

# PAK Study Manual for FETE Spring 2012



## PRODUCT FEATURES

Purposes	Features	PAK Study Manual	PAK Exam Aid	PAK Memorization Aid	PAK Test Aid	PAK Study Manual Package	PAK Online Seminar
Study	Summaries	X				X	
Study	Practice Questions	X				X	
Study	Relevant Past Questions	X				X	
Study	Suggested Study Schedule (Detailed)	X				X	
Study	Email Support	X				X	
Practice	Mock Questions		X			X	
Practice	Past Questions		X			X	
Practice	Case Study Analysis		X			X	
Review	Condensed Summary			X		X	
Review	Electronic Flash Cards			X		X	
Review	Memorization Maps/ Combined Topics			X		X	
Evaluation	Mock Exam				X	X	
Bonus	Bonus materials					X	
Study	Online Video Seminar						X
Study	Past Exam Questions / Case Study						X



## **PAK STUDY MANUAL**

### **1. Summary**

- ◇ Summarize each reading in an organized format
- ◇ Explain logics and calculations in details
- ◇ Provide examples to master the materials

### **2. Key Points**

- ◇ Highlight the key points important for the exam
- ◇ Help you focus on what you need to know to pass this exam

### **3. Relevant Past FETE/8V SOA Exam Questions (List)**

- ◇ Save your time to search what materials are relevant

### **4. Practice Questions (100+ in total)**

- ◇ Available in each reading to refresh the materials learnt

### **5. Mock Exam Questions**

- ◇ Include 5 mock exam questions and solutions

### **6. Suggested Study Schedule (Detailed)**

- ◇ Keep track on your study schedule

### **7. Email Support**

- ◇ Get questions? Send me an email

#### NOTE

1. It is **different** from the Fall 2011 version.

*“Even with my little experience and knowledge of the financial world, I passed the FETE exam on the first try.” By Damien, Montreal*

**[Read the whole story](#)**

## **DO YOU KNOW?**

The PAK Study Manual and related aids are updated EVERY exam sitting.

You will see the most updated materials, examples, and explanations to help you master the concepts and pass this exam in the first attempt.

## **PAK EXAM AID**

### **1. Question Set (200+ in Total) (New)**

- ◇ Includes the conceptual questions and calculation questions that cover the learning objectives in the syllabus
- ◇ Test and strengthen your knowledge base

### **2. Mock Exam Questions and Solutions (30 in Total)**

- ◇ Test your knowledge and review how the topics can be tested
- ◇ Understand how to apply what you learnt in a case
- ◇ Complement with the past exam questions

### **3. Case Study Analysis**

- ◇ Connect the case study materials to the study materials
- ◇ Read what was tested
- ◇ Brainstorm what can be tested
- ◇ Understand the tactics to answer the case study questions

### **4. Past SOA Exam Questions (from All FSA Tracks) Relevant to This Exam (70+ in Total)**

- ◇ Provide a list of relevant past questions
- ◇ Sort the past questions into PDFs
- ◇ Save your time to search what materials are relevant

#### NOTES

1. The availability date of the #1—#3 items above may be different. It depends on when the SOA releases the new case study.
2. The 1<sup>st</sup> release will contain item #4 and will be released on 2/15/12. The 2<sup>nd</sup> release will contain item #1—#3 and will be released on 3/15/12.
3. If the SOA do not release the case study, an additional 5 mock questions will be used to replace the case study analysis.

## **PAK MEMORIZATION AID**

### **1. Electronic Flash Cards**

- ◇ Summarize the key points (with mnemonics) (PDF version is also available)
- ◇ Contain around 250 flash cards (front and back)
- ◇ Work best for any big-screen cellular phone or mp3 player (e.g. iPhone, HTC, Android, Blackberry)

### **2. Condensed Summary**

- ◇ Summarize the key points in outline format
- ◇ Quickly refresh all the important topics in the readings

### **3. Memorization Maps /Combined Topics**

- ◇ Map out the key points across the readings
- ◇ Make the memorization much easier
- ◇ 10 maps are available (in spreadsheet format)

## **DO YOU KNOW?**

You can find the most updated information about the PAK Study Manual and related aids under the “Announcement” section on the front page of the PAK website.

## **PAK TEST AID**

### **1. Mock Exam**

- ◇ This set of mock exam is different from those mock questions available in the PAK Exam Aid. You can write down your answers and send them to me. I will give you detailed feedbacks on how to improve your exam score

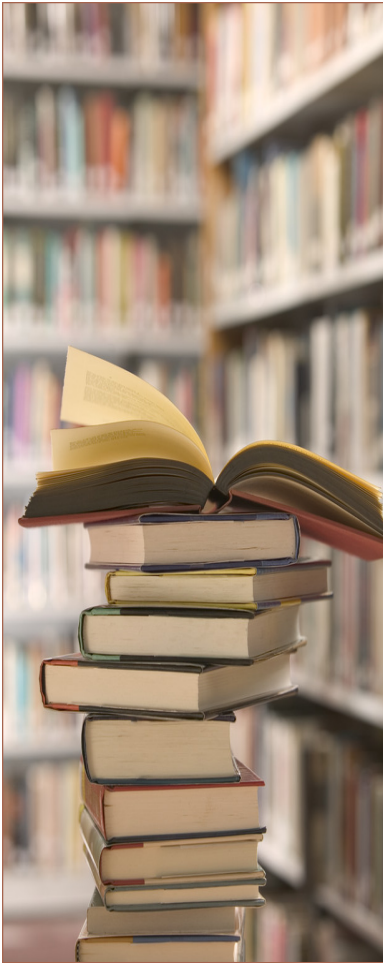
*“I can unequivocally say that it is the best study guide that I have used as I've made my way through the SOA exam system ” By David*

**Read the whole story**

## **RELEASE SCHEDULE**

<b>Features</b>	<b>PAK Study Manual</b>	<b>PAK Exam Aid</b>	<b>PAK Memorization Aid</b>	<b>PAK Test Aid</b>	<b>PAK Study Manual Package</b>	<b>PAK Online Seminar</b>
Summaries	11/15 and 12/30				11/15 and 12/30	
Practice Questions	11/15 and 12/30				11/15 and 12/30	
Relevant Past Questions (List)	11/15 and 12/30				11/15 and 12/30	
Suggested Study Schedule (Detailed)	11/15				11/15	
Email Support	Any time				Any time	
Mock Questions*		3/15			3/15	
Past Exam Questions (Sorted PDFs)		2/15			2/15	
Case Study Analysis*		3/15			3/15	
Condensed Summary			2/15		2/15	
Electronic Flash Cards			2/15		2/15	
Memorization Maps/ Combined Topics			2/15		2/15	
Mock Exam*				3/15	3/15	
Bonus materials					To Be Announced	
Online Video Seminar						12/30
Past Exam Questions / Case Study						3/15

*\*The release date may be changed if the updated case study is released later than 3/1.*



## DO YOU KNOW?

If you are not sure which exam track to take, or how it can advance your career, you can send an email to Eddy and discuss your situation with him. He will share his work experience with you so that you can make your decision informatively.

## PAK STUDY MANUAL PACKAGE

1. PAK Study Manual (Release on 11/15/11 and 12/30/11)
2. PAK Exam Aid (Release on 2/15/12 and 3/15/12)
3. PAK Memorization Aid (Release on 2/15/12)
4. PAK Test Aid (Release on 3/15/12)
5. Bonus materials

## PAK ONLINE SEMINAR

1. Clarify and explain the key concepts/calculations in each reading in the syllabus
2. Discuss the past exam questions (e.g. exam techniques, how to score)
3. Review the new case study
4. Contain condensed outlines for each reading in the syllabus
5. Review the lectures and study at your own pace (PC, Mac, iPhone, HTC, Android, etc)
6. Email support
7. Free access for 2<sup>nd</sup> attempt (only for those who scored 2-5)

### NOTE

1. The online seminar will be released by 2 stages: The 1<sup>st</sup> release containing the readings in the syllabus will be released on 12/30/11. The 2<sup>nd</sup> release containing the past exam questions and new case study analysis will be released on 3/15/12.

## SAMPLES?

You can find more samples on the [PAK](#) website.

## IMPORTANT NOTES

1. Please note that all products are in electronic (PDF) format. **No** hard copy is provided.
2. Once you make a purchase (please use your work email address), I will send you a confirmation email within 1 business days. Once the files are available, I will send them to you through email. Please make sure that you put the correct email address when you purchase the PAK products. **If you do not receive the confirmation email, please send me an email.**
3. Please check your “junk” mailbox. Sometimes, my email is blocked.

## MORE INFORMATION

Want more information? Please contact me at [eddy.chan@pakstudymanual.com](mailto:eddy.chan@pakstudymanual.com) or visit [www.pakstudymanual.com](http://www.pakstudymanual.com)

## COMMENTS FROM THE PAST CANDIDATES

You can find more comments from the past candidates here: [PAK Testimonials](#).

## WHERE TO PURCHASE PAK PRODUCTS

The PAK products are available at [Actex](#), [Actuarial Bookstore](#), and [SlideRule Books](#).

# PAK Study Manual

Financial Economic Theory and Engineering (FETE) Exam  
Spring 2012 Edition



## **PAK Study Manual Overview**

### **Main Goal**

The main goal of the PAK Study Manual is to help you pass this exam in the first attempt. Below briefly describe the features of the PAK Study Manual:

### **Detailed Summary (Different from Fall 2011 Version)**

#### **1. Summarize the key points**

- The syllabus is huge. Many students find it extremely difficult to understand the key points in the readings. The PAK Study Manual summarize the key points in details so you can grasp them quickly and do not miss any important materials.

#### **2. Explain the calculations in details**

- Most of the readings show simplified calculations and many students do not even know how the underlying calculation works. The PAK Study Manual is here to fill in this gap. It provides detailed calculation steps with explanations for students to master the materials.

#### **3. Clarify the confusing concepts**

- There are many important but confusing concepts in this syllabus such as reinsurance, actuarial appraisal value, real options, option pricing, etc). For example, many students get confused in which discount rate should be used, and which valuation framework (risk-neutral vs. real world) should be applied. The PAK Study Manual first clarify these confusing concepts and then explain how their interactions work.

### **Practice Questions and Solutions (100+ in total)**

- High-level key points/calculations (available in each summary)
- Refresh your memory about what you just learnt

### **Past SOA FETE/8V Exam Questions**

- Know which past exam questions correspond to which summary
- Practice them and understand how the topics were tested
- Go back to the readings if you find anything unclear

### **Mock Exam Questions and Solutions**

- Include 5 mock questions and solutions in the PAK Study Manual
- Understand how the materials can be tested

### **Suggested Study Schedule**

- Budget your time efficiently (*what you should do every day*)
- Know your study progress (*beyond or behind study schedule*)

### **Bonus Materials**

- Study Group
- Study Skills
- Many more =)

### **Users' Feedbacks on the PAK Study Manual**

Not convinced? Read others' comments [here](#)

## **Frequent Answer Questions**

### **Do You Need to Read the Source Readings?**

Unlike the preliminary exams, reading the source readings (textbooks, SOA study notes, and online readings) is a must in the FSA exams. PAK Study Manual can help you understand the materials faster and memorize them quickly so that in the time-limited environment, you can be well-prepared for the exam.

### **How Much Time is Needed to Study for This Exam?**

This varies by person. Usually it will take one 350-400 hours to study for this exam. You can read the “Study Schedule” section on next page to better understand how much time is needed.

### **How to Prepare for This Exam?**

Please read the “Study Schedule” section on next page.

### **Which Readings Should You Start First?**

The syllabus contains textbook readings, SOA study notes, and online readings. Which one should you start first? I highly suggest you following the reading order in the syllabus (objective by objective). The SOA usually group the same/similar topics in the same objective. You can understand the materials better by doing this way.

### **What is the PAK Study Manual Format?**

The PAK Study Manual is in electronic format (PDF format).

### **PAK Study Manual Products Available**

1. PAK Study Manual *(Release on 11/15/11 and 12/30/11) (subject to the SOA's syllabus release date)*
2. PAK Exam Aid *(Release on 2/15/11 and 3/15/12)*
3. PAK Memorization Aid *(Release on 2/15/12)*
4. PAK Test Aid *(Release on 3/15/12)*
5. PAK Study Manual Package *(Combine item #1-4)*
6. PAK Online Seminar *(Release on 12/30/11 and upload weekly)*

You can find more information of each product here: <http://www.pakstudymanual.com>

### **Where to Purchase the PAK Study Manual and Other Study Aids**

Please go to [Actex](#), [Actuarial Bookstore](#), or [SlideRule Books](#)

### **How Long Will You Receive the Materials?**

Once you make a purchase, you will receive the materials next business day. Please check your junk mailbox in case your email server blocks my email.

### **More Samples?**

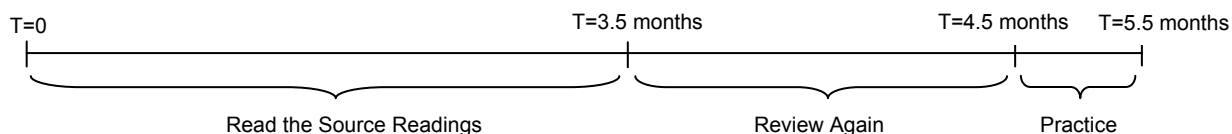
Please go to <http://www.pakstudymanual.com> for samples. If you want more, please feel free to contact me.

### **Any Questions?**

I know you probably have a lot of questions in your mind regarding the exam or choosing study aids. Please feel free to contact me at [eddy.chan@pakstudymanual.com](mailto:eddy.chan@pakstudymanual.com)

## Study Schedule

From the date the SOA release the new syllabus to the exam date, there are around 5.5 months to study. How to plan your study schedule?



### Read the Source Readings

The syllabus is huge!!! Usually it will take 3 to 3.5 months to study the whole syllabus. To study efficiently and make the read-through faster, I highly suggest you following the steps below:

#### Step 1: Define Your Own Study Schedule

- Use the suggested study schedule as a reference
- Prepare your own study schedule (*Target 20 pages @weekday and 40 pages @weekend*)
- Expect to read the whole syllabus and the past exams 2 or 3 times before the exam

#### Step 2: Read the Source Readings Together with the PAK Study Manual

- Write down your notes in the study manual
- Highlight all the key points there (*Will be used for memorization later*)
- Label any calculations that you will go over again later
- Go over the related past exam questions once you finish that reading

#### Step 3: Read the SOA Past Papers

- Read them once you finish your first-round of readings (*use the PAK Exam Aid*)
- Understand how the topics were tested and how the questions were answered

### Review Again

After completing the three steps above, you probably have a general idea about how the exam looks like. Now you should review the source readings again (spend around a month) but this time focus more on the key topics, clarify the confusing concepts/calculations, think of what can be tested and read them carefully (use my mock exam questions)

### Practice

The last month is the most critical month. You will spend most of your time on practicing the questions and memorizing the materials. Here are the steps:

- Practice the past exams and my mock questions to identify what you still do not know
- Go back to the readings and find your answers (*or send me an email if you need help*)
- Start memorizing the key points (*use the PAK Memorization Aid*)
- Use the PAK Test Aid to test your knowledge (*Send me your answers and I will give you detailed feedbacks on how to improve your score in the exams*)

### More Information

I will explain how to prepare for this exam in much more details in the PAK Study Manual.

**Textbook Reading: Options, Futures, and Other Derivatives Ch.12 (8<sup>th</sup> Ed.)** (12.1-7 & 12.9-11) (background only) (by John C. Hull)

**Key Points**

1. Understand how to calculate
  - a. The delta (to construct the riskless portfolio)
  - b. The risk-neutral probability
  - c. The option price (for European and American options, option on futures and option on currencies)

**SAMPLE**

**Binomial Trees**

**Introduction**

Binomial Tree

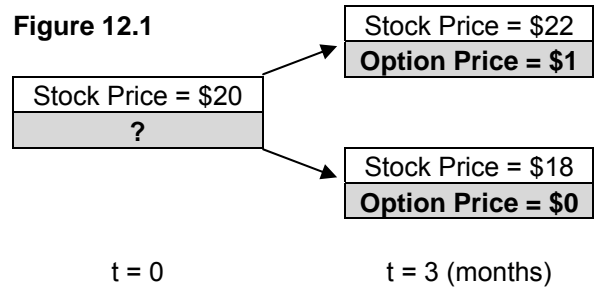
- A diagram representing different possible stock price paths (over the life of an option)
- Assume that the stock price follows a random walk
- The stock price increases (*decreases*) certain percentage amount in a certain probability in each time step

**A One-Step Binomial Model and a No-Arbitrage Argument**

Data

- A stock price is currently \$20
- It will be either \$22 or \$18 at the end of 3 months
- European call at strike price = \$21

**Figure 12.1**



No-Arbitrage Argument

- Arbitrage opportunities do not exist
- If we set up a portfolio of stocks and options and its value is certain at the end of the period
  - o The portfolio has **no risk**
  - o And **the return = the risk-free interest rate**
- Using this argument, we can calculate the cost of setting up the portfolio and the option's price

Data

- Long  $\Delta$  shares of stock and short one call option
- Assume the risk-free rate = 12% (per annum)
- Calculate the value of  $\Delta$  that makes the portfolio riskless

Calculate the Number of Share Needed

- The portfolio is riskless if the value of  $\Delta$  is chosen so that the final value is certain
  - o (The up scenario)  $22\Delta - 1 = 18\Delta$  (the down scenario)  $\rightarrow \Delta = 0.25$
  - o The riskless portfolio is to
    - Long 0.25 shares of stock
    - And short 1 call option
  - o When the stock price moves up to \$22  $\rightarrow$  the value of the portfolio =  $22(0.25) - 1 = 4.5$
  - o When the stock price moves down to \$18  $\rightarrow$  the value of the portfolio =  $18(0.25) = 4.5$

Calculate the PV of the Portfolio

- The riskless portfolio must earn the risk-free interest rate
  - o So the value of the portfolio today =  $4.5e^{-0.12 \times 3/12} = 4.367$

Calculate the Option Price

- The cost of setting up the portfolio today = the value of stock owned – the option cost =  $20 \times 0.25 - f$
- The cost = the PV  $\rightarrow$  it is 4.367  $\rightarrow 20 \times 0.25 - f = 4.367 \Rightarrow f = 0.633$
- The option price = 0.633

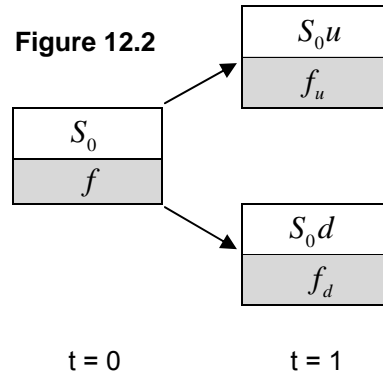
What If the Option Price Changes?

- If  $f > 0.633$ 
  - The value of the portfolio =  $20 \times 0.25 - f \rightarrow V_0 < 4.367$
  - Since  $4.5e^{-0.12 \times 3/12} = V_0 \rightarrow$  If  $V_0 < 4.367$ , need a higher interest rate to discount  $\rightarrow r_{new} > r_{free}$
  - $f \uparrow \Rightarrow V_0 \downarrow \Rightarrow r_{new} \uparrow \Rightarrow r_{new} > r_{free}$
  
- If  $f < 0.633$ , use the same logic as above
  - $f \downarrow \Rightarrow V_0 \uparrow \Rightarrow r_{new} \downarrow \Rightarrow r_{new} < r_{free}$

**A Generalization**

Notations

- $S_0$  = The current stock price
- $f$  = The current option price
- $S_0u$  = The stock price in the “up” scenario
- $f_u$  = The option price in the “up” scenario
- $S_0d$  = The stock price in the “down” scenario
- $f_d$  = The option price in the “down” scenario



Calculate the Number of Share Needed

- Due to no arbitrage, the up scenario = the down scenario  $\rightarrow S_0u\Delta - f_u = S_0d\Delta - f_d \Rightarrow \Delta = \frac{f_u - f_d}{S_0u - S_0d}$

Calculate the PV of the Portfolio

- Since the portfolio is riskless, it must earn the risk-free interest rate
- **PV of the portfolio =  $(S_0u\Delta - f_u)e^{-rT}$**

Calculate the Option Price

- The cost of setting up the portfolio today =  $S_0\Delta - f$
- The cost = the PV  $\rightarrow S_0\Delta - f = (S_0u\Delta - f_u)e^{-rT}$ 
  - Simplifying:  $f = S_0\Delta(1 - ue^{-rT}) + f_ue^{-rT}$
  - **The option price:**  $f = e^{-rT} [pf_u + (1 - p)f_d]$
  - **The (risk-neutral) probability that the stock price moves up:**  $p = \frac{e^{rT} - d}{u - d}$

**Irrelevance of the Stock’s Expected Return**

- The equation does not involve the probabilities of the stock price moving up or down
- The option is calculated in terms of the price of the underlying stock, not in absolute terms
- The probabilities of future up or down movements are already incorporated into the stock price

## Risk-Neutral Valuation

### Risk Neutral Valuation

- When valuing a derivative, we can make the assumption that investors are *risk-neutral*
  - o This assumption means investors do not increase the expected return they require from an investment to compensate for increased risk
  - o A world where investors are risk-neutral is referred to as a *risk-neutral world*
  - o This assumption gives us the right option price in the risk-neutral world and all other worlds

### Two Features that Simplify the Pricing of Derivatives in a Risk-Neutral World

1. The expected return on a stock is the risk-free rate
2. The discount rate used for the expected payoff on an option is the risk-free rate

### Equations

$$\text{Expected payoff from the option at time } t = pf_u + (1-p)f_d$$

$$\text{Expected stock price at time } t = E(S_T) = pS_0u + (1-p)S_0d$$

### Setting the Probability of the Up Movement = p

- Equivalent to setting the return on the stock = the risk-free rate

### The One-Step Binomial Example Revisited

#### Calculation Example (Using the Data in Figure 12.1)

$$\text{The risk-neutral probability (stock price moves up)} = p = \frac{e^{rT} - d}{u - d} = \frac{e^{0.12 \times 3/12} - 0.9}{1.1 - 0.9} = 0.6523$$

$$\text{The expected option price at the end of 3 months} = p \times 1 + (1-p) \times 0 = 0.6523 \times 1 + (1 - 0.6523) \times 0 = 0.6523$$

$$\text{The option price (today)} = f = e^{-0.12 \times 3/12} [0.6523 \times 1 + (1 - 0.6523) \times 0] = 0.633$$

### Real World vs. Risk-Neutral World

- The risk-neutral probability is not equal to the real-world probability

#### Calculation

- The expected stock return = 16%

$$\text{- The real-world probability that the stock price moves up} = p^* = \frac{e^{\mu T} - d}{u - d} = \frac{e^{0.16 \times 3/12} - 0.9}{1.1 - 0.9} = 0.7041$$

- The expected payoff from the option (real world) at the end of 3 months =  $p^* \times 1 + (1 - p^*) \times 0 = 0.7041$
- Since the call option is riskier than the stock, the discount rate used should be greater than 16%
- However, without knowing the option's value, we do not know how much greater than 16%
- The risk-neutral valuation does not have this problem

## Two-Step Binomial Trees

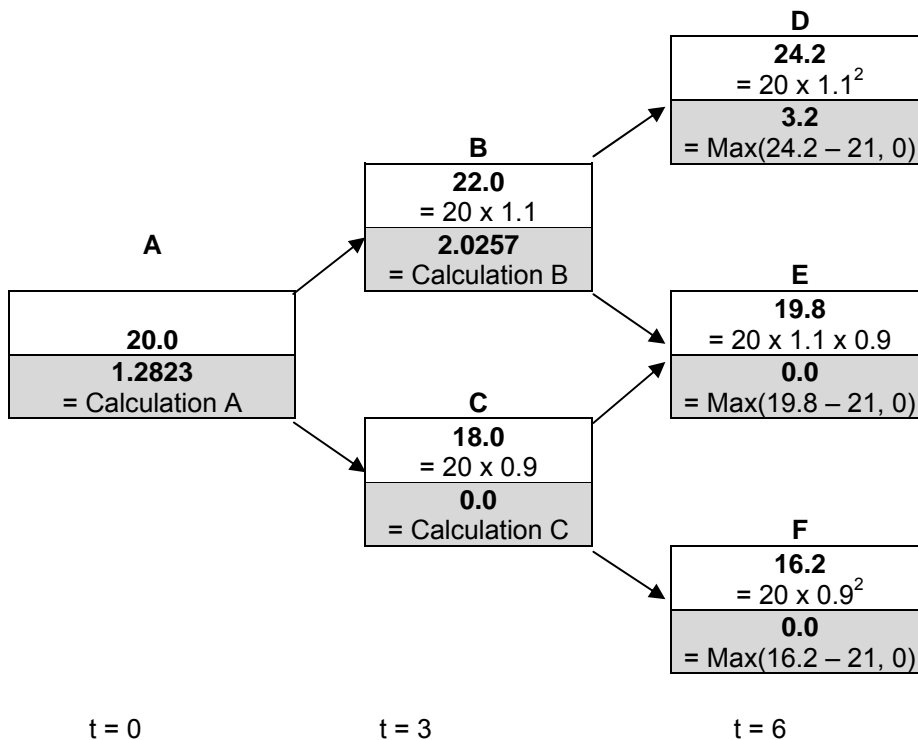
### Objective of the Two-Step Binomial Trees

- Calculate the option price at the initial node of the tree

### Data

- A stock price is currently \$20
- It will move up 10% or down 10% in each of two time steps (3 months each period)
- The risk-free interest rate = 12%
- European call at strike price = \$21

Figure 12.4



### Calculation B

$$p = \frac{e^{rT} - d}{u - d} = \frac{e^{0.12 \times 3/12} - 0.9}{1.1 - 0.9} = 0.6523$$

$$e^{-0.12 \times 3/12} (0.6523 \times 3.2 + 0.3477 \times 0) = 2.0257$$

### Calculation C

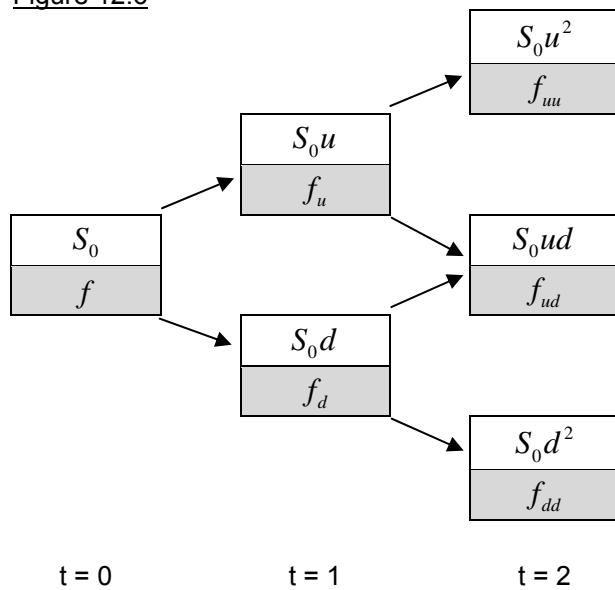
$$e^{-0.12 \times 3/12} (0.6523 \times 0 + 0.3477 \times 0) = 0$$

### Calculation A

$$e^{-0.12 \times 3/12} (0.6523 \times 2.0257 + 0.3477 \times 0) = 1.2823$$

**A Generalization**

Figure 12.6



The risk-neutral probability that the stock price moves up =  $p = \frac{e^{r\Delta t} - d}{u - d}$

**The Equations to Calculate the Option Price**

$$f_u = e^{-r\Delta t} [pf_{uu} + (1-p)f_{ud}]$$

$$f_d = e^{-r\Delta t} [pf_{ud} + (1-p)f_{dd}]$$

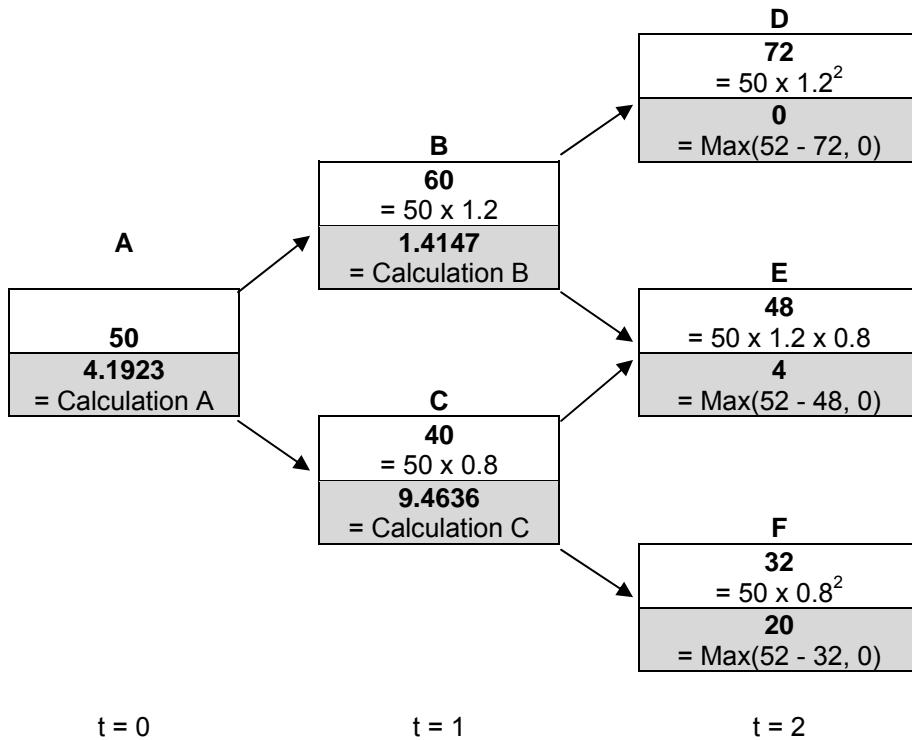
$$f = e^{-r\Delta t} [pf_u + (1-p)f_d]$$

**A Put Example**

Data

- A stock price is currently \$50
- It will move up 20% or down 20% in each of two time steps (1 year each period)
- The risk-free interest rate = 5%
- European put at strike price = \$52

Figure 12.7



Calculation B

$$p = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.05 \times 1} - 0.8}{1.2 - 0.8} = 0.6282$$

$$f_u = e^{-0.05 \times 1} (0.6282 \times 0 + 0.3718 \times 4) = 1.4147$$

Calculation C

$$f_d = e^{-0.05 \times 1} (0.6282 \times 4 + 0.3718 \times 20) = 9.4636$$

Calculation A

$$f = e^{-0.05 \times 1} (0.6282 \times 1.4147 + 0.3718 \times 9.4636) = 4.1923$$

## American Options

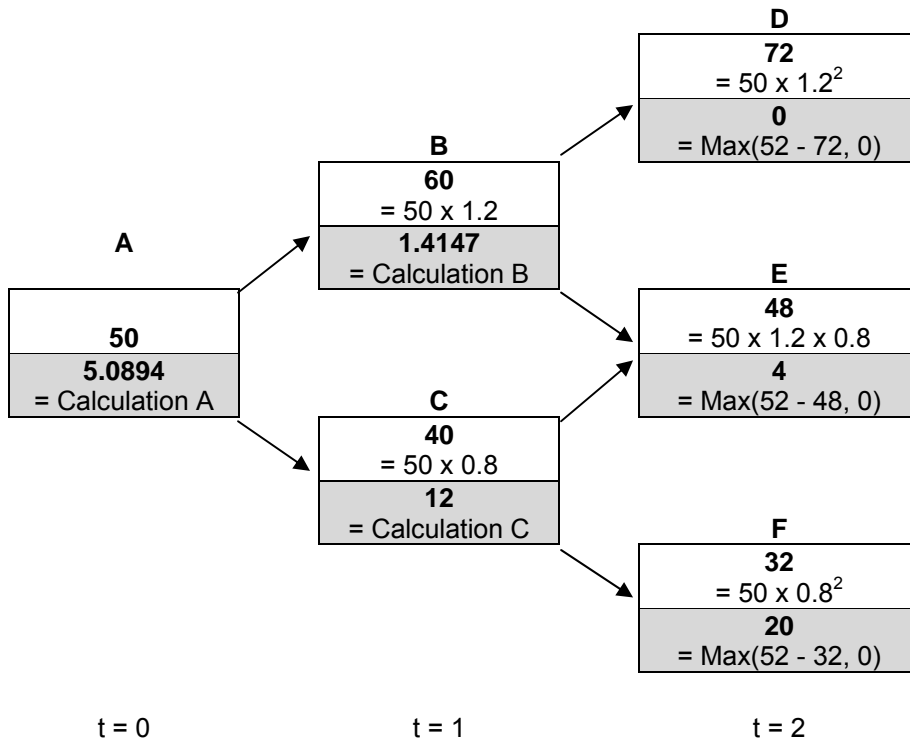
### American Options

- Can be exercised early

### The Procedure

- Work through the tree from the end to the beginning
- Test at each node to see whether early exercise is optimal
- **The option value at each node = Max[ The calculated value, The payoff from early exercise]**

Figure 12.8



### Calculation B

$$p = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.05 \times 1} - 0.8}{1.2 - 0.8} = 0.6282$$

$$f_u = e^{-0.05 \times 1} (0.6282 \times 0 + 0.3718 \times 4) = 1.4147$$

$$\text{The option value at node B} = \text{Max}(1.4147, 52 - 60) = 1.4147$$

### Calculation C

$$f_d = e^{-0.05 \times 1} (0.6282 \times 4 + 0.3718 \times 20) = 9.4636$$

$$\text{The option value at node B} = \text{Max}(9.4636, 52 - 40) = 12$$

### Calculation A

$$f = e^{-0.05 \times 1} (0.6282 \times 1.4147 + 0.3718 \times 12) = 5.0894$$

$$\text{The option value at node A} = \text{Max}(5.0894, 52 - 50) = 5.0894$$

## Delta

### Delta (Greek Letter)

- The ratio of the change in the option value to the change in the underlying stock price
- The number of units of the stock held for each option shorted to create a riskless portfolio
- The riskless portfolio (hedge) is called delta hedging
- The delta of a **call** option is **positive** and the delta of a **put** option is **negative**

$$\Delta = \frac{f_u - f_d}{S_0 u - S_0 d}$$

- The delta changes over time → require to rebalance it periodically

## Matching Volatility with u and d

### Volatility

$\sigma$  = The volatility (standard deviation) of the stock price

$$u = e^{\sigma\sqrt{\Delta t}} \quad d = e^{-\sigma\sqrt{\Delta t}}$$

### Girsanov's Theorem

- When we move from a world with one set of risk preferences to a world with another set of risk preferences
  - o The expected growth rates in variables changes
  - o But their volatilities remain the same

### Changing the Measure

- Move from one set of risk preferences to another
- The **real-world** measure is called **P-measure** ( $p^*$ )
- The **risk-neutral** world measure is called **Q-measure** ( $p$ )

## The Binomial Tree Formulas (Excluded in the Current Syllabus)

### **Note**

The FETE syllabus specifies that section 12.8 (8<sup>th</sup> Ed.) is not in the syllabus but it is "in" the syllabus if we use 7<sup>th</sup> Ed.. To be consistent, I include this section here.

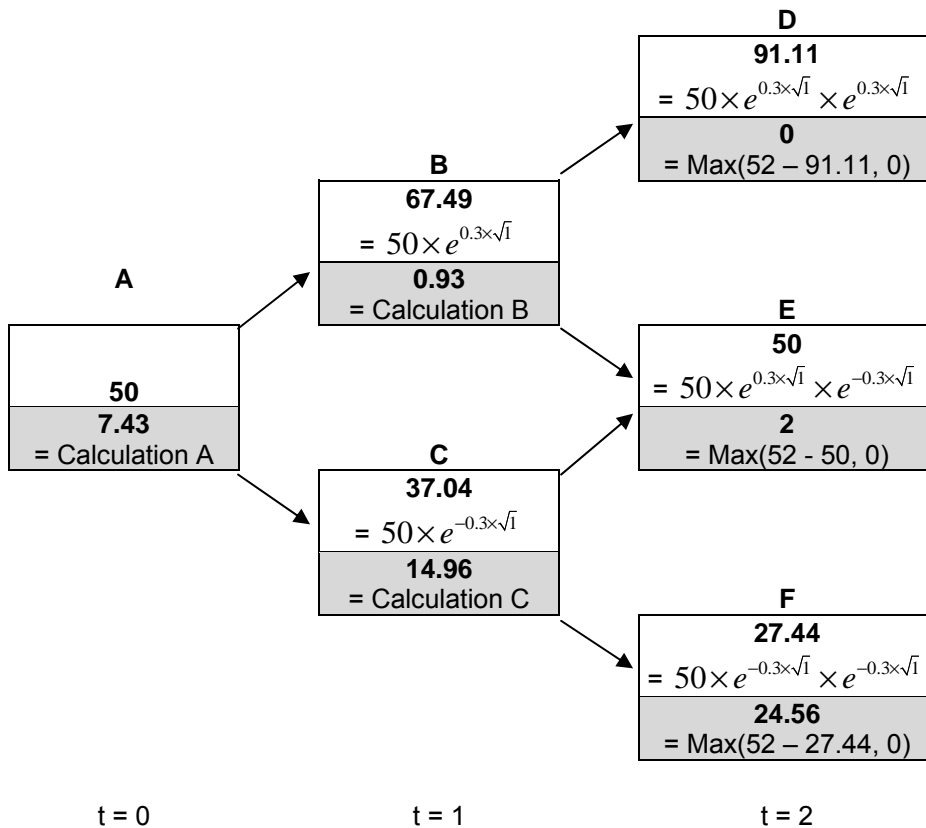
### Binomial Tree Formulas

$$u = e^{\sigma\sqrt{\Delta t}} \quad d = e^{-\sigma\sqrt{\Delta t}} \quad p = \frac{e^{r\Delta t} - d}{u - d}$$

### Data for Figure 12.10

- A stock price is currently \$50
- The volatility  $\sigma = 30\%$
- The risk-free interest rate = 5%
- American put at strike price = \$52
- Expire in 2 years

Figure 12.10



$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.3\sqrt{1}} = 1.3499 \quad d = e^{-\sigma\sqrt{\Delta t}} = e^{-0.3\sqrt{1}} = 0.7408$$

$$p = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.05 \times 1} - 0.7408}{1.3499 - 0.7408} = 0.5097$$

Calculation B

$$f_u = e^{-0.05 \times 1} (0.5097 \times 0 + 0.4903 \times 2) = 0.93$$

$$\text{The option value at node B} = \text{Max}(0.93, 52 - 67.49) = 0.93$$

Calculation C

$$f_d = e^{-0.05 \times 1} (0.5097 \times 2 + 0.4903 \times 24.56) = 12.42$$

$$\text{The option value at node B} = \text{Max}(12.42, 52 - 37.04) = 14.96$$

Calculation A

$$f = e^{-0.05 \times 1} (0.5097 \times 0.93 + 0.4903 \times 14.96) = 7.43$$

$$\text{The option value at node A} = \text{Max}(7.43, 52 - 50) = 7.43$$

**Increasing the Number of Steps / Using DerivaGem**

*Please read these two sections*

**Options on Other Assets**

**Options on Stocks Paying a Continuous Dividend Yield**

- In the risk-neutral world, the risk-free rate = the total return ( $r$ ) = the capital gain + the dividend yield ( $q$ )
- The probability that the stock price moves up:  $pS_0u + (1-p)S_0d = S_0e^{(r-q)\Delta t} \Rightarrow p = \frac{e^{(r-q)\Delta t} - d}{u - d}$

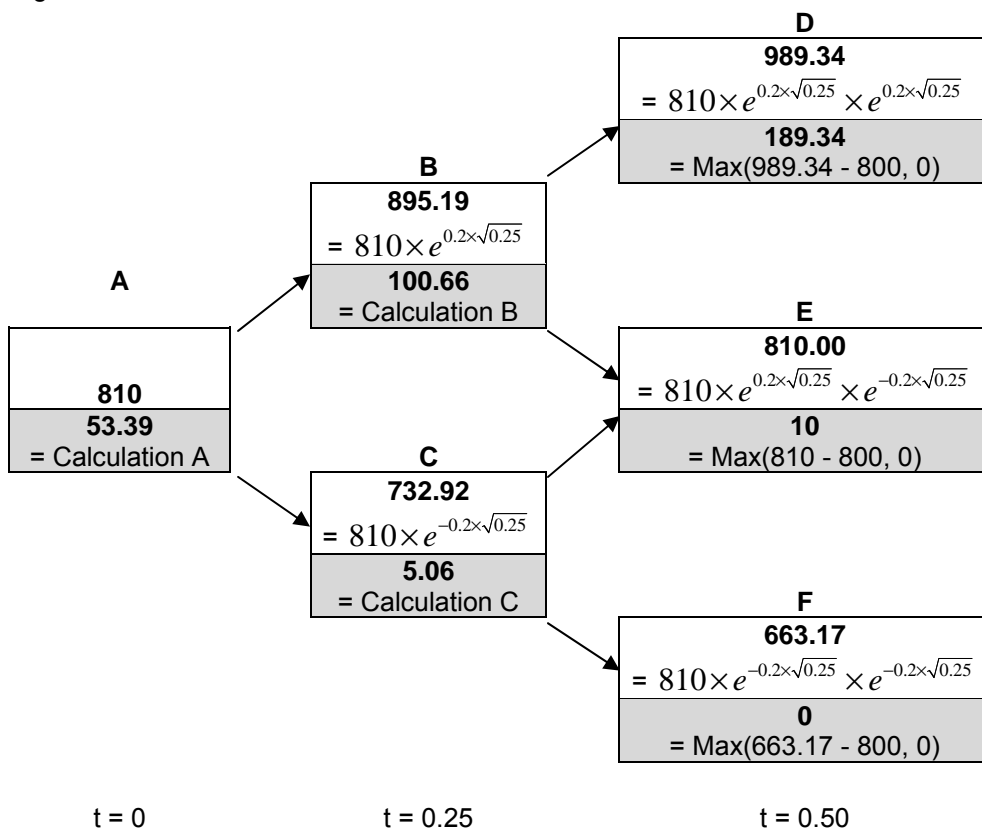
**Options on Stock Indices**

- The valuation of an option on a stock index is similar to the valuation of an option on a stock paying a known dividend yield

**Data**

- $S_0 = 810$ ,  $\sigma = 20\%$ ,  $q = 0.02$ ,  $r = 0.05$
- Calculate European 6-month call with a strike price = \$800 (3 month each period)

Figure 12.11



$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.2\sqrt{0.25}} = 1.1052 \quad d = e^{-\sigma\sqrt{\Delta t}} = e^{-0.2\sqrt{0.25}} = 0.9048$$

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d} = \frac{e^{(0.05-0.02)\times 0.25} - 0.9048}{1.1052 - 0.9048} = 0.5126$$

Calculation B:  $f_u = e^{-0.05 \times 0.25} (0.5126 \times 189.34 + 0.4874 \times 10) = 100.66$

Calculation C:  $f_d = e^{-0.05 \times 0.25} (0.5126 \times 10 + 0.4874 \times 0) = 5.06$

Calculation A:  $f = e^{-0.05 \times 0.25} (0.5126 \times 100.66 + 0.4874 \times 5.06) = 53.39$

**Options on Currencies**

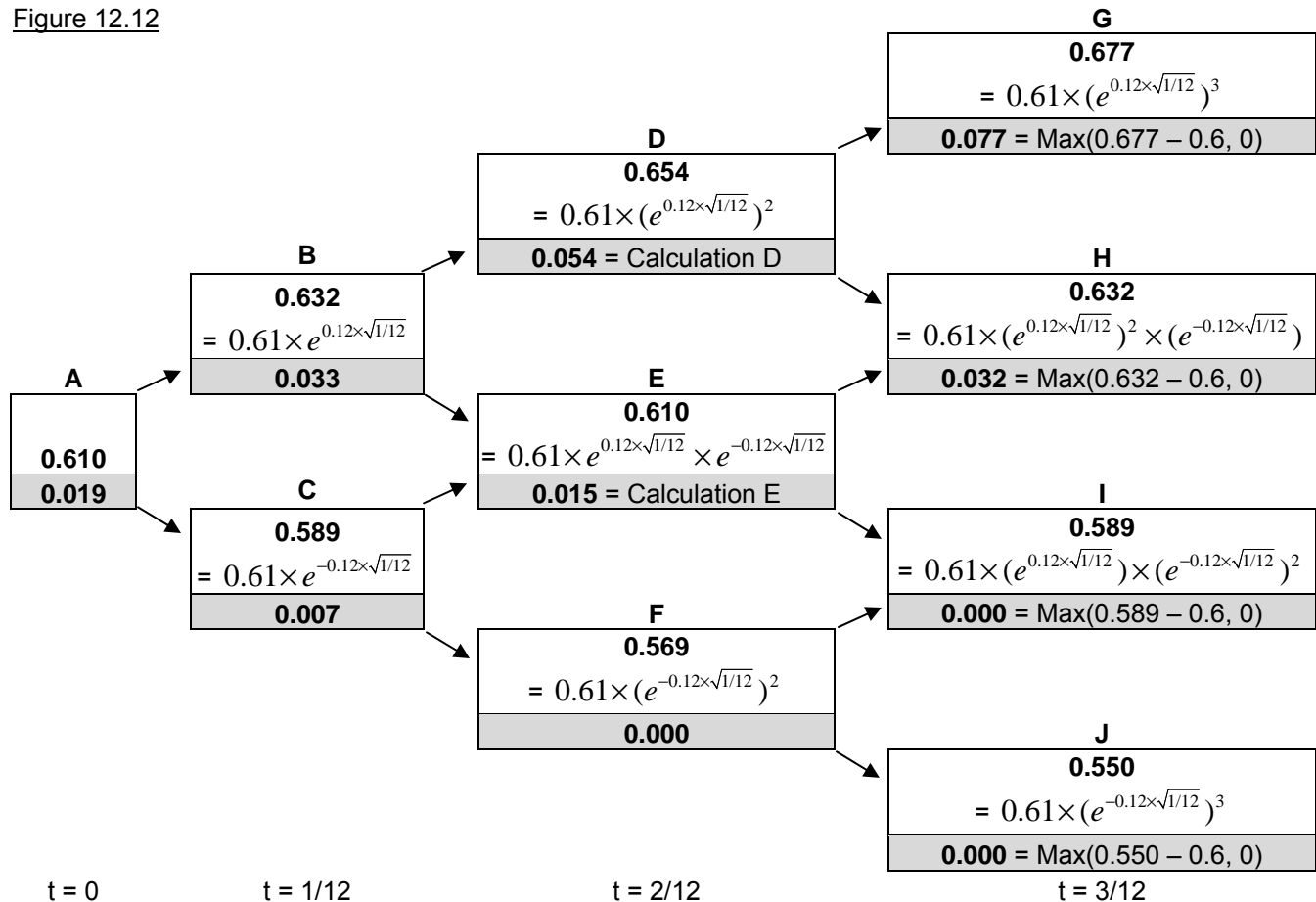
- A foreign currency can be regarded as an asset providing a yield at the foreign risk-free interest rate  $r_f$

(Note: Instead of using  $q$ , we use  $r_f$  in the equation)

**Data**

- Current Exchange Rate (USD per AUD) =  $S_0 = 0.61$ ,  $\sigma = 12\%$ ,  $r_f = 0.07$ ,  $r = 0.05$
- Calculate American 3-month call with a strike price = 0.6 (1 month each period)

Figure 12.12



$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.12\sqrt{1/12}} = 1.0352 \quad d = e^{-\sigma\sqrt{\Delta t}} = e^{-0.12\sqrt{1/12}} = 0.9660$$

$$p = \frac{e^{(r-r_f)\Delta t} - d}{u - d} = \frac{e^{(0.05-0.07)\times 1/12} - 0.9660}{1.0352 - 0.9660} = 0.4673$$

**Calculation D**

$$f_{uu} = e^{-0.05\times 1/12} (0.4673 \times 0.077 + 0.5327 \times 0.032) = 0.053$$

The option price at node D =  $\text{Max}(0.053, 0.654 - 0.600) = 0.054$

**Calculation E**

$$f_{ud} = e^{-0.05\times 1/12} (0.4673 \times 0.032 + 0.5327 \times 0.000) = 0.015$$

The option price at node E =  $\text{Max}(0.015, 0.610 - 0.600) = 0.015$

**Options on Futures**

- In a risk-neutral world, a future price should have an expected growth rate of zero

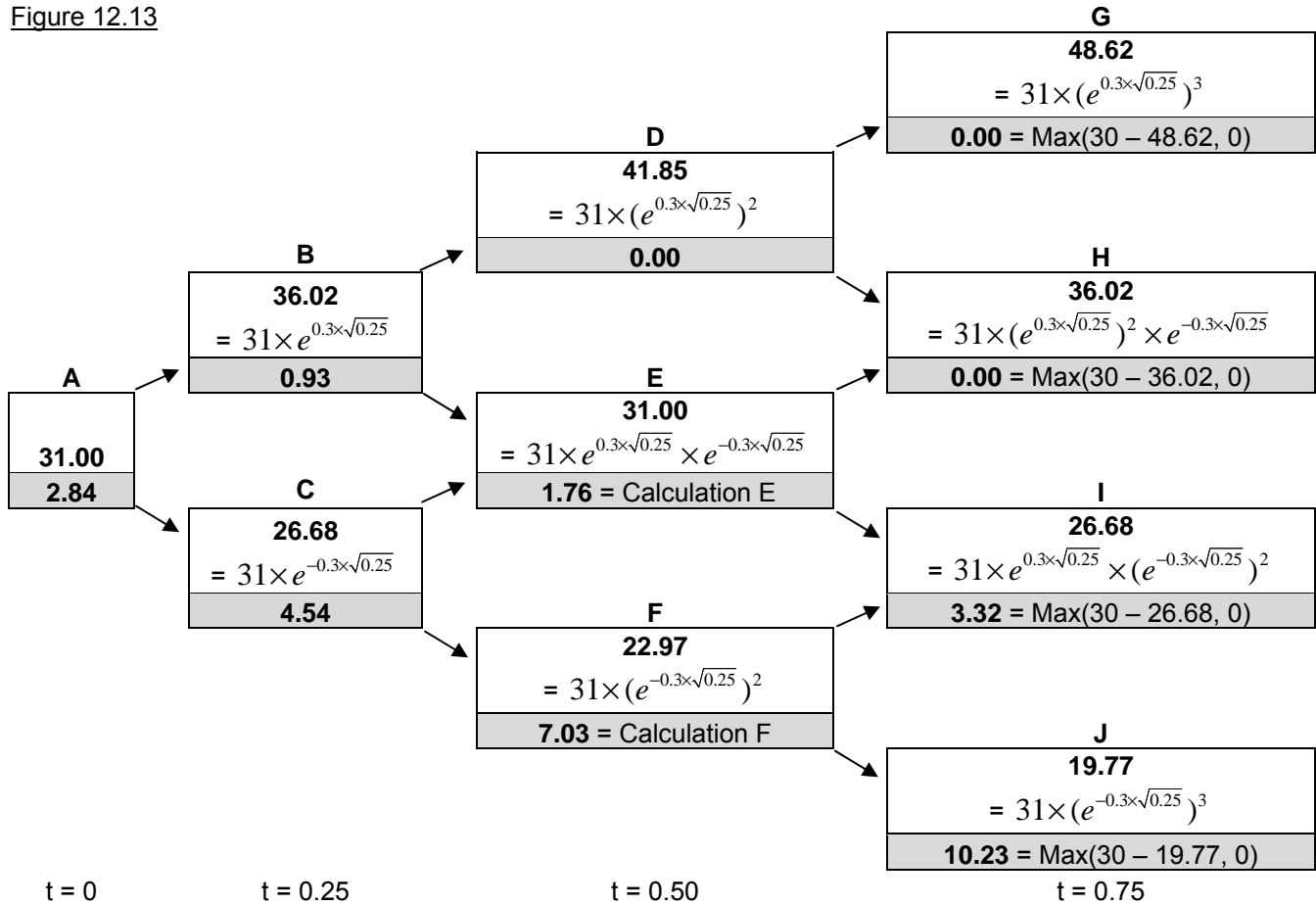
$$- pF_0u + (1 - p)F_0d = F_0 \Rightarrow p = \frac{1 - d}{u - d}$$

**Data**

-  $F_0 = 31$ ,  $\sigma = 30\%$ ,  $r = 0.05$

- Calculate American 9-month put with a strike price = 30 (3 months each period)

Figure 12.13



$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.3\sqrt{0.25}} = 1.1618 \quad d = e^{-\sigma\sqrt{\Delta t}} = e^{-0.3\sqrt{0.25}} = 0.8607$$

$$p = \frac{1 - d}{u - d} = \frac{1 - 0.8607}{1.1618 - 0.8607} = 0.4626$$

**Calculation E**

$$f_{ud} = e^{-0.05 \times 0.25} (0.4626 \times 0 + 0.5374 \times 3.32) = 1.76$$

The option price at node E =  $\text{Max}(1.76, 30 - 31) = 1.76$

**Calculation F**

$$f_{dd} = e^{-0.05 \times 0.25} (0.4626 \times 3.32 + 0.5374 \times 10.23) = 6.95$$

The option price at node E =  $\text{Max}(6.95, 30 - 22.97) = 7.03$

**Summary***Please read this section***Practice Questions****1. Calculate the node A, B, C and D in Figure 12.13**Calculation D

$$f_{uu} = e^{-0.05 \times 0.25} (0.4626 \times 0 + 0.5374 \times 0) = 0$$

The option price at node D =  $\text{Max}(0, 30 - 41.89) = 0$ Calculation C

$$f_d = e^{-0.05 \times 0.25} (0.4626 \times 1.76 + 0.5374 \times 7.03) = 4.54$$

The option price at node C =  $\text{Max}(4.54, 30 - 26.68) = 4.54$ Calculation B

$$f_u = e^{-0.05 \times 0.25} (0.4626 \times 0 + 0.5374 \times 1.76) = 0.93$$

The option price at node B =  $\text{Max}(0.93, 30 - 36.02) = 0.93$ Calculation A

$$f = e^{-0.05 \times 0.25} (0.4626 \times 0.93 + 0.5374 \times 4.54) = 2.84$$

The option price at node A =  $\text{Max}(2.84, 30 - 31) = 2.84$ **Past 8V/FETE SOA Questions Relevant To This SN**

None

**Suggested Questions**

1, 5, 17

**Suggested Solutions****Q12-1**Expected stock price at time t =  $E(S_T) = pS_0u + (1-p)S_0d$ 

$$40e^{0.08 \times 1/12} = p(42) + (1-p)(38)$$

$$p = 0.5669$$

The option price =  $f = e^{-rT} [pf_u + (1-p)f_d]$ 

$$f = e^{-0.08 \times 1/12} [0.5669(3) + (1-0.5669)(0)]$$

$$f = 1.69$$

$$f_u = \max(42 - 39, 0) = 3$$

$$f_d = \max(38 - 39, 0) = 0$$

Alternative Approach

$$\Delta = \frac{f_u - f_d}{S_0u - S_0d} = \frac{3 - 0}{42 - 38} = 0.75$$

The option price:  $S_0\Delta - f = (S_0u\Delta - f_u)e^{-rT}$ 

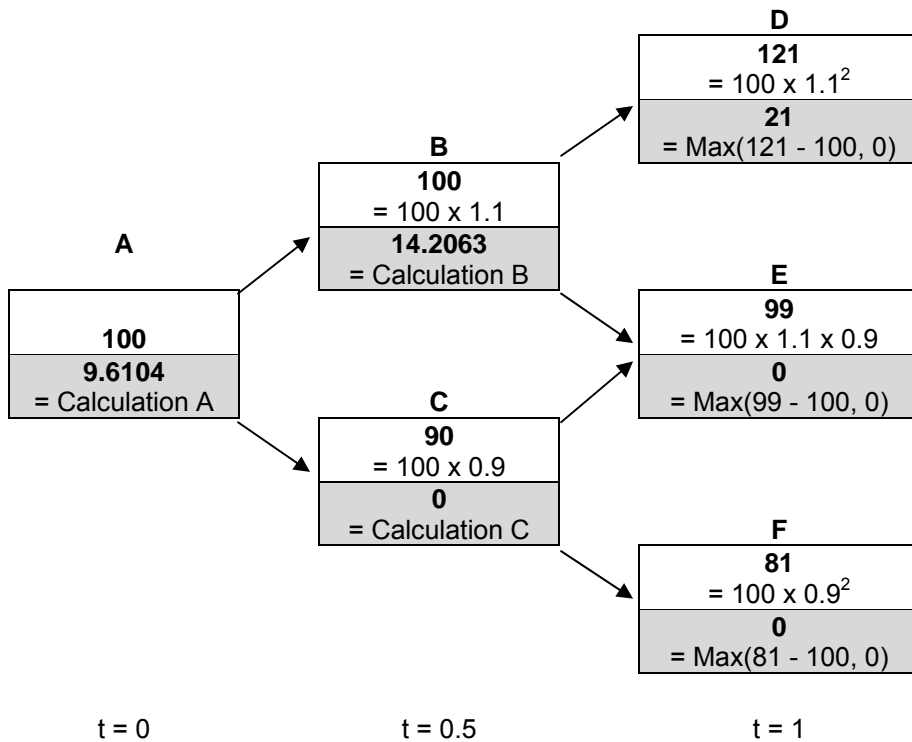
$$40(0.75) - f = [42(0.75) - 3]e^{-0.08 \times 1/12}$$

$$f = 1.69$$

**Q12-5**Data

$S_0 = 100$ ,  $u = 1.1$ ,  $d = 0.9$ ,  $r = 8\%$ ,  $T = 1$  (0.5@ period),  $K = 100$

Calculate the value of an European call

Calculation B

$$p = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.08 \times 0.5} - 0.9}{1.1 - 0.9} = 0.7041$$

$$f_u = e^{-r\Delta t} [pf_{uu} + (1-p)f_{ud}]$$

$$f_u = e^{-0.08 \times 0.5} (0.7041 \times 21 + (1 - 0.7041) \times 0) = 14.2063$$

Calculation C

$$f_d = e^{-r\Delta t} [pf_{ud} + (1-p)f_{dd}]$$

$$f_d = e^{-0.08 \times 0.5} (0.7041 \times 0 + (1 - 0.7041) \times 0) = 0$$

Calculation A

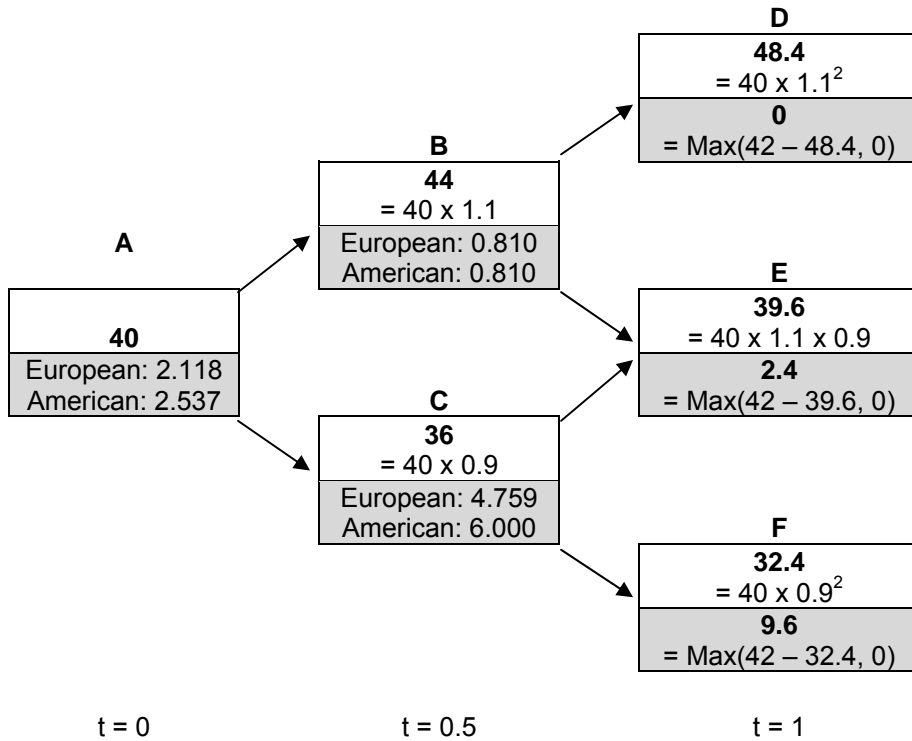
$$f = e^{-r\Delta t} [pf_u + (1-p)f_d]$$

$$f = e^{-0.08 \times 0.5} (0.7041 \times 14.2063 + (1 - 0.7041) \times 0) = 9.6104$$

**Q12-17**Data

$S_0 = 40$ ,  $u = 1.1$ ,  $d = 0.9$ ,  $r = 12\%$ ,  $T = 0.5$  (0.25@ period),  $K = 42$

Calculate the values of an European put and an American put



$$p = \frac{e^{r\Delta t} - d}{u - d} = \frac{e^{0.12 \times 0.25} - 0.9}{1.1 - 0.9} = 0.6523$$

European Put Calculation

$$f_u = e^{-r\Delta t} [pf_{uu} + (1-p)f_{ud}] = e^{-0.12 \times 0.25} (0.6523 \times 0 + (1-0.6523) \times 2.4) = 0.810$$

$$f_d = e^{-r\Delta t} [pf_{ud} + (1-p)f_{dd}] = e^{-0.12 \times 0.25} (0.6523 \times 2.4 + (1-0.6523) \times 9.6) = 4.759$$

$$f = e^{-r\Delta t} [pf_u + (1-p)f_d] = e^{-0.12 \times 0.25} (0.6523 \times 0.810 + (1-0.6523) \times 4.759) = 2.118$$

American Put Calculation

The option price at node B =  $\text{Max}(0.810, 42 - 44) = 0.810$

The option price at node C =  $\text{Max}(4.759, 42 - 36) = 6.000$

$$f = e^{-r\Delta t} [pf_u + (1-p)f_d] = e^{-0.12 \times 0.25} (0.6523 \times 0.810 + (1-0.6523) \times 6) = 2.537$$

The option price at node A =  $\text{Max}(2.537, 42 - 40) = 2.537$

**Do You Know?** If you get a pirate copy of the PAK study manual, you will lose a lot of exam information and benefits. Those are critical to pass the exam. Should you want to have more information or would like to report anybody selling the pirate copy, please feel free to contact me at [eddy.chan@pakstudymanual.com](mailto:eddy.chan@pakstudymanual.com)