Addendum to Spring 2019 Edition
of Practice Problems in Advanced Topics in General Insurance

For each of the parts of the problems for which solutions are identified here, replace the solution in the original Spring 2019 Edition with the one in this Addendum.

Solution 1-37.
(a) From SOA Fall 2018 Exam GIADV Solutions: The assumption does not imply that the observed development factors are independent. While it does imply that the expected development does not depend on past development, other aspects of the distribution (such as the variance) can depend on past development.

The assumption does imply that they are uncorrelated. Correlation relates to expectations, which do not depend on previous observed values.

(b) Mack's other two assumptions:
Assumption 1: Independence of accident years.
Assumption 2: $\text{Var}(C_{i,k+1} \mid C_{i,1}, \ldots, C_{i,k}) = \alpha_k^2 {*} C_{j,k}$ (Proportionality condition)
Verbal description (from SOA GIADV Spring 2014 Solutions): The variance of cumulative claims at lag (k+1) is proportional to cumulative claims at lag k, with the constant depending only on the lag.

From SOA Fall 2018 Exam GIADV Solutions:
• The development in any given accident year is independent of the development in any other accident year.
• The variance of the claim amount in a given development year is proportional to the claim amount at the end of the previous development year. The constant can depend on the development year, but not on the accident year.

(d) From SOA Fall 2018 GIADV Solutions:
Commentary on Question:
Because 0.76 standard deviations could also be considered as an insignificant deviation from zero, it is also acceptable to conclude that the assumption is satisfied.

The standard deviation is $\sqrt{2/(5*4)} = 0.316$. The test statistic is 0.24/0.316 = 0.76 standard deviations below zero. There is some evidence of a negative correlation and hence an alternative model should be considered.

Solution 2-37.
(e) From SOA Fall 2018 Exam GIADV Solutions:
Any two of the following are sufficient for full credit.

• Linear with constant
• Factor times parameter
• Factor times parameter plus a calendar year effect
• Bornhuetter Ferguson
• Parameterized Bornhuetter Ferguson
• Cape Cod
• Additive

Solution 3-76.

(d) The expected payments that will be made during the last three months of 2017 for accident year 2016 will be equal to (2016 On-Level Earned Premium)*ELR*[G(18)-G(15)] = 8,500*0.6291*[0.91033-0.89378] ≈ **88**.

Solution 4-117.

(b) Qualitative techniques are needed to supplement quantitative techniques to ensure that all sources of uncertainty are captured. Qualitative techniques can be integrated into a step-by-step process in order to adjust the results of the quantitative techniques in order to account for the quantitative techniques' known weaknesses. (Marshall et al., p. 9)

From SOA Fall 2018 Exam GIADV Solutions:

- Quantitative methods can be complex and require significant amounts of time, data, and cost.
- Quantitative analysis of historical data cannot capture all sources of future uncertainty.

(c) ***(i)*** **Preparing the claims portfolio for analysis: SOA Fall 2018 Exam GIADV Sample Solution:** Selecting the risk classes to use or the granularity of classes.

Another Possible Answer: Qualitative analysis can be used to determine the appropriate valuation classes into which to split the claims portfolio. If it is impractical to conduct quantitative analysis at the granular level of each valuation class, then qualitative techniques may be used to decide how to allocate the results of a quantitative analysis on the aggregated valuation classes down to the more granular categories (Marshall et al., p. 12)

(ii) **Analyzing internal systemic risk**: **SOA Fall 2018 Exam GIADV Sample Solution:** Developing a balanced scorecard.

Another Possible Answer: Qualitative techniques can be used to assess specification error (e.g., number of models used and range of results, reasonableness checks conducted, subjective adjustments required, and the extent of monitoring and review), parameter selection error (e.g., ability to identify and use predictors, extent to which predictors lead rather than lag claim costs, subjective adjustments required), and data error (e.g., extent, timeliness, and reliability of information from business, access to data, and quality of reconciliations).

(iii) **Analyzing external systemic risk**: **SOA Fall 2018 Exam GIADV Sample Solution:** Identifying key systemic risks or ranking risk categories by expected impact.

Another Possible Answer: Qualitative techniques are useful for reflecting future external systemic risk that differs from the past.

(iv) **Determining correlation effects: SOA Fall 2018 Exam GIADV Sample Solution:** May need to set correlations without data or decide which correlations to model.

Another Possible Answer: Correlation coefficients can be derived following a qualitative analysis of internal systemic risk using a balanced scorecard approach. Qualitative analysis can be used to assume correlations between valuation portfolios and between outstanding claim and premium liabilities. (Marshall et al., p. 22).

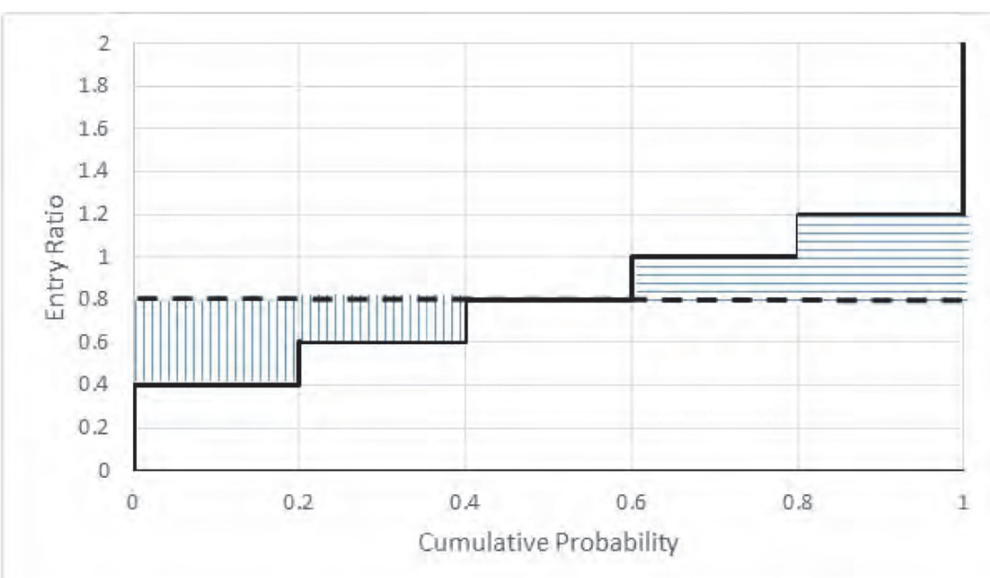
Solution 5-62. (b) The filled-out Table L is provided below.

r	φ*(r)	ψ*(r)
0.00	1.00	0
0.20	0.80	0
0.40	0.60	0
0.60	0.44	0.04
0.80	0.32	0.12
1.00	0.24	0.24
1.20	0.20	0.40
1.40	0.20	0.60
1.60	0.20	0.80
1.80	0.20	1.00
2.00	0.20	1.20

From SOA Fall 2018 Exam GIADV Solutions:
Commentary on Question: *The table is completed above, with further explanation below.*

One approach is to calculate the values directly from the definition. The average limited loss ratio is (20 + 30 + 40 + 50 + 60)/5 = 40. The average loss ratio is then 40/(1 – 0.2) = 50. For r = 0.80 (for example), the expected excess over 0.80(50) = 40 is (0 + 0 + 0 + 10 + 20)/5 = 6. It is then divided by the average loss ratio to obtain 6/50 = 0.12. Finally the loss-elimination ratio is added to obtain the Table L charge of 0.12 + 0.20 = 0.32. The Table L savings is (20 + 10 + 0 + 0 + 0)/5 = 6, divided by 50 to obtain 0.12.

A second approach is to construct a graph. The one below illustrates the calculation for r = 0.80. The cumulative probability increases by 0.2 at each of the entry ratios 20/50, 30/50, 40/50, 50/50, and 60/50. The charge is the area shaded with horizontal stripes, which is 0.08 + 0.04 = 0.12 plus the loss elimination ratio, to give 0.32. The savings is the area shaded with vertical stripes, which is 0.04 + 0.08 = 0.12.



Solution 6-168. (c) The SOA Fall 2018 Exam GIADV Solutions used the assumption that the layer to which the additional multiplicative loading would pertain would be the layer between $3,000,000 and $5,000,000, rather than the layer between $2,777,449 and $5,000,000. If one follows that approach, the answer would be

$$\frac{G(100\%) - G(60\%)}{G(60\%) - G(20\%)} = \frac{95\% - 80\%}{80\% - 50\%} = \frac{15}{30} = 0.5.$$

So the loss cost loaded for the exposure above $3,000,000 would be 25.48%*(1+0.5) ≈ **38.22%** of premium.

Solution 8-55. (a) From SOA Fall 2018 Exam GIADV Solutions:

The mean is 0.05(1500) + 0.03(2000) + 0.02(3000) = 195.
The variance is $0.05(1500)^2 + 0.03(2000)^2 + 0.02(3000)^2 - 195^2 = 374{,}475$.
The risk load is 0.0001(374,475) = **37.45**.

(b) From SOA Fall 2018 Exam GIADV Solutions:

Contract X: Mean = 0.05(500) + 0.03(2000) = 85, Variance = $0.05(500)^2 + 0.03(2000)^2 - 85^2 = 125{,}275$.
Contract Y: Mean = 0.05(1000) + 0.02(3000) = 110, Variance = $0.05(1000)^2 + 0.02(3000)^2 - 110^2 = 217{,}900$.
Renewal risk load for X: 37.45 – 0.0001(217,900) = **15.66**.
Renewal risk load for Y: 37.45 – 0.0001(125,275) = **24.92**.

(c) For the Shapley Method, it is necessary to calculate the shared covariance Cov(X, Y). We know that Var(X+Y) = Var(X) + Var(Y) + 2*Cov(X+Y), implying that
Cov(X+Y) = [Var(X+Y) – Var(X) – Var(Y)]/2 = (374,475 – 125,275 – 217,900)/2 = 15,650.

Renewal Scenario: Each Contract Renewed: Shapley Method		
	Contract X	**Contract Y**
(4) Change in Variance	$\text{Var(X)} + \text{Cov(X, Y)} =$ 125,275 + 15,650 = 140,925	$\text{Var(Y)} + \text{Cov(X, Y)} =$ 217,900 + 15,650 = 233,550
(5) Risk-Load Multiplier	0.0001	0.0001
(6) Risk Load	**14.0925**	**23.355**

Thus, under the Shapley Method, the renewal risk load for Contract X is **14.0925**. The renewal risk load for Contract Y is **23.355**.