Preface

Exam MFE is based on material from chapters 9-14, 18-24 and Appendices B1 and C of the text *Derivatives Markets* by Robert L. McDonald.

There is prerequisite material that must be mastered for these chapters. Exam FM/2 requires the study of material from Chapters 1,2,3,4,5 and 8, and this material must be learned first. **This study guide does not review the prerequisite material*.** The *ACTEX Introduction to Financial Economics* is a separate study guide for prerequisite review. Students who have already had a course on derivative securities will not need this review, but any student who is new to the material will need to cover the FM material in some way.

We will be providing for each chapter of the required text:

1) A preliminary set of lecture notes on the chapter
2) Solutions to odd numbered problems
3) Additional computational review and practice multiple choice problems. (For this material, Chapters 10-11 and 12-13 are treated as a single block of material with problems at the end of the last chapter in the block.)
4) Solutions to relevant problems for the Spring, 2007 exam MFE.

Note that there is a student solutions manual for the text which contains the solutions to all even numbered problems.

This guide also contains seven more practice final exams.

Our recommended procedure for use of this guide is

1) First, read the ACTEX lecture notes
2) Then, read the relevant chapter of the text carefully
3) Work the end-of-chapter problems
4) After you have studied all the chapters, begin working on the practice exam problems.

Pay careful attention to 2). The lecture notes are designed to make the text more understandable, but they do not replace the readings in the textbook. There is only one official textbook for this examination, and the exam writers will be looking at it as they design questions.

A note on rounding. The text *Derivatives Markets* does calculations to the full precision of its computer software. When explaining that text or doing book problems and basic calculation problems, we have done the same thing using

* There is one prerequisite item that is included in this manual because it was needed to work one of the Sample Exam questions released by the Societies. This prerequisite material begins on the next page.

EXCEL. On the exam when you do problems involving the normal distribution, you will work from a normal table with two place z-values and four place probabilities. We have switched to this mode for sample exam problems, even though it can be substantially less accurate. If you use the slower and less accurate table method on book problems, you will most probably not match our answer or the answer in the textbook.

A Note About Errors:

If you find a possible error in this manual, please let us know at the "Customer Feedback" link on the ACTEX homepage (www.actexmadriver.com).

We will review all comments and respond to you with an answer. Any confirmed errata will be posted on the ACTEX website under the "Errata & Updates" link.

A Note On Updates:

Be sure to check the actuarial society websites for additional information. Occasionally exam updates or new sample questions are posted after this guide is published.

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Prerequisite Material Review

Sample Exam Question 3

Overview

We will not review prerequisite material that should only be tested on Exam FM/2. However there is one prerequisite item that was included in a sample question released for MFE, so we will review that here.

Sample Exam question 3 is based on the reasoning similar to that used in section 2.6 of *Derivatives Markets*. The section begins on page 48 and is titled "Example Equity-Linked CDs".

We will begin here by giving you our explanation of this section from the Exam FM/2 study guide.

Example

Equity-Linked CD

In Section 2.6 of *Derivatives Markets*, there is a nice example of how to use call options to create a CD whose return is linked to an index like the S&P 500. To understand why an investor would like such a product, we need to review some basic finance.

The long term return on the S&P 500 is higher than the long term return on safer investments like government bonds. Ibbotson Associates, Inc. reported in their *1998 Yearbook* that from 1927 to 1996 the average annual nominal return was 13% for the S&P 500 and only 5.6% for government bonds.

You can see why an investor would rather have the long run S&P return. However investing in the S&P 500 can be problematic for an investor such as a retiree who needs the return on investment now to pay the bills. The long term return on the S&P 500 results from a long period in which there were some very good years and some very bad years. The retiree does not want be in a decade that has too many bad years. He wants to get something like the S&P return if that return is positive, but not lose his money in the bad years. That is he wants the **option** to get positive return when that is available but not be obliged to accept negative return.

The equity-linked CD described in the text meets this need. It is originally structured when the S&P 500 index is at a current value of 1300. The investor will invest 10,000 and be paid in 5.5 years. The final payment depends on whether the S&P 500 index is above or below 1300 in 5.5 years, as the following table indicates.

Index in 5.5 years	Payoff
Index ≤ 1300	10,000
Index> 1300	$10{,}000(1+.7(\text{percentage gain on index}))$

In other words –if the index goes down, the investor gets his money back, but if the index goes up, he gets the money back and an additional return equal to 70% of the percentage gain on the S&P 500.

Suppose, for example, that the S&P index in 5.5 years has gone up by 40% to 1820. Then the investor will be paid

$$10{,}000(1+.7(.40)) = 10{,}000(1.28) = 12{,}800$$ [1]

The text notes that if S_{final} is used to represent the S&P index in 5.5 years, the percentage gain is $\frac{S_{final}}{1300} - 1$.

Thus the CD pays $10{,}000\left(1+.7\left[\max\left(0, \frac{S_{final}}{1300} - 1\right)\right]\right)$.

We will write this in a different way to give you a different look at it. We will write the percentage gain on the S&P 500 index as

$$\frac{1}{1300}(S_{final} - 1300).$$

Then we write the payment on the CD in 5.5 years as

$$10{,}000 + .7\frac{10{,}000}{1300}\max(0, S_{final} - 1300) = 10{,}000 + 5.3846\max(0, S_{final} - 1300)$$

The final two terms represent
- i) A payoff of the original 10,000, plus
- ii) 5.3846 European call options on the S&P 500 index with strike price of 1300 and expiration in 5.5 years..

Thus what the CD really gives the investor is
Return of Original Amount + Payoff of 5.3846 S&P 1300 strike call options

The first component lets the investor get his money back, and the second links his return to the S&P 500 index.

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[1] The student of interest theory will note that the interest rate per semiannual period here is 2.27%.

Note that an investor does not have to get this payoff from a bank CD. *Derivatives Markets* points out that the investor could buy a zero coupon bond paying 10,000[2] in 5.5 years and also buy the call options on his own. However this is all too complicated for most investors to do. The service the bank provides with this CD is to do all that investing and buying for the investor.

You may do this as an actuary. One of my previous employers had an equity linked annuity that promised a return on investment linked to an index, and it was structured using options. It was designed by one of the actuaries.

The text also points out that this CD does have a cost. If, for example, the investor had an interest rate of 6% per year, a deposit of 10,000 would grow to

$$10{,}000(1.06)^{5.5} = 13{,}777.88$$

Thus he would have earned 3,777.88 in interest, and he has forgone that interest to get the S&P linked return. The present value of that interest accumulation is

$$\frac{3{,}777.88}{1.06^{5.5}} = 2{,}741.99 .$$

Thus as he invests today he has foregone interest with a present value of 2,741.99, and that is the implied cost today of investing in the CD.

Sample Exam Question 3

Sample examination question 3 is given below. It is not a multiple choice question.

An insurance company sells single premium deferred annuity contracts with return linked to a stock index, the time-t value of one unit of which is denoted by $S(t)$.

The contracts offer a minimum guarantee return rate of $g\%$. At time 0, a single premium of amount π is paid by the policyholder, and $\pi \times y\%$ is deducted by the insurance company.

Thus, at the contract maturity date, T, the insurance company will pay the policyholder $\pi \times (1 - y\%) \times \text{Max}[S(T)/S(0), (1 + g\%)^T]$.

[2] The text says this a different way, saying that the investor will buy 7.69 units of a package for which a single unit consists of a zero coupon bond for 1300 and .7 of an index call option.

You are given the following information:

(i) The contract will mature in one year.
(ii) The minimum guarantee rate of return is $g\%$ = 3%.
(iii) Dividends are incorporated in the stock index. That is, the stock index is constructed with all stock dividends reinvested.
(iv) $S(0)$ =100.
(v) The price of a one-year European put option, with strike price of $103, on the stock index is $15.21.

Determine $y\%$, so that the insurance company does not make or lose money on this contract.

Sample Exam Question 3 Solution

The idea here is to see that the amount paid to the policyholder is equivalent to an expression based on the total payoff of a share in the index combined with a put. Using $g=.03, T=1, S_0=100$, the total payoff is

$$\pi(1-y)\max\left(\frac{S_1}{100},1.03\right)=\frac{\pi}{100}(1-y)\max(S_1,103)=\frac{\pi}{100}(1-y)\left[S_1+\max(103-S_1,0)\right]$$

The expression in square brackets is the payoff of a single share of the index and a put, while the two lead terms give the number of units of this combination the company needs to buy to pay off the single premium deferred annuity. The company wants to use the premium π to buy the shares and the options needed. The cost of those shares[3] and options today is

$$\frac{\pi}{100}(1-y)[S_0+\text{put cost}]=\frac{\pi}{100}(1-y)115.21$$

To break even this cost must equal the premium collected.

$$\frac{\pi}{100}(1-y)115.21=\pi\rightarrow y=.13202$$

The required percentage is 13.202%.

Note: This contract could also be funded using a 103-strike call option and a zero coupon bond paying 103. Recall from chapter 2 that

Payoff[Index + K-strike Put] = Payoff[K-strike Call + 0-coupon bond for K]

In this problem it is easier to use the put and the index, since the put price is given directly.

[3] The study note solution points out that the phase "dividends are incorporated in the stock price" means that you can assume $\delta=0$ and buy a share for S_0 without worrying about a tailed position.

Practice Problem

An insurance company sells single premium deferred annuity contracts with return linked to a stock index, the time-t value of one unit of which is denoted by $S(t)$.

The contracts offer a minimum guarantee return rate of *2%*. At time 0, a single premium of amount 10,000 is paid by the policyholder, and $10{,}000 \times y\%$ is deducted by the insurance company.

In one year, when T=1, the insurance company will pay the policyholder

$10{,}000 \times (1 - y\%) \times \text{Max}[S(1)/S(0), (1 + g\%)^1]$., where $S(0) = 100$

You are given the following information:

(i) Dividends are incorporated in the stock index. That is, the stock index is constructed with all stock dividends reinvested.

(ii) The price of a one-year European put option, with strike price of $102, on the stock index is $16.45.

Determine $y\%$, so that the insurance company does not make or lose money on this contract.

Answer 14.126%

Table of Contents

MFE Module 1

Review of *Derivatives Markets*, Chapter 9

Section 1.1

Expressions for the Present Value of Interest Earned

The expressions introduced here are used often in this chapter.

Suppose that you invest an amount K at an effective interest rate of r per period. Then the interest earned over that period will be Kr.

The textbook *Derivatives Markets* uses the expression $K - PV(K)$ in this chapter. Note that

(1.1) $$K - PV(K) = K - \frac{K}{1+r} = \frac{Kr}{1+r} = \text{Present value of interest earned}$$

If you are using continuous interest at a rate r, a dollar invested at time 0 grows to e^{rt} at time t. Thus the interest earned on the dollar over (0, t) is $e^{rt} - 1$.

The present value of this amount at time 0 is

(1.2) $$e^{-rt}\left(e^{rt} - 1\right) = 1 - e^{-rt} = \text{Present value of interest earned}$$

Section 1.2

Put-Call Parity for European Options

This has already been introduced in Chapter 3 of *Derivatives Markets*. The topic is very important, and is explored further here. Chapter 3 is a prerequisite for Exam MFE. We will briefly review some key observations from Chapter 3.

a) You can create a synthetic long forward with forward price K by buying a call with exercise price K and selling a put with exercise price K.
b) The net cost of this synthetic forward is the difference of the call and put prices.

   Net cost of synthetic forward = $C(K,T)-P(K,T)$

c) You can synthetically create a stock using the relationship

   Stock = long forward + zero coupon bond

   If you have a long forward contract and a zero coupon bond today, this is equivalent to having a tailed position in the stock and you will have one full share of the stock at expiration.

d) The cost equation corresponding to the word equation above is the put-call parity equation for European options.

   $PV(F_{0,T}) = C(K,T)-P(K,T)+PV(K)$

The text rewrites the above equation as

(1.3)

$$C(K,T)-P(K,T)=e^{-rT}(F_{0,T}-K) \qquad \text{[DM 9.1]}$$

Since $F_{0,T}^{P}=e^{-rT}F_{0,T}$ we could also rewrite this as

(1.4)

$$C(K,T)-P(K,T)=F_{0,T}^{P}-e^{-rT}K$$

Section 1.3

Options on Stocks

On page 283, *Derivatives Markets* has a section on options on stocks. Recall that for a stock

$F_{0,T}^{P} = S_0$ if there are no dividends

$F_{0,T}^{P} = S_0 - PV_{0,T}(Dividends)$ if there are discrete dividends

$F_{0,T}^{P} = S_0 e^{-\delta T}$ if dividends are paid at the continuous rate δ.

The text uses this to create various new versions of equation **(1.3)**, but these really do not all need to be memorized. If you know **(1.4)**, you can always come up with the alternate versions if you understand $F_{0,T}^{P}$.One version is crucial for understanding Example 9.1 in the text. When there are no dividends, the parity equation is

$$C(K,T) - P(K,T) = S_0 - e^{-rT}K$$

Example 9.1 looks at an instance of the special case where $K = S_0$. We will summarize that in slightly more general terms. When $K = S_0$ we have

$$C(K,T) - P(K,T) = S_0 - e^{-rT}S_0 = S_0\left(1 - e^{-rT}\right).$$

In words this says

Difference in put and call prices = Present value of interest on S_0

Note that if you buy the stock immediately you lose the interest you would have earned on the money used to pay for it. If you do a synthetic forward purchase by buying a call and selling a put you get the stock later but do not lose interest since you did not pay in advance. The text makes the important observation that "*The option premiums differ by interest on the deferral of payment for the stock*." Example 9.1 is designed to make you infer this from a concrete example.

Example 9.2 is designed to demonstrate the difference in put and call prices when $K = S_0$ and there are dividends. In words.

Difference in call and put prices
= Present value of interest on S_0 - PV(Dividends)

Section 1.4

Synthetic Stock

This section re-visits material on synthetic securities that was covered in Chapter 3. If we look at the more general situation where there are dividends and $F_{0,T}^{P} = S_0 - PV_{0,T}(Dividends)$, then the parity equation becomes

(1.5) $$C(K,T) - P(K,T) = S_0 - PV_{0,T}(Dividends) - e^{-rT}K.$$

The terms here can be rearranged to give the pricing of various synthetic securities. Concrete examples are given in Examples 9.3 and 9.4 (we will not repeat these examples). The key point is that it is not necessary to memorize a large collection of equations. Everything follows from Equation **(1.4)**, knowing $F_{0,T}^{P}$ and applying the basic principles of synthetics learned in Chapter 3.

Problem 9.3 of the text asks you to show that there is an arbitrage in a specific case where parity is violated. In that problem the arbitrage involves making use of a synthetic short forward sale to exploit an incorrect price. The problem is worth careful study as it illustrates how to make use of a synthetic for arbitrage.

Section 1.5

Options on Currencies

Note that this section depends on prerequisite material in section 5.6 of *Derivatives Markets*. That material is not included in the Exam FM/2 syllabus.

The text gives the example of an option to buy euros using dollars. We will outline the general conversion option formula and then return to that example. More general thinking will be required in the homework problems, since the options there are not always dollar denominated.

Suppose that you wish to use currency of country A to buy the currency of country B[1]. Let

r_A = interest rate for country A; r_B = country B interest rate
x_0 = exchange rate for currency A/currency B.

Then the price of a prepaid forward denominated in currency A to buy one unit of the currency of country B at time T is

$$F_{0,T}^P = x_0 e^{-r_B T}.$$

The forward price denominated in currency A to buy one unit of the currency of country B at time T is the future value of this amount in country A.

$$F_{0,T} = x_0 e^{(r_A - r_B)T}.$$

Using the prepaid forward value above, the general put-call parity equation becomes the equation for options on a single unit of a currency.

(1.6) $$C(K,T) - P(K,T) = x_0 e^{-r_B T} - e^{-r_A T} K$$

In Example 9.5, the text illustrates currency option parity for options to buy or sell one euro in one year. The equation above becomes equation (9.4) of *Derivatives Markets*.

(1.7) $$C(K,T) - P(K,T) = x_0 e^{-r_{euro} T} - e^{-r_{\$} T} K \qquad \text{[DM 9.4]}$$

Example 9.5 of *Derivatives Markets* illustrates the straightforward application of this parity equation.

[1] Country B can be thought of as the foreign country in this transaction. Then you could use r to denote the interest rate for country A and r_f to denote the interest rate of the foreign country B.

Section 1.6

Options on Bonds

You can also buy an option on a bond which would enable you to call or put a bond for a strike price K. Instead of dividends, bonds pay coupons. The parity relationship for bonds looks like the relationship for stocks with the present value of dividends replaced by the present value of coupons. If B_0 is the bond price today, the parity equation is given by equation 9.5 of *Derivatives Markets*.

(1.8) $$C(K,T) - P(K,T) = B_0 - PV_{0,T}(Coupons) - e^{-rT}K \qquad \text{[DM 9.5]}$$

Section 1.7

Exchange Options and Generalized Parity

When I exercise an option to buy stock for $40, I am actually exchanging dollars for stock. Options can be constructed that exchange items of value that are not dollars.

For example, one of my friends is a homebuilder who owns some land for building. He could arrange a call option that allows him to get another builder's land parcel in exchange for his land. His call option is denominated in land instead of dollars. This type of option is called an **exchange option.** This is a simple idea, but the notation for it is fairly complicated.

Derivatives Markets looks at a general situation involving two assets A and B. It will help you if you think of the example where asset B is my friend's land and asset A is the land he can receive in exchange.

The text uses the following notation. With the familiar calls on stocks, we had a strike price of $$K$ and paid in dollars. Now my friend has a strike asset in asset B (his land), and will pay asset B to get asset A (the other piece of land).

Asset A has price S_t
Asset B has price Q_t

$F_{t,T}^P(S)$: Price of prepaid forward on A at t paying S_T at T.
$F_{t,T}^P(Q)$: Price of prepaid forward on B at t paying Q_T at T.
$C(S_t, Q_t, T-t)$ = price of an option with T-t to expiration to give B to get A
$P(S_t, Q_t, T-t)$ =price of an option with T-t to expiration to give A to get B

With this notation, the call and put payoffs at time T are given by

$$C(S_t, Q_t, 0) = \max(0, S_T - Q_T)$$

$$P(S_t, Q_t, 0) = \max(0, Q_T - S_T).$$

For European options the generalized parity equation is

(1.9) $$C(S_t, Q_t, T-t) - P(S_t, Q_t, T-t) = F_{t,T}^P(S) - F_{t,T}^P(Q) \qquad \text{[DM 9.6]}$$

Note the similarity to the stock equation

$$C(K,T) - P(K,T) = S_0 - e^{-rT}K$$

For a stock the strike asset is a dollar amount, and the right hand side of the equation is

Prepaid Forward Value for stock – Present value of the strike asset.

Equation **(1.9)** (9.6 of the text) has the same form.

Section 1.8

What are Calls and Puts?

On page 288, *Derivatives Markets* gives an example in which the two assets involved are Microsoft stock and Google stock. The text makes the point that an exchange that looks like a call to one person may look like a put to another, and states that: "*The distinction between a put and a call in this example depends on what we label the underlying asset and what we label as the strike asset.*"

This idea is pursued in the following section entitled **"What are Calls and Puts?"** There the text discusses familiar options on stocks and shows how they might be looked as puts or calls.

Consider, for example, a call option to buy a stock for K = \$40. The underlying asset is the stock, and the strike asset is dollars. When we think of this as a call, we think of exchanging \$40 for the stock when the price of the stock is $S > 40$. However we could also think of this as a put option where the strike asset is the stock and the underlying asset is \$40. In this case we would sell 40 to the counterparty in exchange for the stock because the underlying asset is worth $40 < S$.

The discussion seems to be purely academic up to this point, but the text immediately gives an example in a section on **currency options** where all of this has a practical consequence.

Section 1.9

Currency Options

In this section on page 290, *Derivatives Markets* looks at options involving euros and dollars.

Some terminology is needed here. The text states that an option is **dollar denominated** if the strike price and the premium are denominated in dollars and **euro denominated** if the strike price and the premium are denominated in euros.

They proceed to give an example that shows that a dollar denominated call on euros looks very much like a euro denominated put on dollars (This is done in an example on page 290.) The example looks at a time when the current exchange rate was

$$x_0 = \frac{.90 dollars}{1 euro}$$

The exchange rate in one year is denoted by x_1.

Two one-year options are studied:

a) A one year dollar denominated European call option on euros with strike price of $K = .92$ This will have payoff

$$\max(0, x_1 - .92)$$

b) A corresponding one year euro denominated put with a strike price of

$$\frac{1}{K} = \frac{1}{.92} = 1.086957$$

This put will be exercised when the value of the euro is less than the strike. The payoff is

$$\max\left(0, \frac{1}{.92} - \frac{1}{x_1}\right) = \max\left(0, \frac{x_1 - .92}{.92x_1}\right) = \frac{1}{.92x_1}\max(0, x_1 - .92)$$

Note that both the put and the call will have positive payoff if and only if $x_1 > .92$, and that the payoff amounts differ by only an adjusting factor. The text gives a formula relating dollar denominated call and put prices determined in this way on page 292.

We give a slightly more general formula here, since there are 4 currency problems in the exercises and some of them are not dollar denominated.

Relation of call denominated in currency A to buy currency B and put denominated in B to sell A:

$$C_A\left(x_0,\,K,\,T\right)=x_0\,K\,P_B\left(\frac{1}{x_0},\,\frac{1}{K},\,T\right)$$

In the example above we would have

$$C_\$\left(.90,.92,1\right)=.90\left(.92\right)P_{euro}\left(\frac{1}{.90},\frac{1}{.92},1\right)$$

Section 1.10

European vs. American Options

Suppose that you have an American option and a European option created at the same time for the same underlying asset with identical values of K and T.

Then the American option will enable you to do anything that you could do with the European option, but has the added feature of possible early exercise. Thus the American option must have value greater than or equal to the value of the European option due to the additional feature.

$$C_{Amer}(S,K,T) \geq C_{Eur}(S,K,T) \qquad P_{Amer}(S,K,T) \geq P_{Eur}(S,K,T)$$

Section 1.11

Upper and Lower Bounds for Option Prices

Recall that the parity relationship for a European option is

$$C(K,T) - P(K,T) = PV(F_{0,T} - K) = F_{0,T}^{P} - e^{-rT}K$$

This implies that

$$C(K,T) \geq PV(F_{0,T} - K) = F_{0,T}^{P} - e^{-rT}K$$

The call also can never have a negative value. This will give us a lower bound for the value of a call option. In addition, the call option can never be worth more than the price of the stock that you would get by exercising it. This gives us equation 9.9 of *Derivatives Markets*.

(1.10)

$$S \geq C_{Amer}(S,K,T) \geq C_{Eur}(S,K,T) \geq \max\left[0, \mathrm{PV}_{0,T}(F_{0,T}) - \mathrm{PV}_{0,T}(K)\right] \qquad \text{[DM 9.9]}$$

A put can never have value greater than K, the amount you would get by exercising it. Using this fact and similar parity equation reasoning, we get equation 9.10 of *Derivatives Markets*.

(1.11)

$$K \geq P_{Amer}(S,K,T) \geq P_{Eur}(S,K,T) \geq \max\left[0, \mathrm{PV}_{0,T}(K) - \mathrm{PV}_{0,T}(F_{0,T})\right] \qquad \text{[DM 9.10]}$$

Section 1.12

Should an American Option be Exercised Early?

You can do three things with an American call option on a stock at any point in time:

1. Hold onto it for the future
2. Exercise it for $S_t - K$
3. Sell it in the marketplace for its value $C_{Amer}(S,K,T)$.

It would make no sense to exercise it early if $C_{Amer}(S,K,T) \geq S_t - K$. We now look at early exercise separately for puts and calls on stocks with and without dividends.

Calls on Stocks Which do not Pay Dividends

The text proves on page 294 that for stocks without dividends,

$C_{Amer}(S,K,T) \geq S_t - K$.

An American call on a stock with no dividends should never be exercised early.

Calls on Stocks Which Pay Dividends

There may be circumstances in which early exercise makes sense. The text gives the common sense example of a company whose stock is selling for 100 per share and intends to liquidate by paying out a dividend of 99.99 per share. If you have a call with a strike price of 90, it makes sense to call in the stock for 90 immediately before the dividend, get the dividend and pocket 9.99. After the dividend the call would be worthless.

The text mentions that the key issues in deciding on early exercise are the interest you could earn on the strike price *K* if you did not exercise and the present value of the dividends that could be obtained by buying the stock. **Exercise of an American call at time *t* never makes sense for a stock which pays dividends if**

Present value of interest on $K >$ Present value of dividends;

i.e., $K - PV_{t,T}(K) > PV_{t,T}(Div)$

If $K - PV_{t,T}(K) \leq PV_{t,T}(Div)$, you cannot rule out early exercise and need more analysis.

Puts on any Stock

You cannot rule out early exercise of an American put, since it may make sense to get the strike price *K* early so as to earn interest on it. The text gives the example of a firm that has gone bankrupt and had its stock price go to 0. Whenever you exercise the put you will get *K*, so it is better to get *K* now and reinvest it to earn interest.

Section 1.13

Time to Expiration

For otherwise identical American calls or puts, the option with less time to expiration has value less than or equal to the option with more time to expiration, since the longer term option allows you to do anything you could do with the shorter term option –and more.

American Options and Time to Expiration:

$$T_2 > T_1 \rightarrow C_{Amer}(S,K,T_2) \geq C_{Amer}(S,K,T_1) \text{ and } P_{Amer}(S,K,T_2) \geq P_{Amer}(S,K,T_1)$$

For European options the situation is more complex.

When the stock does not pay a dividend, the European call and the American call have the same price and additional time to maturity increases the value of the call.

With dividends, a European call with less time to maturity can have more value than a longer term call if there is need to capture a dividend earlier. The text goes back to the example of a company that is going to issue a liquidating dividend in 2 weeks. A European call with one week to maturity can capture that dividend, but a European call with 3 weeks to maturity will have no value – since the company will be gone when it can be exercised.

With dividends, a European put with less time to maturity can have more value than a longer term put if there is need to capture the strike price K earlier.

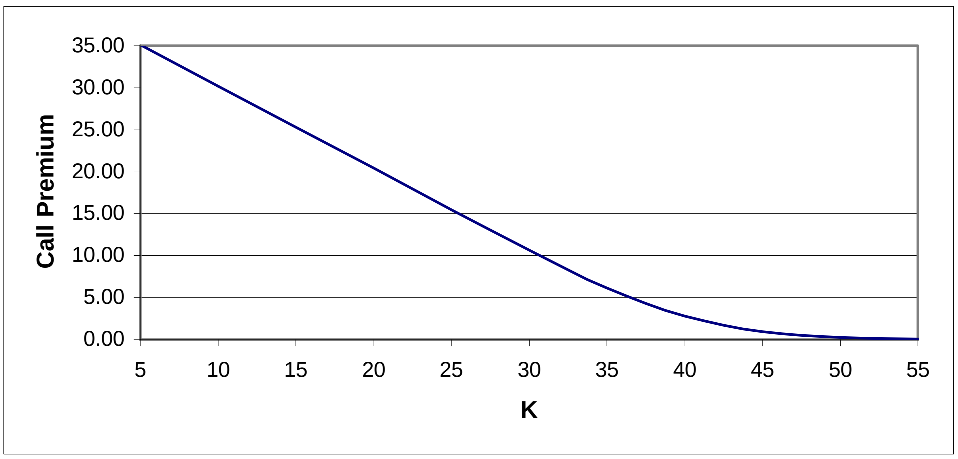
The text goes back to the example of a company that is bankrupt and has zero stock value. A European put can be exercised for K. A European put with one week to maturity can capture that sum sooner than a European put with 3 weeks to maturity and provide additional interest income.

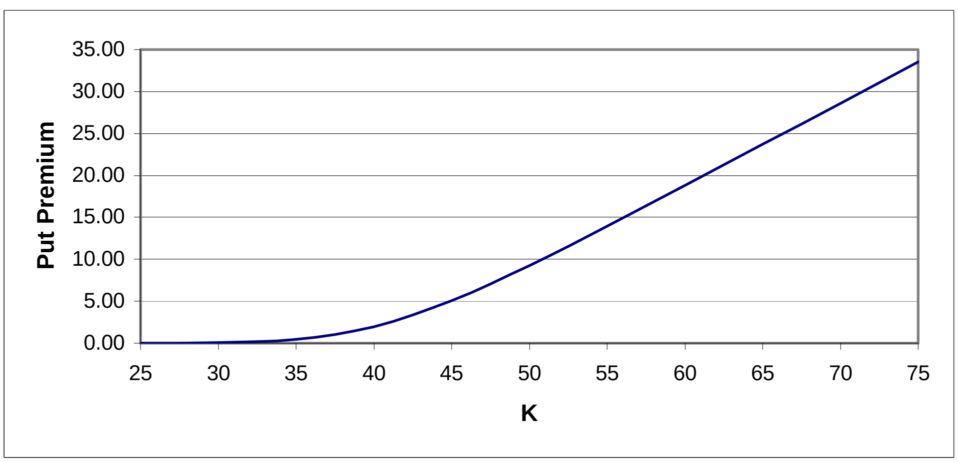
Section 1.14

The Effect of Different Strike Prices on the Values of Calls and Puts

Common sense tells us that a call with strike price 40 is worth more than an otherwise identical call with strike price 50. The final section of Chapter 9 of *Derivatives Markets* gives some relations that must hold for calls and their strike prices.

To give you a picture of how the call price changes with the strike price, we will provide graphs of call price versus strike price and put price versus strike price for a fixed value of the stock. (These were generated in a spreadsheet we created to implement the Black-Scholes pricing model which will be covered in Chapter 12 of *Derivatives Markets*)





These graphs illustrate the properties of put and call prices discussed in *Derivatives Markets* starting on page 299.

1. The call premium is a decreasing function of K and the put function is an increasing function of K.

(1.12) $$K_1 < K_2 \rightarrow C(K_1) \geq C(K_2) \qquad \text{[DM 9.13]}$$

(1.13) $$K_1 < K_2 \rightarrow P(K_1) \leq P(K_2) \qquad \text{[DM 9.14]}$$

2. The difference in premiums between otherwise identical calls with different strike prices cannot be greater than the difference in those strike prices.

(1.14) $$K_1 < K_2 \rightarrow C(K_1) - C(K_2) \leq K_2 - K_1 \qquad \text{[DM 9.15]}$$

(1.15) $$K_1 < K_2 \rightarrow P(K_2) - P(K_1) \leq K_2 - K_1 \qquad \text{[DM 9.16]}$$

If you think in terms of calculus, these identities say that the absolute value of the slope of each curve above is less than or equal to 1.

3. Both the put and call curves are concave up. *Derivatives Markets* states this **convexity** property in terms of the absolute values of the slopes of secant lines.

(1.16) $$K_1 < K_2 < K_3 \rightarrow \frac{C(K_1) - C(K_2)}{K_2 - K_1} \geq \frac{C(K_2) - C(K_3)}{K_3 - K_2} \qquad \text{[DM 9.17]}$$

(1.17) $$K_1 < K_2 < K_3 \rightarrow \frac{P(K_2) - P(K_1)}{K_2 - K_1} \leq \frac{P(K_3) - P(K_2)}{K_3 - K_2} \qquad \text{[DM 9.18]}$$

On page 300, the text gives rules for constructing arbitrages that can be used if any of the previous properties fail. The homework problems require you to know these arbitrages.

Failure of	**Arbitrage**
(1.12) [DM 9.13]	Buy the low strike call and sell the high strike call (call bull spread)
(1.13) [DM 9.14]	Buy the high strike put and sell the low strike put (put bear spread)
(1.14) [DM 9.15]	Sell the low strike call and buy the high strike call (call bear spread)
(1.15) [DM 9.16]	Buy the low strike put and sell the high strike put (put bull spread)
(1.16) [DM 9.17]	Construct an asymmetric butterfly spread with calls (see Example 9.8)
(1.17) [DM 9.18]	Construct an asymmetric butterfly spread with puts (see Example 9.9)

The use of asymmetric butterfly spreads seems forbidding, but remember that there is a simple way to think of how to execute this. For example, on page 301 Example 9.8 has $K_1 = 50,\ K_2 = 59,\ K_3 = 65$. The distance from K_1 to K_3 is 15, and the distance from K_2 to K_3 is 6, which is 40% of 15. Thus the asymmetric butterfly spread is based on a value of $\lambda = .40$. You construct the spread by:

a) Selling 10 calls with strike price of $K_2 = 59$.
b) Buying $.4(10) = 4$ calls with strike price of $K_1 = 50$
c) Buying 6 calls with strike price of $K_3 = 65$.

This pattern is not hard to remember. It is required for the homework.

Section 1.15

Module 1 Summary

Put-Call Parity for European Options

The put-call parity relation is obtained by noting that the costs for the two different ways of guaranteeing ownership of the index at time T should be the same.

$$PV\left(F_{0,T}\right) = F_{0,T}^{P} = C(K,T) - P(K,T) + PV(K)$$

$$C(K,T) - P(K,T) = PV\left(F_{0,T} - K\right) = F_{0,T}^{P} - e^{-rT}K$$

Generalized Parity

Asset A has price S_t
Asset B has price Q_t

$F_{t,T}^{P}(S)$: Price of prepaid forward on A at t paying S_T at T.
$F_{t,T}^{P}(Q)$: Price of prepaid forward on B at t paying Q_T at T.

$C\left(S_t, Q_t, T-t\right)$ = price of an option with T-t to expiration to give B to get A
$P\left(S_t, Q_t, T-t\right)$ = price of an option with T-t to expiration to give A to get B

$$C\left(S_t, Q_t, 0\right) = \max\left(0, S_T - Q_T\right)$$
$$P\left(S_t, Q_t, 0\right) = \max\left(0, Q_T - S_T\right)$$

$$C\left(S_t, Q_t, T-t\right) - P\left(S_t, Q_t, T-t\right) = F_{t,T}^{P}(S) - F_{t,T}^{P}(Q)$$

Currency Options:

Use currency of A to buy currency of B

r_A = interest rate for country A;
r_B = country B interest rate
x_0 = exchange rate for currency A/currency B

$$C(K,T) - P(K,T) = x_0 e^{-r_B T} - e^{-r_A T}K$$

Relation of call denominated in A to buy B and put denominated in B to sell A.

$$C_A\left(x_0, K, T\right) = x_0 K P_B\left(\frac{1}{x_0}, \frac{1}{K}, T\right)$$

European Versus American Options

$$C_{Amer}(S,K,T) \geq C_{Eur}(S,K,T) \qquad P_{Amer}(S,K,T) \geq P_{Eur}(S,K,T)$$

Bounds for Option Prices

$$S \geq C_{Amer}(S,K,T) \geq C_{Eur}(S,K,T) \geq \max\left[0, \mathrm{PV}_{0,T}(F_{0,Y}) - \mathrm{PV}_{0,T}(K)\right]$$

$$K \geq P_{Amer}(S,K,T) \geq P_{Eur}(S,K,T) \geq \max\left[0, \mathrm{PV}_{0,T}(K) - \mathrm{PV}_{0,T}(F_{0,Y})\right]$$

Should an American Option be Exercised Early?

a) ***An American call on a stock with no dividends should never be exercised early.***

b) Exercise of an American call at time t never makes sense for a stock which pays dividends if

Present value of interest on K $= K - PV_{t,T}(K) > PV_{t,T}(Div)$
= Present value of dividends

If $K - PV_{t,T}(K) \leq PV_{t,T}(Div)$ you cannot rule out early exercise and need more analysis.

c) You cannot rule out early exercise of an American put, since it may make sense to get the strike price K early so as to earn interest on it.

Time to Expiration

American Options and Time to Expiration

$$T_2 > T_1 \rightarrow C_{Amer}(S,K,T_2) \geq C_{Amer}(S,K,T_1) \text{ and } P_{Amer}(S,K,T_2) \geq P_{Amer}(S,K,T_1)$$

European Options

When the stock does not pay a dividend, the European call and the American call have the same price and additional time to maturity increases value for the call.

With dividends, a European call with less time to maturity can have more value than a longer term call if there is need to capture a dividend earlier.

With dividends, a European put with less time to maturity can have more value than a longer term put if there is need to capture a the strike price K earlier.

The Effect of Different Strike Prices on the values of Calls and Puts

$$K_1 < K_2 \rightarrow C(K_1) \geq C(K_2) \quad \text{[DM 9.13]}$$

$$K_1 < K_2 \rightarrow P(K_1) \leq P(K_2) \quad \text{[DM 9.14]}$$

$$K_1 < K_2 \rightarrow C(K_1) - C(K_2) \leq K_2 - K_1 \quad \text{[DM 9.15]}$$

$$K_1 < K_2 \rightarrow P(K_2) - P(K_1) \leq K_2 - K_1 \quad \text{[DM 9.16]}$$

$$K_1 < K_2 < K_3 \rightarrow \frac{C(K_1) - C(K_2)}{K_2 - K_1} \geq \frac{C(K_2) - C(K_3)}{K_3 - K_2} \quad \text{[DM 9.17]}$$

$$K_1 < K_2 < K_3 \rightarrow \frac{P(K_2) - P(K_1)}{K_2 - K_1} \leq \frac{P(K_3) - P(K_2)}{K_3 - K_2} \quad \text{[DM 9.18]}$$

Failure of	**Arbitrage**
(1.12) [DM 9.13]	Buy the low strike call and sell the high strike call (call bull spread)
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(1.15) [DM 9.16]	Buy the low strike put and sell the high strike put (put bull spread)
(1.16) [DM 9.17]	Construct an asymmetric butterfly spread with calls (see Example 9.8)
(1.17) [DM 9.18]	Construct an asymmetric butterfly spread with puts (see Example 9.9)

Section 1.16

Solutions to odd-numbered problems

9.1.

We use the relationship $P(35,0.5) = C(35,0.5) - F^P_{0,0.5} + PV(35)$.

Recall that $F^P_{0,T} = S_0 e^{-\delta T}$. Thus, $P(35,0.5) = 2.27 - 32e^{-.06(.5)} + e^{-.04(.5)}35 = 5.52$.

9.3

Note in advance that put-call parity is violated. Here we have
$C(815,1) - P(815,1) = 75 - 45 = 30 \geq F^P_{0,T} - e^{-rT}K = 800 - 815e^{-.05} = 24.748$

Thus we know that there is an arbitrage, and the problem describes what it is –buy the index, sell the 815-strike call and buy the 815-strike put. You are really buying the index and creating a synthetic short forward sale. The arbitrage exists because you will net 30 in the short forward sale and that amount should have been 24.748. You are being overpaid.

a) The required investment is

Buy index	-800
Sell Call	75
Buy Put	-45
Total	-770

In one year, you will definitely have 815. If the index is above 815 it will be called for 815, and if it is below 815 you can put it for 815.Thus the annual return is $\frac{815}{770} - 1 = .05844$

The continuously compounded rate of return is $\ln\left(\frac{815}{770}\right) = .0568$.

This is greater than the risk free rate.

b) You can borrow 770 at the risk free rate of 5% to implement the strategy above. In one year, you will have to repay the loan with an amount of $770e^{.05} = 809.48$. You will collect 815, for a risk free arbitrage profit of 815.00 – 809.48 = 5.52.

c) Put call parity would eliminate the arbitrage. As we saw above, the difference in put and call prices required for put-call parity is 24.748.

d) K=780 $\rightarrow F^P_{0,T} - e^{-rT}K = 800 - 780e^{-.05} = 58.04$

K=800 $\rightarrow F^P_{0,T} - e^{-rT}K = 800 - 800e^{-.05} = 39.02$

K=820 $\rightarrow F^P_{0,T} - e^{-rT}K = 800 - 820e^{-.05} = 19.99$

K=840 $\rightarrow F^P_{0,T} - e^{-rT}K = 800 - 840e^{-.05} = 0.97$

9.5

This problem requires the formula for the relation of a call denominated in currency B to buy currency A and a put denominated in B to sell A.

$$C_A(x_0,K,T)=x_0KP_B\left(\frac{1}{x_0},\frac{1}{K},T\right)$$

In this case A is the Euro and B is the yen and the formula becomes

$$C_{Euro}(x_0,K,T)=x_0KP_{yen}\left(\frac{1}{x_0},\frac{1}{K},T\right)$$

The currency exchange rate for using a euro to buy a yen is

$$x_0=\frac{\left(\frac{1}{95}\right)euro}{1yen}=.$$

The strike price given for the put in the question is

$$\frac{1}{K}=100\rightarrow K=\frac{1}{100}$$

Thus we have

$$C_{Euro}\left(\frac{1}{95},\frac{1}{100},T\right)=\frac{1}{95}\left(\frac{1}{100}\right)P_{yen}(95,100,T)=\frac{8.763}{9500}=.00092242211$$

9.7

Note that the calls give are dollar denominated to buy yen. The interest rates denominated in dollars and yen are $r_{\$} = .05$, $r_{yen} = .01$

a) This part of the question requires you to find the put price from the call price using the currency put-call parity formula

$$C(K,T) - P(K,T) = x_0 e^{-r_{yen}T} - e^{-r_{\$}T} K$$

Thus $.0006 - P(.009,1) = .009e^{-.01} - e^{-.05}.009$

$P(.009,1) = .0002506163$

b) If the put is actually priced at .0004, it is priced too high. You can arbitrage this by selling the put at the excessive price. Remember that you can create a synthetic forward purchase by selling a put and buying a call. Recall also that: **Forward + zero coupon bond = Asset.**

Thus you can buy the asset using a synthetic long forward and a bond (lending), and then sell it with a forward short sale. The arbitrage is

Sell put	.0004
Buy call	-.0006
Lend PV(strike)	$-.00856 = -.009e^{-.05}$
Sell a prepaid forward	$.00891 = .009e^{-.01}$
Total	.00015

You have pocketed .00015 but have no risk. In one year you will have the right to buy 1 yen for $.009 from the synthetic forward purchase and the repayment of the loan will give you $.009 to pay for it. You can buy the yen to satisfy the prepaid forward short sale, and have no further obligations.

9.7, continued

c) This problem is a reversal of part a), since now the foreign currency is the dollar. We are asked for both the price of the yen denominated at the money call and put. It is necessary to find the second one first. Since we are given that the price of the at the money dollar denominated call to buy yen is .0006, we can use the relationship from Equation (9.7) to find the price of the yen denominated put.

$$C_{\$}\left(x_0,K,T\right)=x_0KP_{Yen}\left(\frac{1}{x_0},\frac{1}{K},T\right).$$

$$.0006=.0009(.0009)P_{Yen}\left(\frac{1}{.0009},\frac{1}{.0009},1\right)$$

$$P_{Yen}\left(\frac{1}{.009},\frac{1}{.009},1\right)=\frac{.0006}{(.009)^2}=7.407407Yen$$

Next we use parity to find the price of the yen denominated call.

$$C_{Yen}\left(\frac{1}{.009},1\right)-P_{Yen}\left(\frac{1}{.009},1\right)=x_0e^{-r_{\$}T}-e^{-r_{Yen}T}K$$

$$C_{Yen}\left(\frac{1}{.009},1\right)-7.407407=\frac{1}{.009}e^{-.05}-e^{-.01}\frac{1}{.009}$$

$$C_{Yen}\left(\frac{1}{.009},1\right)=3.094029$$

9.9.

Note that the difference in exercise prices is $K_2 - K_1 = 55 - 50 = 5$. However the Equations (9.15) and (9.16) of *Derivatives Markets* both fail, since

$$C(K_1) - C(K_2) = 16 - 10 > 5$$

$$P(K_2) - P(K_1) = 14 - 7 > 5$$

The text points out on page 300 that you can arbitrage the failure of 9.15 by buying the high strike call and selling the low strike call. We will look at that arbitrage first.

Buy 55 strike call	-10
Sell 50 strike call	16
Total	6

Initially you have 6. Now we look at the possibilities at expiration.

- $S_T < 50$.
  Both calls expire worthless, and you have the original 6 plus interest.

- $50 \le S_T \le 55$.
  The 55-strike call pays nothing, but the written 50-strike call requires you to pay $S_T - 50 \le 5$. You got 6 to start and must pay at most 5. You have made money.
- $S_T > 55$
  You pay $S_T - 50$ for the written 50 strike call and are paid $S_T - 55$ on the purchased 55 strike call. You net -5 from the 2 calls, but you got 6 initially. You have made money.

In each case above, you have invested nothing and made a profit.

You could also arbitrage the put discrepancy by selling the high (55) strike put and buying the low (50) strike put.

9.11

Both the call and put convexity conditions are violated.

$$\frac{C(80)-C(100)}{100-80}=.65<.8=\frac{C(100)-C(105)}{105-100}$$

$$\frac{P(100)-P(80)}{100-80}=.85>.76=\frac{P(105)-P(100)}{105-100}$$

Convexity violations are arbitraged using asymmetric butterfly spreads. The value of λ to use is

$$\lambda=\frac{105-100}{105-80}=.2.$$

To arbitrage the violation of convexity for calls, buy 2 80-strike calls, sell 10 100 strike calls and buy 8 105 strike calls. The tables below demonstrate the arbitrage. First we look at cash flow at time 0.

Buy 2 80-strike calls	-44
Sell 10 100-strike calls	90
Buy 8-105 strike calls	-40
Total	6

Thus we pocket +6 at time 0. Next we look at the future result of the spread by cases. We will denote the time of expiration by *T*.

	$S_T<80$	$80\le S_T\le 100$	$100\le S_T\le 105$	$S_T>105$
Buy 2 80-strike calls	0	$2(S_T-80)$	$2(S_T-80)$	$2(S_T-80)$
Sell 10 100-strike calls	0	0	$10(100-S_T)$	$10(100-S_T)$
Buy 8- 105 strike calls	0	0	0	$8(S_T-105)$
Total	0	$2(S_T-80)\ge 0$	$840-8S_T\ge 0$	0

Thus we get 6 immediately from the spread, and at expiration have a non-negative amount. This is an arbitrage.

To arbitrage the violation of convexity for puts, buy 2 80-strike puts, sell 10 100 strike puts and buy 8 105 strike puts. The arbitrage is demonstrated in a similar fashion.

9.13

We will look at the call first. The interest rate is 0, so there is no interest loss from early exercise. Recall that if $K - PV_{t,T}(K) \le PV_{t,T}(Div)$ you cannot rule out early exercise and need more analysis.

You may exercise early to get a liquidating dividend as described in the text example on page 296, but in other cases you may want to keep the option for its insurance protection.

Since the only benefit of early exercise of the put is the interest earned on the strike price K, in this case you would not exercise the put early since no interest will be earned.

9.15

We can make .424 today if we buy the 1.5 year call with strike price 107.788 for a premium of 11.50, and we sell the 1 year call with strike price 105.127 for a premium of 11.924. It turns out that you cannot lose money in the future if you do this. To understand this we need to look at the possibilities for in one year at time 1.

If $S_1 \leq 105.127$ at time 1, the sold call will not be exercised and we cannot lose on it. The purchased call has a minimum value of 0, so we cannot lose on it. This is summarized in the first table below.

If $S_1 > 105.127$ At time 1, we short sell the stock for settlement in 0.5 years, and lend 105.127 at 5% for 0.5 years. Again we cannot lose, but the analysis is a bit more complex. It is summarized in the second table below.

Case 1: $S_1 \leq 105.127$

Time	t=0	t=1	$t=T$ & $S_T \leq$ 107.788	$t=T$ & $S_T >$ 107.788
Transaction				
Sell 1-year Call	11.924	0	------	-----
Buy 1.5 year call	-11.50	---	0	S_T - 107.788
Total	0.424	0	0	S_T - 107.788$\geq$0

Case 2: $S_1 > 105.127$

Time	t=0	t=1	$t=T$ & $S_T \leq$ 107.788	$t=T$ & $S_T >$ 107.788
Transaction				
Sell 1-year Call	11.924	105.127- S_1	------	-----
Buy 1.5 year call	-11.50	---	0	S_T - 107.788
Sell stock short	---	S_1	-S_T	-S_T
Lend money @5%	---	-105.127	107.788	107.778
Total	0.424	0	107.788- $S_T \geq$0	0

Since there is a positive payoff at time 0 and no negative payoffs in the future, we have demonstrated an arbitrage strategy.

9.17

This problem is a bit confusing as stated, since many of the numbers given in it are wrong.

For example, the problem says to assume that the stock price is 106.79, but Table 9.1 says that the current price is 84.85. The option prices stated in table 9.1 were quoted on October 15, 2004 for options expiring in November 2004 and January 2005. However the problem refers to a dividend of .14 in May, which is not relevant to these options.

If you are able to look at the instructor's manual you can see that the authors meant to tell you that there would be a dividend in November and that it would be .18, not .14.

In addition to the confusion, the full problem here is not a possible exam problem. The full solution has 16 cases, and you need a computer spreadsheet to do all the required work. We will go through one case carefully so you can at least see what the logic is.

The idea of the problem is to for you to demonstrate that the strategies given in a) and b) will not lead to an arbitrage. **The key behind demonstrating this is to observe that each of the two strategies has 0 payoff at expiration.**

Thus the only way to get an arbitrage would be to pocket money at the beginning. The heart of the problem is to show that there is an out-of-pocket cost at the beginning.

We will begin by showing that the strategy in part a) has 0 payoff. At expiration the combination of a purchased call and a written put gives you a forward purchase at the exercise price K. You will have K available from the payoff of the loan, so you can buy the stock from available funds and deliver it to cover the short sale. The payoff is 0. The demonstration for b) is similar.

We will show that strategy a) requires an out of pocket investment when the strike price of the options is K = 75.

First we need to go over some relevant dates that were not provided. Options typically expired on the Saturday following the third Friday of the month. If you make this assumption you get:

November 2004 expiration: November 20, 2004.
January 2005 expiration: January 22, 2005

The text did not give a date for the dividend to be paid. The instructor's manual says to assume that the dividend will be paid on November 8, 2004, before the expiration of the November options.

9.17, continued

Now let's look at what needs to be done at time 0 to set up strategy a) with $K = 75$ with expiration in November.

- Sell the stock short and receive **84.85**.
- Buy the call for -10.30 and sell the put for +0.20. Net **-10.10**
- Lend the present value of 75 for repayment on 11/20/2004. This will be a 36 day loan with repayment at time $t = 36/365 = .09863$. (The repayment of 75 will be used for K). The present value is $75e^{-.09863(.019)} = 75.8596$**.** You are out of pocket **-74.8596**.
- Lend the present value of the anticipated dividend so that you will be able to cover the dividend payment. The dividend will be payable in 24 days at time t=.06575. The amount loaned is $.18e^{-.06575(.019)} = .1798$. You are out of pocket **-.1798.**

The total proceeds are 84.85 – 10.10 – 74.8596 – 0.1798 = -.2894.

Practice Problems

Corresponding to Chapter 9 of *Derivatives Markets*

Computational Review

1. A stock currently sells for $ 33.5. A 6-month call option with strike price of $ 33 has a premium of $ 2.49. Assuming a 4 % continuously compounded risk-free rate and a 10 % continuous dividend yield, what is the price of the associated put option?

2. A stock currently sells for $ 32.8. A 6-month call option with strike price of $ 31.35 has a premium of $ 2.34, and a 6-month put with the same strike has a premium of $ 1.59.

Assuming a 3 % continuously compounded risk-free rate, find the present value of dividends payable over the next 6 months?

3. The premium of a 100-strike yen-denominated put on the euro is 8.639 Yen. The current exchange rate is 91.55 Yen/euro.

What is the premium of 1/100 strike euro denominated call with the same time to maturity?

4. Suppose the exchange rate is 0.982 $/euro, the euro-denominated continuously compounded interest rate is 8% , the dollar-denominated continuously compounded interest rate is 2% , and the price of a 1-year 0.971-strike European call on the euro is $ 0.0575.

What is the price of a 0.971-strike European put?

5. Suppose the A and T index is 775, the continuously compounded risk-free rate is 7 %, and the dividend yield is 0 %. A 1-year 797 - strike European call costs $ 77.25 and a 1-year 797 - strike European put costs $ 47. Consider the strategy of buying the index, selling the 797 - strike call, and buying the 797 -strike put.

What is the continuous rate of return on this position held until the expiration of the options?

Answers:

1. 2.97 **2.** 1.167 **3.** 0.000944 **4.** 0.102773 **5.** 6.781

Additional Practice Problems

1.

Consider a European call option and a European put option on a nondividend-paying stock. You are given:

(i) The current price of the stock is $ 50 .
(ii) The call option currently sells for $ 0.55 more than the put option.
(iii) Both the call option and the put option will expire in 5 years.
(iv) Both the call option and put option have a strike price of $81 .

What is the continuously compounded risk-free interest rate?

a) 4.93% b) 9.87 c) 7.56% d) 3.78% e) 4.50%

2.

A share of XYZ company stock is now valued at $50. The risk-free rate is 4%. The company has just paid its $1 semiannual dividend. A 50-strike European call maturing in 1 year sells for 8.50. What is the price of a 1-year 50-strike European put?

a) 4.60 b) 5.23 c) 8.48 d) 9.20 e) none of these

3.

A share of ABC company stock is now valued at $40 per share. ABC pays no dividends. The risk-free rate is 4%.What is the difference between the values of a 6-month 45-strike put and a 6-month 45-strike call?

a) 3.56 b) 4.11 c) 4.20 d) 4.54 e) 5.00

4.

A share of ABC company stock is now valued at $40 per share. ABC pays no dividends. The risk-free rate is 4%. For a strike price of 40.808 the 6-month put is priced at 3.38 and the 6-month call is priced at 3.30. Which of the following is true?

a) No arbitrage is possible.
b) An arbitrage profit may be made by selling the call, buying the put and entering into a short forward contract.
c) An arbitrage profit may be made by buying the call, selling the put and entering into a short forward contract.
d) An arbitrage profit may be made by buying the call, selling the put and entering into a long forward contract
e) None of the above

5.

The dollar exchange rate for euros is .91 $/euro. The dollar interest rate is 4.5% and the euro interest rate is 4%. The price of a six month dollar denominated $.90-strike put option for euros is .062. Find the price of the corresponding dollar denominated six month call option.

a) .043 b) .056 c) .068 d) .074 e) .086

6.

The dollar exchange rate for euros is .92 $/euro. The dollar interest rate is 4% . The price of a six month dollar denominated $.90-strike put option for euros is .058 and the price of a six month dollar denominated $.90-strike call option for euros is .079. Find the implied euro interest rate.

a) 2.5% b) 3.7% c) 4.0% d) 4.3% e) 4.9%

7.

Consider the following set of call option prices.

Strike *K*	45	50
Call option price	6	8

Which of the following is true?

a) No arbitrage is possible.

b) The prices violate the convexity rules

c) An arbitrage profit can be made by buying the low strike call and selling the high strike call.

d) An arbitrage profit can be made by selling the low strike call and buying the high strike call

e) An arbitrage profit can be made using a box spread.

8.

Consider the following set of put option prices.

Strike *K*	45	50
Put option price	4	x

Which of the following values of *x* would lead to an arbitrage?

a) 4.5 b) 6 c) 7 d) 9 e)11

9.

Consider the following set of call option prices.

Strike K	45	50	55
Call option price	10	x	3

Which of the following values of x would lead to an arbitrage?

a) 5 b) 5.5 c) 6 d) 7 e) none of these

10.

A stock is currently priced at 40. Which of the following would have the highest price?

a) An American call with strike price 38 and $T = .5$
b) An American call with strike price 42 and $T = .5$
c) An American call with strike price 38 and $T = 1$
d) An American call with strike price 42 and $T = 1$
e) A European call with strike price 38 and $T = 1$

11.

The underlying asset of an exchange option is the stock of company A, which pays no dividend and has a current price of 40. The strike asset is the stock of company B, which pays no dividend and has a current price of 45. The three month European exchange call option has a price of 6. What is the value of the corresponding exchange put option?

a) 7 b) 8 c) 9 d) 10 e) 11

12.

The dollar exchange rate for euros is .91 \$/euro. The dollar interest rate is 4.5% and the euro interest rate is 4%. The price of a six month dollar denominated \$.90-strike call option for euros is .074. Find the price of the euro denominated six month call option with strike price $1/.90$.

a) .076 b) .094 c) .10 d) .103 e) .105

Additional Practice Problem Solutions

1.

This is a parity problem. When there are no dividends, the parity equation is
$C(K,T)-P(K,T)= S_0 - e^{-rT}K$

In this problem, $.55 = 50-81e^{-.5r} \rightarrow r = .0987$

Answer B

2.

This is a parity problem with dividends. The appropriate equation here is
$C(K,T)-P(K,T)= S_0$-PV(Dividends) - $e^{-rT}K$

For this problem, we have $8.50-P(K,T)=50-\left(1e^{-.04(.5)}+1e^{-.04(1)}\right)-50e^{-.04(1)}$
$P(K,T)=8.48$

Answer C

3.

Again we use $C(K,T)-P(K,T)= S_0 - e^{-rT}K$.
$C(K,T)-P(K,T)= 40-45e^{-.04(.5)} = -4.11$

Answer B

4.

Put call parity is violated here. We should have
$C(K,T)-P(K,T)= S_0 - e^{-rT}K = 40-40.808e^{-.02} = 0$.

Thus the call and put should have the same price, but they do not. Note that if you sell the 40.808 put and buy the call two things happen.

1) You pocket 3.38 – 3.30 = .08.
2) You have created a synthetic forward purchase for a price of 40.808.

Note that the actual forward price for 6 months is $40e^{.02} = 40.808$. Thus you can offset the synthetic forward purchase by entering into a forward sale contract for a price of 40.808 in 6 months. These offsetting obligations lead to a net payoff of 0 in 6 months. You have earned .08 at time 0 with no obligation at time .5. This is the arbitrage described in answer c.

Answer C

5.

This is a currency option parity problem. We use the relationship

$$C(K,T)\ -\ P(K,T)\ =\ x_0 e^{-r_{euro}T} - e^{-r_{\$}T}K$$
$$C(K,T)\ -\ .062 =\ .91e^{-.02} - e^{-.0225}(.90)\ \rightarrow\ C = .074$$

Answer D

6.

Using the basic equation displayed in problem 5, we have
$.079 - .058 = .92e^{-.5r_f} - .90e^{-.04(.5)} \rightarrow r_{euro} = .037$

Answer B

7.

Call prices decrease, not increase, as the strike price rises. Thus an arbitrage is possible. If you buy the 45-strike call and sell the 50 strike call with the given prices, you will have \$2 at time 0. At expiration time T you will always have a non-negative payoff:

$S_T \le 45$: Both calls expire worthless, for a payoff of 0.
$45 < S_T \le 50$: The 50-strike call is worthless, and the total payoff is $S_T - 45 > 0$.
$S_T > 50$: The total payoff is $S_T - 45 + 50 - S_T = 5$

Answer C

8.

A price of 11 would violate the requirement that
$P(50,T) - P(45,T) \le 50 - 45$

Answer E

9.

A price of 7 would violate the call convexity requirement that
$$\frac{C(55)-C(50)}{55-50} \le \frac{C(45)-C(50)}{50-45}$$

The arbitrage would utilize a butterfly spread.

Answer D

10.

Calls are worth more with lower strike prices. At the same strike price, the American call is worth more than the European call. At the same strike price, an American call with longer maturity is worth more than an American call with shorter maturity.

Answer C

11.

The relevant exchange option parity equation is

$$C(S_t, Q_t, 0) - P(S_t, Q_t, T-t) = F^P_{t,T}(S) - F^P_{t,T}(Q)$$

Recall that S denotes the underlying asset and Q the strike asset. Thus

$$6 - P(S_t, Q_t, T-t) = 40 - 45 \rightarrow P(S_t, Q_t, T-t) = 11$$

Answer E

12.

We first use the relationship

$$C_\$(x_0, K, T) = x_0 K P_{euro}\left(\frac{1}{x_0}, \frac{1}{K}, T\right)$$

For this problem we have

$$.074 = .91(.90) P_{euro}\left(\frac{1}{.91}, \frac{1}{.90}, .5\right) \rightarrow P_{euro}\left(\frac{1}{.91}, \frac{1}{.90}, .5\right) = .090354$$

Then we can find the value of the euro-denominated call using parity.

$$C_{euro}\left(\frac{1}{.91}, \frac{1}{.90}, .5\right) - .090354 = \frac{1}{.91}e^{-.0225} - \frac{1}{.90}e^{-.02}$$

$$C_{euro}\left(\frac{1}{.91}, \frac{1}{.90}, .5\right) = .075696$$

Answer A

Exam Problems and Solutions

1. (MFE Sample Questions #1)
Consider a European call option and a European put option on a nondividend-paying stock. You are given:

(i) The current price of the stock is $60.
(ii) The call option currently sells for $0.15 more than the put option.
(iii) Both the call option and put option will expire in 4 years.
(iv) Both the call option and put option have a strike price of $70.

Calculate the continuously compounded risk-free interest rate.

(A) 0.039 (B) 0.049 (C) 0.059 (D) 0.069 (E) 0.079

Preliminary comment

This is a straightforward put-call parity question. When there are no dividends, the parity equation is

$$C(K,T)-P(K,T)= S_0 - e^{-rT}K$$

Solution

We must find the unknown interest rate *r*. We are given $S_0 = 60$, $C(K,T)-P(K,T)=.15$, $K=70$ and $T=4.$. Thus $.15 = 60-70e^{-4r} \rightarrow r=.039$.

Answer A

Additional comments:

a) We noted in our guide that in general

$$C(K,T)-P(K,T)= F_{0,T}^{P} - e^{-rT}K.$$

Thus we could use this formula to establish parity for dividend paying stocks. The SOA note states this, using $F_{0,T}^{P} = S_0 - PV(Dividends)$.

b) The SOA note comments that the put-call parity relation above holds for European options, but cannot be used for American options. That note points out that for American options we do know that

$$S_0 - PV(Dividends) - K \le C(K,T)-P(K,T) \le S_0 - e^{-rT}K$$

2. (MFE Sample Questions #2)
Near market closing time on a given day, you lose access to stock prices, but some European call and put prices for a stock are available as follows:

Strike Price	Call Price	Put Price
40	11	3
50	6	8
55	3	11

All six options have the same expiration date.

After reviewing the information above, John tells Mary and Peter that no arbitrage opportunities can arise from these prices.

Mary disagrees with John. She argues that one could use the following portfolio to obtain arbitrage profit: Long one call option with strike price 40; short three call options with strike price 50; lend $1; and long some calls with strike price 55.

Peter also disagrees with John. He claims that the following portfolio, which is different from Mary's, can produce arbitrage profit: Long 2 calls and short 2 puts with strike price 55; long 1 call and short 1 put with strike price 40; lend $2; and short some calls and long the same number of puts with strike price 50.

Which of the following statements is true?
(A) Only John is correct.
(B) Only Mary is correct.
(C) Only Peter is correct.
(D) Both Mary and Peter are correct.
(E) None of them is correct.

Preliminary Comment

This problem is based on the possibility of arbitrage if option prices do not satisfy equations 9.15-9.18 of *Derivatives Markets*. The property that is not satisfied here is the convexity property for calls.

(DM 9.17) $K_1 < K_2 < K_3 \rightarrow \dfrac{C(K_1)-C(K_2)}{K_2-K_1} \geq \dfrac{C(K_2)-C(K_3)}{K_3-K_2}$

Solution

If you use $K_1 = 40, K_2 = 50, K_3 = 55$

$$\frac{C(K_1)-C(K_2)}{K_2-K_1} = .5 < .6 = \frac{C(K_2)-C(K_3)}{K_3-K_2}.$$

Thus we know that an arbitrage exists, and John is wrong. To check if Mary and Peter are right, you must also verify that their arbitrages work. This is done in tables on page 5 of the SOA note. There are some tricky points here.

For Mary's proposed arbitrage the number of long calls at $K = 55$ is not given. However you can quickly figure out what it is. The arbitrage lends \$1, so in order to have 0 outlay at the beginning there must be \$1 of excess cash obtained from the sale and purchase of calls. It there are n long calls at $K = 55$ we have the following proceeds from options.

Strike	40	50	55
Position	Long 1	Short 3	Long n
Proceeds	-11	+6	-3n

Since total proceeds are 1 to lend, we have $-11 + 18 - 3n = 1 \rightarrow n = 2$.

This combination of long and short calls is the asymmetric butterfly spread which the text states can be used to arbitrage a failure of call convexity. If you know that, there is no need to build the first table to check the arbitrage. This butterfly spread method is shown in table 9.7 on page 302 of the text.

For Peter's proposed arbitrage the number of options at $K = 50$ is not given either. The arbitrage lends \$2, so there must be \$2 of excess cash obtained from the sale and purchase of calls. It there are n puts and n calls at $K = 55$ we have the following proceeds from options.

Strike	40	50	55
Position	Buy one call Sell one put	Sell *n* calls Buy *n* puts	Buy 2 calls Sell 2 puts
Proceeds	-11+3=-8	*n*(6-8)=-2n	2(-3+11)=16

Since total proceeds are 2, we have $-8 - 2n + 16 = 2 \rightarrow n = 3$.

We have set *n* so that total proceeds at time 0 are 0, since the excess cash of 2 from the options was loaned and will be repaid with interest.

Note that the positions at different strike prices are synthetic forwards. We can summarize the total proceeds at time *T* using this observation.

Strike	40	50	55	
Position	Buy one call Sell one put	Sell 3 calls Buy 3 puts	Buy 2 calls Sell 2 puts	Lend 2
Equivalent	One long forward at 40	3 short forwards at 50	2 long forwards at 55	
Payoff	$S_T - 40$	$3(50 - S_T)$	$2(S_T - 55)$	$2e^{rT}$

The total payoff is $2e^{rT}$. Thus we have invested 0 and made a positive profit. Similar analyses for the cases $S_T \leq 40$, $40 < S_T \leq 50$ and $50 < S_T \leq 55$ show positive profit. Thus we have invested 0 and made a positive profit.

Answer D

3. (Spring '07 CAS3, #3)

For a dividend paying stock and European options on this stock, you are given the following information:

- The current stock price is \$49.70
- The strike price of options is \$50.00.
- The time to expiration is 6 months.
- The continuous risk-free rate is 3% annually.
- The continuous dividend yield is 2% annually.
- The call price is \$2.00.
- The put price is \$ 2.35.

Using put-call parity, calculate the present value arbitrage profit per share that could be generated, given these conditions.

(A) Less than \$0.20
(B) At least \$0.20 but less than \$0.40
(C) At least \$0.40 but less than \$0.60
(D) At least \$0.60 but less than \$0.80
(E) At least \$0.80

Solution

In this question, we can find the arbitrage by checking the put-call parity relationship $C(K,T)$ - $P(K,T)$ = $PV\left(F_{0,T}-K\right)=F_{0,T}^{P}-e^{-rT}K$

We are given $S=49.7, K=50, T=.5, r=.03, \delta=.02$. The quoted call and put prices are $C=2.00, P=2.35$. Thus
$C-P=-.35$ and $F_{0,T}^{P}-e^{-rT}K=S_0e^{-\delta T}-Ke^{-rT}=49.70e^{-.01}-50e^{-.015}=-.05$.

This implies that the call is priced 0.30 too low relative to the put. To take advantage of this mispricing, sell the put and buy the call and you will have a net gain of 0.35. When you sell the put and buy the call you are creating a synthetic long forward purchase at $K=50$. This can be offset with a synthetic short forward. Recall that our word equation for this is
-Forward = -Stock + Bond.

Sell a tailed position in the stock for $49.70e^{-.01}$ and invest $50e^{-.015}$ in a zero-coupon bond paying 50 in 6 months. The net cost of this is $49.70e^{-.01}-50e^{-.015}=-.05$, so the result is a total net gain of 0.35 – 0.05 = .30. In six months you will have 50 from the bond. If the stock price in six months is above 50 you can use your bond proceeds to exercise the call and get the stock to cover the short position. If the stock price in 6 months is below 50, you can use your bond proceeds to pay the 50 required by the put and the stock will be put to you. In either case you covered the short sale and have no gain or loss in 6 months. Thus you have an arbitrage profit of 0.30.

Answer B

4. (Spring '07 CAS3, #4)

The price of a European call that expires in six months and has a strike price of $30 is $2. The underlying stock price is $29, and a dividend of $0.50 is expected in two months and in five months. The term structure is flat, with all continuously compounded, risk-free rates being 10%. Calculate the price of a European put option on the same stock with expiration in six months and a strike price of $30.

(A) 0.57
(B) 1.06
(C) 1.54
(D) 2.02
(E) 2.51

Solution

Here we will use parity to price the put. We are given $S_0 = 29, K = 30, r = .1, C = 2$. The parity relation is again

$$C(K,T) - P(K,T) = PV\left(F_{0,T} - K\right) = F_{0.T}^{P} - e^{-rT}K$$

With discrete dividends,

$$F_{0,T}^{P} = S_0 - PV\left(Div\right) = 29 - .5e^{-.1/6} - .5e^{-.1(5/12)} = 29 - .97 = 28.03$$

Thus $2 - P = 28.03 - 30e^{-.05} \rightarrow P = 2.51$

Answer E

5. (Spring '07 CAS3, #12)

Which of the following effects are correct on the price of a stock option?

I. The premiums would not decrease if the options were American rather than European.
II. For European put, the premiums increase when the stock price increases.
III. For American call, the premiums increase when the strike price increases.

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

Solution

I. True. American options always have a value greater than or equal to that of otherwise equivalent European options.
II. False. The value of the put decreases as the stock price increases
III. False. The call is worth more as the stock price increases.

Answer A

6. (Spring '07 CAS3, #13)

The price of a non-dividend paying stock is $85. The price of a European call with a strike price of $80 is $6.70 and the price of a European put with a strike price of $80 is $1.60. Both options expire in three months.

Calculate the annual continuously compounded risk-free rate on a synthetic T-Bill created using these options.

(A) Less than 1%
(B) At least 1%, but less than 2%
(C) At least 2%, but less than 3%
(D) At least 3%, but less than 4%
(E) At least 4%

Solution

We use the relationship **Bond = Stock – Forward.**

Buy the stock for 85 and create a synthetic forward sale at 80 by buying the put and selling the call. The net investment is

85 – (6.70 – 1.60) = 79.90

In three months there are two possibilities.

i) $S_{.25} \geq 80$.
Then the stock you own will be called for a price of 80, and you will have 80.

ii) $S_{.25} < 80$.
Then you can use the put to sell your stock for 80, and you will have 80.

In any case, you have invested 79.90 and received 80 in three months. The return is given by

$$80 = 79.90e^{.25r} \rightarrow \quad r = .005$$

Answer A

7. (Spring '07 SOA MFE, #1)

On April 30, 2007, a common stock is priced at $52.00. You are given the following:

i) Dividends of equal amounts will be paid on June 30, 2007 and September 30, 2007.

ii) A European call option on the stock with strike price of $50.00 expiring in six months sells for $4.50.

iii) A European put option on the stock with strike price of $50.00 expiring in six months sells for $2.45.

iv) The continuously compounded risk-free interest rate is 6%.

Calculate the amount of each dividend.

(A) $0.51
(B) $0.73
(C) $1.01
(D) $1.23
(E) $1.45

<u>Solution</u>
We are given $S_0 = 52, K = 50, r = .06$. The call and put prices are $C = 4.50, P = 2.45$. We need to find the unknown dividend D. We use the relationship

$$C - P = S_0 - PV(Div) - Ke^{-rT}$$

$$2.05 = 52 - D\left(e^{-.06/6} + e^{-.06(5/12)}\right) - 50e^{-.06/2} \;\; \rightarrow \;\; D = .73$$

Answer B

8. (Spring '07 SOA MFE, #6)

Consider a model with two stocks. Each stock pays dividends continuously at a rate proportional to its price.

$S_j\ (t)$ denotes the price of one share of stock j at time t.

Consider a claim maturing at time 3. The payoff of the claim is Maximum (S1(3),S2(3))

You are given:

(i) $S_1(0) = \$100$
(ii) $S_2(0) = \$200$
(iii) Stock 1 pays dividends of amount $(0.05)S_1\ (t)\ dt$ between time t and time $t+dt$.
(iv) Stock 2 pays dividends of amount $(0.1)\ S_2\ (t)\ dt$ between time t and time $t+dt$.
(v) The price of a European option to exchange Stock 2 for Stock 1 at time 3 is \$10.

Calculate the price of the claim.

(A) \$96
(B) \$145
(C) \$158
(D) \$200
(E) \$234

Solution

Note first that at time 3 $\max\left(S_1(3), S_2(3)\right) = S_2(3) + \max\left(S_1(3) - S_2(3), 0\right)$.

The term $\max\left(S_1(3) - S_2(3), 0\right)$ represents the payoff of a call enables us to give up a a share of stock 1 in exchange for a share of sock 2. Thus we can obtain the payoff $\max\left(S_1(3), S_2(3)\right)$ by purchasing a prepaid forward for S_2 and an exchange call. The cost of this combination is $F_{0,3}^P(S_2) + C(S_1, S_2)$

To price a prepaid forward for a stock we need its dividend yield. Note the (iii) and (iv) imply that the dividend yields on S_1 and S_2 are $\delta_1 = .05$ and $\delta_2 = .10$ It follows that $F_{0,3}^P(S_2) = 200e^{-.10(3)} = 148.16$.

We are given $C(S_1, S_2) = 10$. Thus the price of the claim is $F_{0,3}^P(S_2) + C(S_1, S_2) =$ $148.16 + 10.00 = 158.16$

Answer C