Digital Actuarial Resources

Equation Guide (Sample)

Society of Actuaries
Exam FM: Financial Mathematics

Introduction:

This compilation of equations is designed to aid the student in preparing for Exam FM (Financial Mathematics) offered through the Society of Actuaries starting in the spring of 2005. The comprehensive list contains equations for most of the theory of interest subjects that the exam covers. This formula sheet includes topics from compound interest to annuities to stocks. The list is not guaranteed to cover all the material that exists on the exam. Equations that are unlikely to show up on an exam are not included in this compilation. The 180+ formulas are neatly organized into several categories. Refer to the index for guidance.

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Section 1: Introductory Interest Equations

$a(t) =$ accumulation function

$\mathrm{A(t) = k * a(t)} =$ amount function

$I_n = A(n) - A(n-1) =$ interest earned in n$^{\text{th}}$ period

$i_n = \dfrac{A(n) - A(n-1)}{A(n-1)} = \dfrac{a(n) - a(n-1)}{a(n-1)} =$ effective rate of interest during n$^{\text{th}}$ period

$d_n = \dfrac{A(n) - A(n-1)}{A(n)} = \dfrac{a(n) - a(n-1)}{a(n)}$ effective rate of discount during n$^{\text{th}}$ period

$v = (1+i)^{-1}$
$v^n = (1+i)^{-n}$

$d = \dfrac{i}{i+1} = iv$

$v + d = 1$

$(1-d)(1+i) = 1$

$d = 1 - v$

$i - d = id$

Simple Interest:

$i =$ annual rate of simple interest
$t =$ number of years
$ac.v. = a(t) = 1 + i * t$
$p.v. = a^{-1}(t) = (1 + i * t)^{-1}$

Simple Discount:

$d =$ annual rate of simple discount
$t =$ number of years
$ac.v. = a(t) = (1 - d * t)^{-1}$
$p.v. = a^{-1}(t) = 1 - d * t$

Compound Interest:

$i =$ annual effective rate of interest

$ac.v. = a(t) = (1+i)^t$
$p.v. = a^{-1}(t) = (1+i)^{-t} = v^t$

Compound Discount:

$d =$ annual effective rate of discount

$ac.v. = a(t) = (1-d)^{-t}$
$p.v. = a^{-1}(t) = (1-d)^t = v^t$

Nominal, Compound Interest:

$i^{(m)} =$ nominal rate of interest compounded m times per year

$$1+i = \left(1+\frac{i^{(m)}}{m}\right)^m$$

$$ac.v. = a(t) = \left(1+\frac{i^{(m)}}{m}\right)^{mt}$$

$$p.v. = a^{-1}(t) = \left(1+\frac{i^{(m)}}{m}\right)^{-mt}$$

$\frac{i^{(m)}}{m}$ = effective rate of interest per m^{th} of a year

$\frac{d^{(m)}}{m}$ = effective rate of discount per m^{th} of a year

Nominal, Compound Discount:

$d^{(m)} =$ nominal rate of discount compounded m times per year

$$d = 1-\left(1-\frac{d^{(m)}}{m}\right)^m$$

$$ac.v. = a(t) = \left(1-\frac{d^{(m)}}{m}\right)^{-mt}$$

$$p.v. = a^{-1}(t) = \left(1 - \frac{d^{(m)}}{m}\right)^{mt}$$

Force of Interest:

$$\delta_t = \frac{A'(t)}{A(t)} = \frac{a'(t)}{a(t)} = \text{ force of interest}$$

If δ is a constant, then $a(t) = e^{\delta t}$

If δ_r is a function of r, then $a(t) = e^{\int_0^t \delta_r \, dr}$

$e^{\delta} = 1 + i$, $\delta = \ln(1+i)$

Forms of Short-Term Interest:

Exact Simple Interest: exact number days, 365 days per year
Ordinary Simple Interest: each month has 30 days, 360 days per year
Banker's Rule: exact number of days, 360 days per year

Linear Interpolation:

*Use for finding an unknown rate of interest (j)
* $j_1 =$ first rate of interest
* $j_2 =$ second rate of interest
* $\Delta j = j_2 - j_1$
* $f(j) =$ a function of j with one side of the equation set to 0
  *Must solve for j (the unknown rate of interest) that makes the equation true

$$j = j_1 + \Delta j * \left(\frac{-f(j_1)}{f(j_2) - f(j_1)}\right)$$

Other Equations:

For compound interest, $a(t+s) = a(t) * a(s)$

$$\frac{i^{(m)}}{m} - \frac{d^{(m)}}{m} = \frac{i^{(m)}}{m} * \frac{d^{(m)}}{m}$$

If the length of time is n + k, where n is an integer and 0<k<1, use compound interest over the n periods and apply simple interest over time k:

ac.v. = $(1+i)^{n+k} \approx (1+i)^n (1+ki)$
p.v. = $v^{n+k} = (1-d)^{n+k} \approx v^n (1-kd)$

$$i > i^{(2)} > i^{(3)} > .... > \delta > .... > d^{(3)} > d^{(2)} > d$$