}{500^2}}$, which is an increasing function.
II is true.

III. According to the hazard rate test, a random variable with a decreasing hazard rate has a heavy right tail, and a if the hazard rate is increasing the right tail is light. X has a decreasing hazard rate so it has a heavy right tail, and Y has an increasing hazard rate so it has a light right tail. III is true. Answer: E

2. The randomly chosen policy is a mixture of the three policy types.
The expected number of claims per year for a randomly chosen policy is
$(.6)(.2) + (.3)(1) + (.1)\lambda = .42 + .1\lambda$.

The second moment of a Poisson random variable is $E[N^2] = Var[N] + (E[N])^2 = \lambda + \lambda^2$.
The second moment of number of claims for a low risk policy is $.2 + (.2)^2 = .24$.
The second moment of number of claims for a medium risk policy is $1 + (1)^2 = 2$.
The second moment of number of claims for a high risk policy is $\lambda + (\lambda)^2$.

The second moment of the number of claims per year for a randomly chosen policy is
$(.6)(.24) + (.3)(2) + (.1)(\lambda + \lambda^2) = .744 + .1(\lambda + \lambda^2)$.
The variance of the number of claims in a year for a randomly chosen policy is
$.744 + .1(\lambda + \lambda^2) - (.42 + .1\lambda)^2 = .5676 + .016\lambda + .09\lambda^2$.
We are given that this is 1.1701 , so that $.5676 + .016\lambda + .09\lambda^2 = 1.1701$.
This is the quadratic equation $.09\lambda^2 + .016\lambda - .6025 = 0$.
The two roots of the equation are 2.5 and $-$ 2.67. We ignore the negative root. $\lambda = 2.5$.
Answer: C