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Bolded chapters signify that chapter was covered in an earlier section of the syllabus.

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Note: FET-174-10: *Stochastic Calculus for Finance*, Chapters 1-3 are for background reading only and therefore have not been included in this study note.

IV. FINANCIAL MODELING

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We acknowledge the contributions of others in order to assure the timely availability of this manual.

Other authors include:

Nick Mocciolo – material that was originally included on the Course 8V exam syllabus
Matt Modisett – material that was new to the syllabus for the 2007 Financial Economics exam
Robin Cunningham – material that was original to the Course 8E exam syllabus
Vince Granieri – material that was new to the syllabus for the 2007 Advanced Finance exam
Frank Bensics – material that was originally included on the Course 8F exam syllabus

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FET-173-10
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"Variable Annuities"

I. Introduction

A. While variable annuities (VA's) are relatively new to the European market place, similar types of guarantees have been offered before:
   1. Unit linked pension contracts with Guaranteed Annuity Option (GAO)
   2. Unit linked pensions with death benefit protection
   3. Ratchet funds (unit funds with a step up lock in of value)

B. What's new with VA's is the understanding of the embedded guarantees and the monitoring of and management of them.

C. VA's have been popular in the US for decades
   1. 1990's: sales grew, in large part, due to strong equity markets and the introduction of living benefit guarantees
   2. Since the 1990's: market share of selected companies has increased through consolidation but total industry sales have remained flat

D. So why the sudden interest from the UK market?
   1. Replacement product for DB plans
   2. The demand for quality retirement savings schemes means these products might actually be bought rather than sold

E. However, some challenges still exist
   1. Guaranteed annuity rates (in GMIB products, discussed below) can look poor in a low rate environment
   2. Independent Financial Advisers (IFA's) have concerns about cost and value of the guarantees
   3. Providers must learn from the challenges encountered in the US:
      a. Uncovered guarantees – certain exposures were left unhedged and were sources of financial strain when markets and rates collapsed
      b. Economic capital costs – not usually calculated, thus a poor understanding of the type and size of risks
      c. Policyholder behaviour – failure to acknowledge policyholders would understand the value of the guarantees and not lapse when in-the-money
      d. Management understanding – management did not have a clear understanding of the risks their balance sheets were exposed to
      e. Lack of technical skills – companies did not have the proper knowledge, processes and controls in place to manage the risks

II. Product Features

A. Essential components of all VA's
   1. Unit linked single premium or regular premium product providing exposure to equity, bonds and other real assets
   2. Structure that allows conversion to an annuity or to otherwise provide a regular income

3. One or more guarantees providing protection against falling markets, lower interest rates, longevity or early death

B. Varieties of VA's

1. GMIB: Guaranteed Minimum Income Benefit
   a. Guarantees an annuitization factor ($\ddot{a}_x$: mortality and interest rates) at contract inception for when the policyholder annuitizes their account value in the future
   b. Payout to policyholder upon annuitization:

$$\max(\text{guaranteed payout, current payout})$$

where,

$$\text{guaranteed payout} = \frac{\text{GMIB Base}}{\ddot{a}_x^G} = \frac{\text{Contractual Nominal Amount}}{\text{Guaranteed Annuity Factor}}$$

$$\text{current payout} = \frac{\text{Account Balance}}{\ddot{a}_x^C} = \frac{\text{Accumulated Amount}}{\text{Current Annuity Factor}}$$

   c. The GMIB Base starts out equal to initial account balance and may grow at some prescribed rollup rate and/or ratchet in up markets

2. GMDB: Guaranteed Minimum Death Benefit
   a. Guarantees at least a return of premium to a beneficiary upon death of policyholder. The guaranteed premium may also grow at some prescribed rollup rate and/or ratchet in up markets.
   b. Payout to beneficiary upon death of policyholder:

$$\max(\text{Account Balance, GMDB})$$

3. GMAB: Guaranteed Minimum Accumulation Benefit
   a. Guarantees at least a return of premium to the policyholder if he lives to contract maturity (account balance is usually paid out to a beneficiary if death occurs before maturity). The guaranteed premium may also grow at some prescribed rollup rate and/or ratchet in up markets.
   b. Payout to policyholder at contract maturity:

$$\max(\text{Account Balance, GMAB})$$

4. GMWB: Guaranteed Minimum Withdrawal Benefit
   a. Guarantees a minimum periodic withdrawal, usually expressed as a percentage of some nominal base which equals account balance at inception but may also grow at some prescribed rollup rate and/or ratchet in up markets.
   b. This minimum withdrawal continues until the nominal base is depleted, in the case of a standard GMWB, or for the life of the policyholder, in the case of GLWB (guaranteed living withdrawal benefit), even if the account balance is depleted.
   c. Payout to policyholder:

$$\text{GMWB: } \max\left(\text{Account Balance, GMWB Base}_t \text{ x Withdrawal \%}\right)$$

$$\text{where GMWB Base}_t = \text{GMWB Base}_{t\text{-}1} - \sum_{i=1}^{t-1} \text{Withdrawal}_i$$

$$\text{GLWB: } \max\left(\text{Account Balance, GMWB Base x Withdrawal \%}\right)$$

where withdrawals continue until death

III. Product Design and Distribution

A. Designing a successful product
1. Must address needs of targeted customers
2. The most suitable distribution channels should be considered; alternatively, given the available distribution channels, what markets can be targeted
3. Availability of simple, flexible models for pricing, accounting, hedging, capital requirements
4. Experienced hedging team
5. Improved understanding of risks by senior management
6. Tax regime the product will be offered in
7. Coordination across legal, admin, actuarial and IT to put appropriate controls and processes in place

B. Modeling requirements
1. Pricing model needs to be stochastic
2. Policyholder behaviour model needs to recognize that policyholders will act differently when their guarantee is in or out of the money
3. Whether to differentiate price by age band and/or by fund should be assessed
4. Model should be able to accurately determine capital requirements

C. Ongoing product maintenance
1. Customer support training on product details
2. How frequently to reprice the guarantees needs to be decided prior to launch

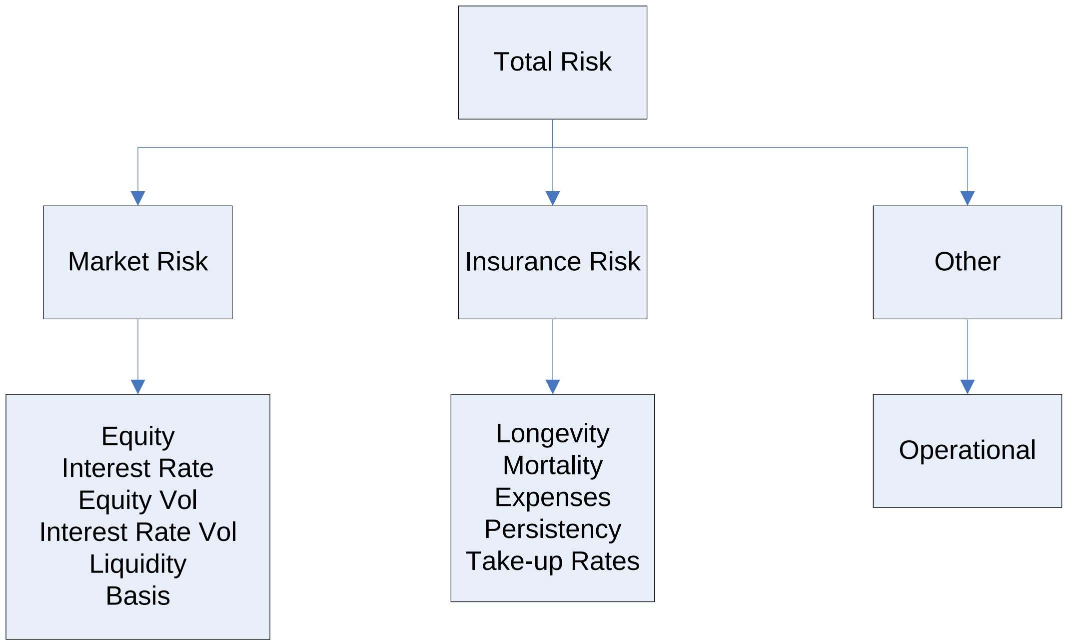
D. IFA views on a simplified product offering (results from research carried out by NMG)
1. <u>Awareness</u>: 60% have heard of VA's; 13% have a reasonable understanding
2. <u>Provider Awareness</u>: 41% of IFA's who were aware of VA's could not name a provider of VA's
3. <u>Sales Activity</u>: <10% have sold a VA; only 2% have sold more than one
4. <u>Structure</u>: 60% believe VA's should be offered in both life and pension environments
5. <u>Age Suitability</u>: 38% believed ages 60+ were best suited for VA's; 19% for ages 50+
6. <u>Offering to Clients</u>: 34% likely to offer a VA in the next 12 months
7. <u>Reasons to offer</u>: 31% stated clients like downside protection
8. <u>Reasons not to offer</u>: 45% concerned about lack of knowledge; 12% concerned about product complexity

E. Distribution
1. VA sales in the US are dominated by independent agents, career agents and banks

2. Financial planners, stock brokers and direct sales are more important to VA sales than they are to fixed annuity sales which is dominated by independent agents

IV. Managing Risks

The concept of economic capital (EC), as defined in Solvency II, is used to put the risks of VA's into context. Solvency II defines EC as a 1 in 200 year risk of default over a 1 year horizon (i.e. 99.5% probability of survival over 1 year). Risks under Solvency II are classified as follows (*note, the headings for Insurance Risk and Market Risk are incorrectly reversed in the syllabus*):



A. Market Risks
1. These risks are systematic and so can be hedged
2. Stresses can be derived from historical data or using market agreed calibrations

B. Insurance Risks
1. Mortality/Longevity – The latest projections from industry bodies is the best source for determining stress levels of mortality
2. Policyholder Behaviour – Depending on the financial sophistication of advisers and consumers, persistency and/or take-up rates can be correlated to market performance. This risk is currently unhedgable.

C. Risk Mitigation
1. Avoidance of risk through product features
   a. Caps on benefit levels
   b. Limit on funds and switching between funds
   c. Ratchets on benefits to discourage lapses
   d. Persistency carrots: return of fees, loyalty bonuses
2. Transfer of risk through reinsurance
   a. Removes risk but at a price. Introduces credit risk of reinsurer.
3. Reduction of risk through hedging

a. While possible for financial risks, this is currently not possible for risks such as basis risk, model error risk, assumptions risk

4. Retention of risk
   a. With risk comes reward but how much risk to retain depends on the risk capacity and appetite of the insurer

D. Risk Management (direct hedging of market risks is discussed in the next section; the risks listed here can fall out as a consequence of hedging – for example basis risk)
1. Lapse and mortality risk – usually modelled as a static assumption; the risk is of actual experience deviating from assumptions
2. Basis risk – mismatch between the performance of the underlying VA funds and the portfolio of hedge assets
3. Policyholder behaviour risk – the risk is of actual experience deviating from assumptions
4. Grouping risk – if policies are grouped for the purposes of hedging, mismatching can result
5. Non linearity risk – fewer policyholders lapses when markets are low and more volatile
6. Monitoring risk – the risk of allowing assumptions to stray too far from actual experience; regular and frequent monitoring of experience is vital
7. Liquidity risk – the availability to trade when needed may be an overly optimistic assumption

V. Hedging Techniques

A. Matching Greeks
1. A standard approach to hedging the market risks within VA's is to determine the sensitivity of guarantees (known as "the greeks") to various underlying risk factors and create a hedge portfolio with an equal sensitivity to that same risk factor. The suite of greeks typically calculated are as follows:

$V =$ option value or guarantee value

$S =$ equity price or price of underlying asset

$q =$ equity dividend

$\sigma =$ equity volatility

$$Delta = S \cdot \frac{\partial V}{\partial S} = \frac{\partial V}{\partial S / S};\ Gamma = S^2 \cdot \frac{\partial^2 V}{\partial S^2} = \frac{\partial^2 V}{\partial S^2 / S^2};\ Lambda = \frac{\partial V}{\partial q}$$

$$Vega = \frac{\partial V}{\partial \sigma};\ Volga = \frac{\partial^2 V}{\partial \sigma^2};\ Vanna = \frac{\partial^2 V}{\partial S / S \cdot \partial \sigma}$$

$$Rho = \frac{\partial V}{\partial r}$$

2. Deciding which greeks to hedge depends on what a life insurer wishes to target (P&L level, P&L volatility, regulatory capital, economic capital) and how much risk they are willing to accept
3. Other greeks can be calculated for informational purposes: for example, change in liability with respect to changes in policyholder behaviour
4. Just delta hedging introduces some limitations because, by definition, delta only covers the linear change in the liability whereas the actual change is not linear and exhibits convexity (gamma). For small market movements delta hedging is sufficient but not so for large movements, particularly if the market is moving faster than one is able to rebalance.

B. Trading
1. Rebalancing of asset greeks to liability greeks does happen continuously as this would be impractical and costly
2. A threshold is typically set whereby no rebalancing trades are made until the market moves sufficiently far to breach this threshold. Where to set this threshold depends on balancing the unhedged exposure between trades with the cost of trading more frequently.

C. Hedging Mortality/Longevity Risk
1. Longevity in annuities can be naturally offset with life insurance
2. Reinsurance is an alternative route
3. The market for longevity bonds and swaps continues to evolve and grow

D. Hedging Policyholder Options
1. Where a policyholder has an option to exercise early, American style or Bermudan style options can be used to hedge this exposure
2. Most policyholder behaviour models exhibit a similar relationship between lapses and in-the-moneyness. The more in-the-money the guarantee is (when markets fall), the less likely a policyholder is to lapse.
3. The big question is whether actual evidence supports this correlation with financial markets.
   a. A European study showed little correlation of lapses with financial markets and the biggest correlation with market sentiment about the company itself.
   b. A fully market correlated model can be viewed as a worst case scenario that assumes all policyholder act efficiently and rationally. An alternative approach known as “dice-rolling” models uses historical experience that captures other reasons for lapses such as liquidity needs, or churning by advisers.
4. A workable definition of ratinonality must take account of relevant information without allowing policyholders a crystal ball unavailable to the rest of the market. In other words, assuming policyholder are omniscient is unrealistic and overly conservative.
5. Because of the inverse relationship between markets and policyholder behaviour, it is important that pricing be done stochastically rather than deterministically. In a down market, higher persistency means more claims **and** more fees (although the lower market means a smaller base against which to charge fees) and vice versa in an up market.
   Note, the authors ramble on for 2 pages on this topic in a very confusing manner which I would suggest not spending too much time on.

VI. Economic Capital

A. EC should be determined with without hedging using a full economic balance sheet
B. A useful exercise is to determine the capital requirements for just those risks being hedged to investigate the adequacy of a hedging program under extreme market conditions
C. Quite often a delta and rho hedging strategy will not materially reduce EC because of the nonlinearity of the liability previously mentioned. One solution is to add derivatives that provide offsetting nonlinearity such as equity options or swaptions.
D. The sensitivity of EC to certain underlying risk factors is influenced by the term of the underlying contracts. Contracts near maturity will have a different risk profile than one with 20 years still to go.

VII. Using Risk Geographies to find the Least Solvent Likely Event (LSLE)

A. LSLE is defined as the most onerous event (a combined stress test) at a given level of probability
B. Risk Geographies is a technique used to determine the LSLE for issues of VA contracts. When the probability is set equal to 99.5% the LSLE can be used to define EC.
    1. A traditional correlation matrix approach will not capture the affect of non-linearity when combining capital requirements from different risk drivers
    2. A common approach to allow for non-linearity where diversification is present is to reduce the level of confidence of each individual stress test such that the sum of the individual stress tests, when applied simultaneously, equals the diversified capital amount (referred to by the author as "the medium bang approach")
    An obvious question left unanswered by the authors' is why you would use this approach if you already know the diversified capital amount to begin with.
    3. A problem with this approach is that it implicitly makes allowance for the level of diversification around the individual base stresses rather than around the actual combined-stressed scenario of interest. Thus it overestimates (underestimates) non-linearity, and thus capital, if there is greater (less) diversification around the actual combined-stressed scenario.
    4. The technique is iterative:
        a. Determine the impact to net asset value for all combinations of a pair of risk factors (for example, interest rates and equities)
        b. Lay a joint probability density function over the potential outcomes
        c. Determine the intersection of the first graph with the second at the desired level of probability
        d. Continue to pair off risks and rerun steps a through c
        e. This will eventually converge at the point most onerous and likely for the life insurer

VIII. Cost of Hedging

A. As mentioned earlier, due to trading friction costs trading is not done continuously. Thus, companies need to consider the following:
    1. frequency of rebalancing
    2. optimal position limits to set for the hedge
    3. allowance to make pricing for cost of capital and cost of hedging

B. The authors perform an analysis and conclude that it is optimal to not rebalance back to zero (i.e. delta neutral) when trading limits are breached. This is because transaction costs are cheaper if you only rebalance to the edge of the limit, i.e. it is more cost effective to not rebalance back to zero.

C. The trade off, as illustrated graphically on page 37, is between cost of hedging and cost of capital. The more you trade the higher the cost of hedging but the lower the cost of capital. The optimization formula shown on page 37 attempts to identify how large to set position trading limits to minimize the combination of these two costs.

Practice Problem 13
Source: Options, Futures, and Other Derivatives, Chapter 21

(6 Points)

You are given the following information regarding two capital market securities, which is reflective of the conditions as of the close of business yesterday:

Security	Daily Volatility	Closing Price
A	1.50%	25
B	1.80%	30

As of the close of business today, the security prices were as follows:

Security	Closing Price
A	26
B	31

The correlation between their returns as of the close of business yesterday was 0.60.

Using this information:

(a) Assuming a EWMA model with parameter $\lambda = .92$:

(i) Compute the estimated covariance as of the end of yesterday

(ii) Compute the updated correlation estimate as of today's close

(b) Assuming a GARCH(1,1) model with $\alpha = .03, \beta = .95$, and $\omega = .000002$ for both the correlation and volatilities:

(i) Compute the updated correlation estimate as of today's close

Solution to Practice Problem 13

Points	Statement
	(A) EWMA
1	$\sigma_{A,n-1} = 1.5\%, \sigma_{B,n-1} = 1.8\%, A_{i-1} = 25, B_{i-1} = 30, \rho = .60, \lambda = .92$
3	$\rho = \dfrac{\text{cov}_{n-1}}{\sigma_{A,n-1}\sigma_{B,n-1}} \Rightarrow \text{cov}_{n-1} = \rho\sigma_{A,n-1}\sigma_{B,n-1} = .60(.015)(.018) = .000162$
2	$a_{n-1} = \dfrac{26-25}{25} = \dfrac{1}{25} = .04, b_{n-1} = \dfrac{31-30}{30} = \dfrac{1}{30}$
4	$\text{cov}_n = \lambda \text{cov}_{n-1} + (1-\lambda)a_{n-1}b_{n-1} =$ $.92(.000162) + (1-.92)\left(\dfrac{1}{25}\right)\left(\dfrac{1}{30}\right) = .000255707$
4	$\sigma^2_{A,n} = \lambda\sigma^2_{A,n-1} + (1-\lambda)a^2_{n-1} = (.92)(.015)^2 + (1-.92)\left(\dfrac{1}{25}\right)^2 = .000335$ $\sigma_{A,n} = .01830301$
1	$\sigma^2_{B,n} = \lambda\sigma^2_{B,n-1} + (1-\lambda)b^2_{n-1} = (.92)(.018)^2 + (1-.92)\left(\dfrac{1}{30}\right)^2 = .00038697$ $\sigma_{B,n} = .01967152$
1	$\rho = \dfrac{\text{cov}_n}{\sigma_{A,n}\sigma_{B,n}} = \dfrac{.000255707}{(.01830301)(.01967152)} = .71020156$
	(B) GARCH (1,1)
1	$\sigma_{A,n-1} = .015, \sigma_{B,n-1} = .018, A_{n-1} = 25, B_{n-1} = 30, \alpha = .03, \beta = .95,$ $\omega = .000002(\textit{correlation}), \omega = .000002(\textit{volatilities}), X_n = 26, Y_n = 31$
5	$\sigma^2_{A,n} = \omega + \alpha u^2_{A,n-1} + \beta\sigma^2_{A,n-1} = .000002 + (.03)\left(\dfrac{1}{25}\right)^2 + (.95)(.015)^2 = .0002638$ $\sigma_{A,n} = .0162404$
1	$\sigma^2_{B,n} = \omega + \alpha u^2_{B,n-1} + \beta\sigma^2_{B,n-1} = .000002 + (.03)\left(\dfrac{1}{30}\right)^2 + (.95)(.018)^2 = .0003431$ $\sigma_{y,n} = .0185239$
4	$\text{cov}_n = \omega + \alpha x_{n-1}y_{n-1} + \beta\,\text{cov}_{n-1} = .000002 + .03\left(\dfrac{1}{25}\right)\left(\dfrac{1}{30}\right) + .95(.000162)$ $= .0001959$
1	New correlation estimate = $\dfrac{.0001959}{(.0162404)(.0185239)} = .651189$
28	**TOTAL POINTS**