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The five exams contained in this booklet were developed in three difficulty levels with Level 3 being the most difficult.

Practice Exam 1

Question 1-1

Suppose $P(A \cup B) = 0.60$, $P(A \cap B) = 0.30$ and $P(A \cap B^C) = 0.25$.

Calculate $P(B)$.

A) 0.30

B) 0.35

C) 0.45

D) 0.55

E) 0.85

Question 1-2

You are given $P(A) = 0.30$, $P(B) = 0.25$ and $P(B \mid A) = 0.50$

Calculate $P(A^C \cap B^C)$.

A) 0.40

B) 0.58

C) 0.60

D) 0.85

E) 0.88

Question 2-9

Suppose $P(A)=0.6$, $P(B)=0.52$, $P(A\cap B)=0.14$ and $P(A^C\cap B^C)=0$. Furthermore, suppose $P\left(C\middle|\left(A\cap B^C\right)\right)=0.35$, $P\left(C\middle|\left(A^C\cap B\right)\right)=0.55$ and $P\left(C\middle|\left(A\cap B\right)\right)=0.85$.

Calculate $P(C)$.

A) 0.37 B) 0.49 C) 0.50 D) 0.62 E) 0.65

Question 2-10

Assume X and Y are random variables where $E(X)=4$, $E\left(x^2\right)=80$, $E(Y)=6$, $E\left(Y^2\right)=72$ and $Var(X+Y)=150$.

Calculate the $Cov(X,Y)$.

A) 11.0 B) 25.0 C) 50.0 D) 125.0 E) 400.0

Question 2-11

Suppose T follows a distribution with the following probability density function:

$$f(t)=\begin{cases}\dfrac{2}{t^3} & 1\le t\\ 0 & otherwise\end{cases}$$

Let X be a random variable which is the maximum of 4 and T. Calculate $E(X)$.

A) $\frac{1}{4}$ B) $\frac{23}{16}$ C) 2 D) $\frac{61}{16}$ E) $\frac{17}{4}$

Question 3-5

Suppose X follows a probability distribution such that $P\left(X=0\right)=0.75$ and for each $i\geq 1$, $P\left(X=i+1\right)=\frac{1}{6}P\left(X=i\right)$.

Calculate $P\left(X\leq 2\right)$.

A) 0.208 B) 0.243 C) 0.250 D) 0.900 E) 0.993

Question 3-6

An insurance company has issued 75 policies. The number of claims filed under each policy follows a Poisson distribution with a mean 3.

Assuming that the claims filed by each policyholder are independent of each other, what is the approximate probability that more than 250 claims will be filed by the group of policyholders?

A) 0.048 B) 0.168 C) 0.424 D) 0.576 E) 0.952

Question 3-7

Let Y = 650X and let X have the following probability density function:

$$f(x)=\begin{cases} c\sqrt{x} & 0<x \textbf{ and } x<1 \\ 0 & \textit{otherwise} \end{cases}$$

where c is constant.

Calculate $P(Y>400 \mid Y>100)$.

A 0.34 B) 0.52 C) 0.55 D) 0.63 E) 0.94

Practice Exam 4

Question 4-1

Suppose $P(A)=0.48$, $P(B)=0.42$ and $P(A\cap B)=0.17$ and $P(A^C\cap B^C)=0$. Furthermore, suppose $P\left(C\middle|\left(A\cap B^C\right)\right)=0.37$, $P\left(C\middle|\left(A^C\cap B\right)\right)=0.52$ and $P\left(C\middle|\left(A\cap B\right)\right)=0.81$.

Calculate $P(C)$.

A) 0.24

B) 0.38

C) 0.40

D) 0.44

E) 0.53

Question 4-2

Let X and Y represent the number of hours that a person spends cooking and doing the dishes, respectively, during a 30-day period. Assume that during a 30-day period, the expected number of hours that a person spends cooking is 20, with a variance of 40. Also assume that during a 30-day period, the expected number of hours that a person spends doing the dishes is 10, with a variance of 25. Suppose the covariance of X and Y is 10.

If 100 people are selected at random and observed for a 30-day period, what is the probability that the total number of hours spent by the group doing the dishes and cooking exceeds 2900?

A) 0.140

B) 0.505

C) 0.860

D) 0.869

E) 0.875

Question 5-6

Suppose $P(A \cap B) = 0.32$, $P\left(A^C \cap B^C\right) = 0.25$ and that $P(A) = P(B) + 0.19$.
Find $P(A)$.

A) 0.12

B) 0.31

C) 0.38

D) 0.44

E) 0.63

Question 5-7

An auto insurance company's review of claims yields the following table of data.

Model Year	Proportion of Insured Vehicles	Probability of Being Involved in Accident
2000	0.10	0.06
2001	0.15	
2002	0.25	0.03
Other	0.50	0.07

The probability of an insured car being model year 2001, given it was involved in an accident, is 0.134.

What value belongs in the missing cell of the table above?

A) 0.01

B) 0.02

C) 0.05

D) 0.13

E) 0.29

Solutions to Practice Exam 1

Solution 1-1

For this problem it is important to know that $P(A) = P(A \cap B) + P(A \cap B^C)$.
We start by writing out the equation for the union

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Now we plug in our important identity, which gives us

$$P(A \cup B) = P(A \cap B) + P(A \cap B^C) + P(B) - P(A \cap B)$$

The $P(A \cap B)$ and $-P(A \cap B)$ cancel out, and we can solve for $P(B)$

$$P(B) = P(A \cup B) - P(A \cap B^C)$$
$$\Rightarrow \quad P(B) = 0.60 - 0.25 = 0.35$$

Answer: B

Solution 1-2

First, DeMorgan's law states that

$$P(A^C \cap B^C) = P\left[(A \cup B)^C\right] = 1 - P(A \cup B).$$

Plugging in the equation for the union, we get

$$P(A^C \cap B^C) = 1 - (P(A) + P(B) - P(A \cap B)).$$

We also use that $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)}$. Multiplying both sides by $P(A)$, we get

$$P(B \cap A) = P(A \cap B) = P(B \mid A)P(A).$$

Now we can plug this back into our second equation, which gives us

$$P(A^C \cap B^C) = 1 - (P(A) + P(B) - P(B \mid A)P(A))$$
$$\Rightarrow \quad P(A^C \cap B^C) = 1 - (0.30 + 0.25 - (0.50)(0.30))$$
$$\Rightarrow \quad P(A^C \cap B^C) = 0.60$$

Answer: C

Solution 2-9

To solve, we are going to use

$$P(C) = P\left(C \middle| \left(A \cap B^C\right)\right) P\left(A \cap B^C\right) + P\left(C \middle| \left(A^C \cap B\right)\right) P\left(A^C \cap B\right) + P\left(C \middle| \left(A \cap B\right)\right) P\left(A \cap B\right)$$

$$+P(C|(A^C \cap B^C))P(A^C \cap B^C)$$

So we only need to find $P\left(A \cap B^C\right)$ and $P\left(A^C \cap B\right)$ to find $P(C)$.

$$P(A) = P(A \cap B) + P\left(A \cap B^C\right)$$
$$P\left(A \cap B^C\right) = P(A) - P(A \cap B) = 0.6 - 0.14 = 0.46$$

$$P(B) = P(A \cap B) + P\left(A^C \cap B\right)$$
$$P\left(A^C \cap B\right) = P(B) - P(A \cap B) = 0.52 - 0.14 = 0.38$$

Now we just plug in our known values and get

$$P(C) = (0.35)(0.46) + (0.55)(0.38) + (0.85)(0.14) + (0) = 0.489$$

Answer: B

Solution 2-10

We will solve for $Cov(X,Y)$ using the following formula:
$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$.

First, we need to find the $Var(X)$ and the $Var(Y)$.

The formula for *Var(X)* $= E\left(X^2\right) - \left[E(X)\right]^2 = 80 - [4]^2 = 64$.
The $Var(Y) = 72 - [6]^2 = 36$.

Using the $Var(X+Y)$ formula, we will have

$$150 = 64 + 36 + 2\ Cov(X,Y)$$

Thus, $Cov(X,Y) = 25.0$

Answer: B

Solution 3-5

Note that $\sum_{i=0}^{\infty} P(X=i)=1$ and $P(X \le 2)=P(X=0)+P(X=1)+P(X=2)$.

Since $P(X=0)=0.75$ and $\sum_{i=0}^{\infty} P(X=i)=1,$ then 1-$P(X=0)=\sum_{i=1}^{\infty} P(X=1)=0.25.$

According to the question, $P(X=2)=\frac{1}{6}P(X=1)$ and

$$P(X=3)=\left(\frac{1}{6}\right)P(X=2)=\left(\frac{1}{6}\right)^2 P(X=1).$$

It follows,

$$P(X=4)=\left(\frac{1}{6}\right)^3 P(X=1), \text{ etc.}$$

Notice, the question does not explicitly state the value of $P(X=1)$. However, we can see that $P(X=1)+P(X=2)+P(X=3)+\cdots$ is a sum of a geometric series with a starting value of $P(X=1)$ and factor of $\frac{1}{6}$. Thus, we can use the formula for the sum of a geometric series to determine $P(X=1)$.

$$\sum_{i=1}^{\infty} P(X=i)=\frac{P(X=1)}{1-\frac{1}{6}}.$$

It was shown earlier that $\sum_{i=1}^{\infty} P(X=i)=.25$

which implies $P(X=1)=.208.$

Then $P(X=2)=\frac{1}{6}P(X=1)=.035.$

Thus $P(X \le 2)=P(X=0)+P(X=1)+P(X=2)=.75+.208+.035=.993$

Answer: E

Solution 3-6

Let the total number of claims be defined as $S = \sum_{i=1}^{n} X_i$ where X follows a Poisson distribution with $E(X) = 3$ and $Var(X) = 3$.

Since each X_i is independent and identically distributed, then according to the Central Limit Theorem, $E(S) = nE(X)$ and $Var(S) = nVar(X)$ and S is approximately a Normal distribution for a large *n*.

In addition, the $StDev(S) = \sqrt{n}StDev(X)$.

Thus, $E(S) = (75)(3) = 225$ and $Var(S) = (75)(3) = 225$.

The $StDev(S) = (225)^{\frac{1}{2}} = 15$.

Since S is normally distributed, we can standardize S and use the Standard Normal Table to find the answer.

$$P(S \le 250) = P\left(\frac{S - E(S)}{StDev(S)} \ge \frac{250 - 225}{15}\right) = P(Z \ge 1.667)$$

$$= 1 - P(Z \le 1.667) = 1 - 0.952 = 0.048$$

Answer: A

Solution 3-7

First, we must find *c*. So we integrate $f(x)$ and set it equal to 1, and then solve for *c*.

$$\int_0^1 c\sqrt{x}\,dx = \frac{2}{3}x^{3/2}\,c\Big|_0^1 = \frac{2}{3}c = 1$$

This implies $c = \frac{3}{2}$.

Now, we have

$$P(Y > 400 | Y > 100) = P(650X > 400 | 650X > 100)$$

$$= P(X > 0.615 | X > 0.154).$$

Using the formula for conditional probability, we have

$$P(X > 0.615 | X > 0.154) = \frac{P(0.615 < X \textbf{ and } 0.154 < X)}{P(0.154 < X)} = \frac{P(0.615 < X)}{P(0.154 < X)}$$

So now we find $P(0.615 < X)$ and $P(0.154 < X)$.

$$P(0.615 < X) = \int_{0.615}^1 \tfrac{3}{2}\sqrt{x}\,dx = 0.517$$

$$P(0.154 < X) = \int_{0.154}^1 \tfrac{3}{2}\sqrt{x}\,dx = 0.94$$

Finally, our answer is 0.55.

Answer: C

Solution 3-8

In general, for constants a and b,
$Var(X+a+bY)=Var(X)+b^2Var(Y)+2bCov(X,Y)$. For this question, $a=25$ and $b=1.15$.

However, we first need to find $Cov(X,Y)$. To do so, use the formula:
$Var(X+Y)=Var(X)+Var(Y)+2Cov(X,Y)$.

$650=450+175+2Cov(X,Y)$

Thus, $Cov(X,Y)=\frac{25}{2}$.

Now, we can solve for the variance at the end of the year,

$$Var\left[X+25+(1.15)Y\right]=Var(X)+(1.15)^2Var(Y)+2(1.15)Cov(X,Y)$$

$$=450+(1.15)^2(175)+2(1.15)(12.5)=710$$

Answer: E

Solution 3-9

This problem can be solved by simply integrating the density function over
$1.5<X<2$ and $1.5<Y<2$.

$$\int_{1.5}^{2}\int_{1.5}^{2}\left(\frac{1}{8}x+\frac{1}{8}y\right)dx\,dy=\int_{1.5}^{2}(.109+.062y)\,dy=.109$$

Answer: B

Solutions to Practice Exam 4

Solution 4-1

To solve, we are going to use

$$P(C) = P\left(C \middle| \left(A \cap B^C\right)\right) P\left(A \cap B^C\right) + P\left(C \middle| \left(A^C \cap B\right)\right) P\left(A^C \cap B\right) + P\left(C \middle| (A \cap B)\right) P(A \cap B)$$

$$+ P(C|(A^C \cap B^C))P(A^C \cap B^C)$$

So we only need to find $P\left(A \cap B^C\right)$ and $P\left(A^C \cap B\right)$ to find $P(C)$.

$$P(A) = P(A \cap B) + P\left(A \cap B^C\right)$$
$$P\left(A \cap B^C\right) = P(A) - P(A \cap B) = 0.48 - 0.17 = 0.31$$

$$P(B) = P(A \cap B) + P\left(A^C \cap B\right)$$
$$P\left(A^C \cap B\right) = P(B) - P(A \cap B) = 0.42 - 0.17 = 0.25$$

Now we can just plug in our known values and we get

$$P(C) = (0.37)(0.31) + (0.52)(0.25) + (0.81)(0.17) + (0) = 0.382$$

Answer: B

Solution 4-2

Let T_i be the total number of hours spent by individual i cooking and doing the dishes. Thus, $T_i = X_i + Y_i$.

For an individual $i, E(T_i) = E(X_i + Y_i) = E(X_i) + E(Y_i) = 20 + 10 = 30.$

In addition,
$$Var(T_i) = Var(X_i + Y_i) = Var(X_i) + Var(Y_i) + 2\,Cov(X_i, Y_i) = 40 + 25 + 2(10) = 85$$
Let S be the total time spent by the group cooking and doing the dishes, thus S is defined to be $S = \sum_{i=1}^{n} T_i$, where $n = 100.$

Since each T_i are identical and identically distributed, then

$$E(S) = E\left(\sum_{i=1}^{n} T_i\right) = \sum_{i=1}^{n} E(T_i) = (100)(30) = 3{,}000.$$

In addition, $Var(S) = Var\left(\sum_{i=1}^{n} T_i\right) = \sum_{i=1}^{n} Var(T_i) = (100)(85) = 8500.$

Since n is large, according to the Central Limit Theorem, S is approximately a Normal distribution. Therefore we can use the Standard Normal Table to find $P(S > 2900)$.

We will first find $StDev(S) = (8500)^{1/2} = 92.195.$

Now we can standardize S, where
$$P(S \geq 2900) = P\left(\frac{S - E(S)}{StDev(S)} \geq \frac{2900 - 3000}{92.195}\right) = P(Z \geq -1.08)$$

Now by using the Standard Normal Table, we find $P(Z \leq -1.08) = 0.140$

Thus, by using the property, $P(Z \geq a) = 1 - P(Z \leq a) = 1 - 0.140 = 0.860$

Answer: C

Solution 5-5

We will need to use the conditional density function, $f\left(y|x=1\right)=\dfrac{f\left(1,y\right)}{f_x\left(1\right)}$.

First, we calculate $f_x\left(1\right)=\int_0^{2-x}\left(\frac{1}{2}x\,y+\frac{1}{2}x\right)dy=\int_0^{1}\left(\frac{1}{2}y+\frac{1}{2}\right)dy=\frac{3}{4}$

Next, $f\left(1,y\right)=\frac{1}{2}y+\frac{1}{2}$

Now, we have that $f\left(y|x=1\right)=\dfrac{f\left(1,y\right)}{f_x\left(1\right)}=\frac{2}{3}y+\frac{2}{3}$ for $0<y<1$.

Finally, we integrate from 0 to 1 since we are trying to find the probability that $Y<X$ given that X=1.

$$P\left(Y<X\,|X=1\right)=\int_0^1\left(\frac{2}{3}y+\frac{2}{3}\right)dy=1$$

Answer: C

Solution 5-6

First, note $P(A^C\cap B^C)=P\left[(A\cup B)^C\right]=1-P(A\cup B)=0.25$, so $P(A\cup B)=0.75$.

We will use $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ to solve.

We are given that $P(A)=P(B)+0.19$, but we are solving for $P(A)$ so we want to get $P(B)$ in terms of $P(A)$, that is, $P(B)=P(A)-0.19$.

Now we plug our values into the equation:

$$0.75=P(A)+P(A)-0.19-0.32$$
$$0.75=2\,P(A)-0.51$$
$$P(A)=0.63$$

Answer: E

Solution 5-7

We use Bayes' Theorem to solve this problem. Let A denote an insured car being involved in an accident, and let Y_0, Y_1, Y_2 and Y_{other} denote model years 2000, 2001, 2002 and Other, respectively. Then

$$P(Y_1 \mid A) = \frac{P(A \mid Y_1)P(Y_1)}{P(A \mid Y_0)P(Y_0) + P(A \mid Y_1)P(Y_1) + P(A \mid Y_2)P(Y_2) + P(A \mid Y_{other})P(Y_{other})}$$

and the value we are trying to find is $P(A \mid Y_1)$. Filling in the probabilities we already know, we have

$$(0.134) = \frac{P(A \mid Y_1)(0.15)}{(0.06)(0.10) + P(A \mid Y_1)(0.15) + (0.03)(0.25) + (0.07)(0.50)}$$

$$0.134 = \frac{0.15\,P(A \mid Y_1)}{0.15\,P(A \mid Y_1) + 0.0485}$$

Solving for $P(A \mid Y_1)$, the missing value is 0.05.

Answer: C