Errata and Updates for the 2022 ACTEX Manual for Exam P

(Last updated 5/5/2023) sorted by page

Page 106 **Solution to Problem 1. Fifth line.**

Change $\binom{8}{2}$ to $\binom{8}{3}$.

Page 108 **Solution to Problem 11. Last line.**

Change **Answer A** to **Answer C**.

Page 140 **Last line.**

Change the formula to:

$$\begin{aligned} Var[X] &= E[(X - E[X])^2] = E\left[\left(X - \frac{7}{2}\right)^2\right] \\ &= \left(1 - \frac{7}{2}\right)^2 \times \frac{1}{6} + \left(2 - \frac{7}{2}\right)^2 \times \frac{1}{6} + \cdots + \left(6 - \frac{7}{2}\right)^2 \times \frac{1}{6} = \frac{35}{12}. \end{aligned}$$

Page 163 **Solution to Problem 21. Third and Fourth line.**

Change "low-risk drivers" to "high-risk drivers".

Page 180 **4th Line from the bottom.**

The numerical answer should be 0.000833524 instead of 0.0750, namely

$$\frac{10!}{2! \times 1! \times 0! \times 3! \times 1! \times 3!} \times \left(\frac{1}{6}\right)^2 \left(\frac{1}{6}\right)^1 \left(\frac{1}{6}\right)^0 \left(\frac{1}{6}\right)^3 \left(\frac{1}{6}\right)^1 \left(\frac{1}{6}\right)^3 = 0.000833524.$$

Page 254 **At the begining of the page**

Add

Order statistics for a continuous random variable X

and at the end of first line, change "The density or probability function" to "In the continuous case the density function".

Page 256 **At the begining of the page, before Section 9.3**

Add the following paragraphs:

Order statistics for a discrete random variable X

If X has a discrete distribution, the distributions of the order statistics for a random sample from the distribution of X are found using combinatorial methods. An illustration based on the die toss example described will help to explain this.

Suppose that $Y_1, Y_2, ..., Y_{10}$ are the order statistics for a random sample of the outcomes of 10 independent tosses of a fair die (stating that the tosses are independent is redundant since the definition of a random sample includes the requirement that the outcomes are independent). Suppose we wish to find the probability $P[Y_8 = 4]$, the probability that the third from the largest toss (eighth from the smallest toss) is a 4.

In the case of X being discrete, it is easier to look at the distribution functions of the Y_k's than the probability functions. Since X is discrete, each Y_k is also discrete so that
$P[Y_8 = 4] = P[Y_8 \leq 4] - P[Y_8 \leq 3]$. Finding the cdf of Y_8 is a combinatorial problem.

To have $Y_8 \leq 4$ we must have at least 8 tosses ≤ 4, which means either 8, 9 or all 10 of the tosses are ≤ 4. The probability of any one of the tosses of a fair die being ≤ 4 is $\frac{4}{6} = \frac{2}{3}$. The probability that exactly 8 out of 10 independent tosses of a fair die are ≤ 4 is $\binom{10}{8} \times (\frac{2}{3})^8 \times (\frac{1}{3})^2$. In a similar way we get the probability that exactly 9 of the 10 tosses are ≤ 4 to be $\binom{10}{9} \times (\frac{2}{3})^9 \times (\frac{1}{3})^1$, and the probability that exactly 10 of the tosses is ≤ 4 is $\binom{10}{10} \times (\frac{2}{3})^10 \times (\frac{1}{3})^0$. The sum of these three probabilities is $P[Y_8 \leq 4]$. In a similar way we can find $P[Y_8 \leq 3]$ as the sum of the probabilities of exactly 8, 9 or 10 tosses are ≤ 3.

In general for a discrete integer-valued random variable X, the k-th order statistic probability $P[Y_k = j]$ is found as $P[Y_k \leq j] - P[Y_k \leq j-1]$, where $P[Y_k \leq j]$ and $P[Y_k \leq j-1]$ are found using combinatorial methods.

This level of detail involving order statistics is unlikely to arise on Exam P.

Page 266 **Solution to Problem 14. Last line.**

Change $\Phi(2.28) = 0.9987$ to $\Phi(2.28) = 0.9887$.

Page 290 **Problem 2. Second rom of the table.**

Change 200,00 to 200,000.

Page 300 **Solution to Problem 4. Seventh line.**

Add the missing e so that the formula becomes:

$$\int_2^\infty t \times \frac{1}{3} e^{-t/3}\, dt = \int_2^\infty t\, d(-e^{-t/3}) = -te^{-t/3}\Big|_2^\infty - \int_2^\infty -e^{-t/3} dt = 2e^{-2/3} + 3e^{-2/3}$$
$$= 5e^{-2/3}.$$

Page 320 **Problem 8.**

Change the first sentence to:

Two players put one dollar each into a pot.

Page 322 **Problem 21.**

Change the choices to:

(A) $\dfrac{1}{3(9a+3)}$ (B) $\dfrac{2}{3(9a+3)}$ (C) $\dfrac{1}{3(3a+1)}$ (D) $\dfrac{4}{3(9a+3)}$ (E) $\dfrac{5}{3(9a+3)}$

Page 327 **Solution to Problem 5.**

The last number should be 3 and the correct answer should be D instead of E.

Page 331 **Solution to Problem 16.**

Change the final answer from **Answer C** to **Answer B**.

Page 332 **Solution to Problem 21.**

Change the last formula to:

$$E[X^2] - (E[X])^2 = \frac{a+1}{3 \times (3a+1)} - \frac{1}{9} = \frac{2}{3(9a+3)}.$$

Page 514 **Problem 9. Third line.**

Change it to:

$P[K = 0] = P\left[X \leq \frac{1}{2}\right]$ and $P[K = k] = P\left[k - \frac{1}{2} \leq X \leq k + \frac{1}{2}\right]$ for $K = 1,\ 2,\ 3,\ \ldots$

Page 523 **Solution to Problem 8.**

Replace the solution by:

Note that if a fair coin is tossed $X = x$ times, the number of heads Y has a Binomial$(n = x, p = 1/2)$ distribution. This means

$$E(Y|X) = \frac{1}{2}X$$
$$E(Y^2|X) = Var(Y|X) + [E(Y|X)]^2 = \left(\frac{1}{2}X\right)\frac{1}{2} + \left(\frac{1}{2}X\right)^2 = \frac{X + X^2}{4}.$$

To calculate the desired variance, we use the formula

$$Var[Y|X \text{ is even}] = E[Y^2|X \text{ is even}] - (E[Y|X \text{ is even}])^2.$$

The conditional mean :

$$\begin{aligned} E[Y|X \text{ is even}] &= E(Y|X=2)\,P(X=2|X \text{ is even}) \\ &+ E(Y|X=4)\,P(X=4|X \text{ is even}) \\ &+ E(Y|X=6)\,P(X=6|X \text{ is even}) \end{aligned}$$

When X is even, probability of X being 2,4, or 6 must be the same. Therefore,

$$P(X=2|X \text{ is even}) = P(X=4|X \text{ is even}) = P(X=6|X \text{ is even}) = \frac{1}{3}.$$

Using the $E(Y|X) = X\frac{1}{2}$, we obtain

$$E[Y|X \text{ is even}] = \left(2 \times \frac{1}{2}\right)\frac{1}{3} + \left(4 \times \frac{1}{2}\right)\frac{1}{3} + \left(6 \times \frac{1}{2}\right)\frac{1}{3} = 2.$$

Similarly, using $E(Y^2|X) = \frac{X+X^2}{4}$,

$$\begin{aligned} E[Y^2|X \text{ is even}] &= E[Y^2|X=2]\,P[X=2|X \text{ is even}] \\ &+ E[Y^2|X=4]\,P[X=4|X \text{ is even}] \\ &+ E[Y^2|X=6]\,P[X=6|X \text{ is even}] \\ &= \left(\frac{2+4}{4}\right)\frac{1}{3} + \left(\frac{4+16}{4}\right)\frac{1}{3} + \left(\frac{6+36}{4}\right)\frac{1}{3} = \frac{68}{12}. \end{aligned}$$

Conbining, we get $Var[Y|X \text{ is even}] = \frac{68}{12} - 2^2 = \frac{5}{3}$.

Answer E

Page 523 **Solution to Problem 9.**

Replace the solution by:

The pdf of X is $f_X(x) = e^{-x}$ for $x > 0$.

$P[K=0] = P\left[X \leq \frac{1}{2}\right] = \int_0^{1/2} e^{-x}\, dx = 1 - e^{-1/2}$.

$P[X=k] = P\left[k - \frac{1}{2} < X \leq k - \frac{1}{2}\right] = \int_{k-\frac{1}{2}}^{k+\frac{1}{2}} e^{-x}\, dx = e^{-\frac{k-1}{2}} - e^{-\frac{k+1}{2}} = e^{-k} \times \left[e^{\frac{1}{2}} - e^{-\frac{1}{2}}\right]$.

$$\begin{aligned} E[K] &= \sum_{k=0}^{\infty} k \times P[K=k] = \sum_{k=1}^{\infty} k \times P[K=k] \\ &= \left[e^{\frac{1}{2}} - e^{-\frac{1}{2}}\right] \times \sum_{k=1}^{\infty} k \times e^{-k} = \left[e^{\frac{1}{2}} - e^{-\frac{1}{2}}\right] \times \frac{e^{-1}}{(1-e^{-1})^2} = .9595. \end{aligned}$$

Note that we have used the increasing geometric series summation relationship

$$\sum_{k=1}^{\infty} k \times a^k = \frac{a}{(1-a)^2} \text{ for } -1 < a < 1.$$

Answer B