SOA Exam P
CAS Exam 1

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Flashcards

Basic Probability Rules

(i) $P(\emptyset) = 0$

(ii) For any event *A*, $P(A) \le 1$

(iii) If $A \subset B$ then $P(A) \le P(B)$

(iv) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(v) If *A* and *B* are disjoint then $P(A \cup B) = P(A) + P(B)$

(vi) If $A_1, A_2, ..., A_n$ are mutually exclusive then $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$

(vii) For any event *A*, $P(A') = 1 - P(A)$

(viii) For any events *A* and *B*, $P(A) = P(A \cap B) + P(A \cap B')$

(ix) If events $B_1, B_2, ..., B_n$ are mutually exclusive and exhaustive they are called a partition of the probability space, and for any event *A*,

$$P(A) = \sum_{i=1}^{n} P(A \cap B_i)$$

(x) For any events *A*, *B*, and *C*, $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$$-P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Bayes Rule

If A and B are any events, then

$$P(B \mid A) = \frac{P(A \mid B) \cdot P(B)}{P(A)} = \frac{P(A \mid B) \cdot P(B)}{P(A \mid B) \cdot P(B) + P(A \mid B') \cdot P(B')}$$

If $B_1, B_2, ..., B_n$ forms a partition of the probability space, then

$$P(B_j \mid A) = \frac{P(A \mid B_j) \cdot P(B_j)}{\sum_{i=1}^{n} P(A \mid B_i) \cdot P(B_i)}$$

Moment Generating Function of a Random Variable

The moment generating function of X is $M_X(t) = E\left[e^{tX}\right]$.

For a discrete random variable, $M_X(t) = \sum_{\text{all } x_i} e^{tx_i} p(x_i)$.

For a continuous random variable, $M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x)dx$.

$$\frac{d}{dx} M_X(t)\Big|_{t=0} = M'_X(0) = E[X] \ , \quad \frac{d^2}{dx^2} M_X(t)\Big|_{t=0} = M''_X(0) = E[X^2] \ ,$$

$$\frac{d^n}{dx^n} M_X(t)\Big|_{t=0} = M_X^{(n)}(0) = E[X^n] \ .$$

Geometric Distribution Y With Parameter p ($0 < p < 1$)

Defined For $y = 1, 2, 3, \ldots$

In this version of the geometric distribution, Y represents the trial number of the first success, so $Y = X + 1$, where X is number of failures until the first success.

$$p(y) = (1 - p)^{y-1} p \quad \text{for} \quad y = 1, 2, 3, \ldots$$

$$E[Y] = \frac{1}{p} \quad , \quad Var[Y] = \frac{1-p}{p^2} .$$

Standard Normal Distribution

The standard normal random variable has mean 0 and variance 1.

The pdf is $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$. The distribution function is $\Phi(x) = P[X \le x]$. In the graph below $\Phi(1.96) = .975$, so that 1.96 is the 97.5 percentile of the standard normal distribution.