Practice Exam 1

Questions

1-1. A share of AOK company stock is now valued at $40. The risk-free rate is 3.5%. The company has just paid its $0.50 semiannual dividend. A 40-strike European call maturing in 1 year sells for 8.10.

What is the price of a 1-year 40-strike European put?

A) 4.60 B) 7.70 C) 8.48 D) 9.18 E) none of these

1-2. Near market closing time on a given day, the European call and put prices for a stock are available as follows:

Strike Price	Call Price	Put Price
40	11	3
50	6	8
55	3	11

The options have expiration time $T = .5$. The continuously compounded annual interest rate is $r = .04$.

Mary constructs the following portfolio:

- Long two call options with strike price 40;
- Short six call options with strike price 50;
- Lend $2; and
- Long some calls with strike price 55.

The $2 she lends is obtained from the sale and purchase of the options.

What is her arbitrage profit at $T = .5$ if the price of the stock is 52 at that time?

A) 0 B) 5.02 C) 4 D) 6.08 E) 14.04

1-3. The dollar exchange rate for euros is .92 $/euro. The dollar interest rate is 4% and the euro interest rate is 3.5%. The price of a six month dollar denominated $.90-strike call option for euros is .076.

Find the price of the euro denominated six month call option with strike price $1/.90$.

A) .065 B) .094 C) .10 D) .103 E) .105

1-4. The current price of a stock is 30. Given $r = .08, \delta = .01$ and $\sigma = .2$, use a two period binomial tree to find the price of a one year European call with strike $K = 31$.

A) 2.87 B) 2.98 C)3.201 D) 3.510 E) 3.732

1-5. The following tree shows the option values resulting from a binomial tree valuation of a 1-year call option–except for the final answer C_0. The interest rate used was $r = .05$.

Find C_0.

		C_{uu}	10.4314
	C_u		
C_0		C_{ud}	0.2243
	C_d 0.1017		
		C_{dd}	0

A) 2.09 B) 2.19 C) 2.25 D) 2.58 E) 2.86

1-6. The current price of a stock is 35. In six months the stock will either go up to 40 or down to 31. The stock pays no dividends and the risk-free rate is 4%. The current price of a put option with strike price 34 is 1.45.

Which of the following is true?

A) There is no arbitrage opportunity.

B) You can make an arbitrage profit of 0.05 if you sell a put option, sell short .3333 shares of stock and lend 13.0693.

C) You can make an arbitrage profit of 0.05 if you buy a put option, buy .3333 shares of stock and borrow 13.0693.

D) You can make an arbitrage profit of 0.07 if you sell a put option, sell short .3823 shares of stock and lend 13.2515.

E) You can make an arbitrage profit of 0.07 if you buy a put option, sell short .3823 shares of stock and borrow 13.2515.

1-7. The current price of a stock is 32. You wish to buy a 32-strike three month put option on the stock. You are given

(i) δ δ = 0.01.
(ii) σ = 0.25
(iii) The continuously compounded risk-free interest rate is 0.035

Calculate the price of the option.

A) 1.34 (B) 1.43 C) 1.57 D) 1.69 E) 1.83

1-8. You wish to buy a dollar denominated six month put option to sell euros at a strike price of .80 dollars per euro. The current exchange rate is .81 dollars per euro. The continuously compounded risk-free interest rates are 4.5% for euros and 4% for dollars. The annual volatility of the exchange rate is 12%.

Find the price in dollars of the put option.

A) .017 B) .019 C) .021 D) .023 E) .025

1-9. A stock is currently selling for \$32. A three month 32-strike call option is available. You are given that $\sigma = 0.3$ and the delta of the call option is 0.5567.

Given that $r = .04$ and $d_2 = .0175$, which of the following represents the Black-Scholes price of the option?

A) $17.81 - 12.64\int_{-\infty}^{.0175} e^{-x^2/2}dx$

B) $17.81 - 32.31\int_{-\infty}^{.0175} e^{-x^2/2}dx$

C) $17.81 + 32.32\int_{-\infty}^{.0175} e^{-x^2/2}dx$

D) $12.89\int_{-\infty}^{.0175} e^{-x^2/2}dx - 17.81$

E) None of the above

1-10. The delta of a three month call option on a stock is $\Delta = .5464$, and you are given that $d_1 = .12$.

Find the continuous dividend yield δ.

A) .005 B) .01 C) .015 D) .02 E) .025

1-11. A stock has current price of 95, with $\sigma = .25$. The continuously compounded interest rate and dividend yield are $r = .03$ and $\delta = .01$. You are given that $p^*=.455921$.

Use a two period binomial tree to find the price of a one year (arithmetic) average strike call option.

A) 3.58 B) 3.97 C) 4.36 D) 4.58 E) 4.79

1-12. A stock has current price of 95, with $\sigma = .25$. The continuously compounded interest rate and dividend yield are $r = .03$ and $\delta = .01$. You are given that $p^*=.455921$.

Use the two period binomial tree in problem 11 to find the price of a gap call option which pays $S - 100$ if $S > 105$.

A) 7.38 B) 7.67 C) 7.96 D) 8.38 E) 8.59

1-13. Suppose S follows a mean reverting process so that $dS(t) = \lambda[\alpha - S(t)]dt + \sigma dZ(t)$. In this case $a(S,t) = \lambda[\alpha - S]$ and $\sigma(S,t) = \sigma$.

Let $F(S) = S^{3/2}$.

Find $dF(S)$.

A) $\left(\frac{3}{2}S^{1/2}\lambda(\alpha - S) + \frac{3}{8\sqrt{S}}\sigma^2\right)dt + \frac{3}{2}S^{1/2}\sigma dZ$

B) $\left(\frac{3}{2}S^{1/2}\lambda(\alpha - S) + \frac{3}{4\sqrt{S}}\sigma^2\right)dt + \frac{3}{2}S^{1/2}\sigma dZ$

C) $\left(\frac{3}{2}S^{1/2}\lambda(\alpha - S) + \frac{3}{8\sqrt{S}}\sigma^2\right)dt + \frac{3}{2}S^{1/2}\lambda(\alpha - S)dZ$

D) $\left(\frac{3}{2}S^{1/2}\lambda(\alpha - S) + \frac{3}{4\sqrt{S}}\sigma^2\right)dt + \frac{3}{2}S^{1/2}\lambda(\alpha - S)dZ$

E) None of these

1-14. Suppose that a stock price $S(t)$ follows the geometric Brownian motion $\frac{dS(t)}{S(t)} = .13dt + .2dZ(t)$, with $S(0) = 1$.

Let $R(t)$ be the ratio of the mean of $\ln(S(t))$ to the variance of $\ln(S(t))$. What is $R(2)$?

A) 0 B) 1.79 C) 2.75 D) 4.58 E) undefined

1-15. Let $\{Z(t)\}$ be a pure Brownian motion. You are given:

(i) $U(t) = 2Z(t) - 3$

(ii) $V(t) = \left[Z(t)\right]^3 - t$

(iii) $W(t) = \left[Z(t)\right]^2$

Which of the processes defined above has / have zero drift?

A) (i) only B) (ii) only C) (iii) only D) all E) none

1-16. What is the missing number in the three year Black-Derman-Toy tree below?

		0.13843
	0.10362	
0.08000		0.10635
	0.07676	
		?

A) .05133 B) .06237 C) .07143 D) .07505 E) .08170

1-17. Use the tree in problem 16 to find the price of a 3-year zero-coupon bond.

A) .513 B) .623 C) .767 D) .868 E) .904

1-18. The current price of a stock is 25, and its continuously compounded dividend yield is $\delta = .03$. The continuously compounded risk-free rate is $r = .04$ and the volatility is $\sigma = .3$.

Find the elasticity Ω for a three month ($T = .25$) 26-strike call.

A) 6.47 B) 7.32 C) 9.83 D) 10.03 E) 11.31

1-19. Bond forward prices are lognormally distributed. You are given the following information.

Bond Maturity	1	2	3
Bond Price	.927	.852	.784
1-year forward price σ		.10	.135

Find the price of a call to buy the one-year zero-coupon bond forward for a price of .94 in two years.

A) .027 B) .039 C) .043 D) .049 E) .052

1-20. The one-year futures price for a commodity is $F = 3$, and the volatility is .25. The risk-free rate is 4%.

Find the price for a 3-strike call option on the future.

A) .287 B) .274 C) .259 D) .243 E) .227

1-21. The current price of a stock is 46. You wish to buy 200 44-strike six month call options on the stock. You are given

(i) δ = 0.02.
(ii) σ = 0.25
(iiii) The continuously compounded risk-free interest rate is 0.05.

Calculate the price of the block of 200 options.

A) \$1305 B) \$1208 C) \$1150 D) \$1951 E) \$898

1-22. You wish to buy a block of one million euro-denominated six month call options to purchase dollars at a strike price of .92 euros per dollar. The current exchange rate is .90 euros per dollar. The continuously compounded risk-free interest rates are 5% for euros and 4% for dollars. The annual volatility of the exchange rate is 10%.

Find the price in euros of the block of call options.

A) 12,800 B) 15,600 C) 21,460 D) 29,200 E) 33,700

Solutions to Practice Exam 1

1-1. This is a parity problem with dividends. The appropriate equation here is

$C(K,T)-P(K,T)=$ S_0 -PV(Dividends) - $e^{-rT}K$

For this problem, we have

$8.10-P(40,1)=40-\left(0.5e^{-.035(.5)}+0.5e^{-.035(1)}\right)-40e^{-.035(1)}$

$P(K,T)=7.70$

Answer B

1-2. For Mary's portfolio the number of long calls at $K=55$ is not given. However you can quickly figure out what it is. The arbitrage lends \$2, so in order to have 0 outlay at the beginning there must be \$2 of excess cash obtained from the sale and purchase of calls.

If there are *n* long calls at $K=55$ we have the following proceeds from options.

Strike	40	50	55
Position	Long 2	Short 6	Long *n*
Proceeds	-22	+36	-3n

Since total proceeds are 1 to lend, we have $-22+36-3n=2\rightarrow n=4$.

Mary has no out-of-pocket cost at time 0. She earns \$2 and invests it at the continuous rate $r=.04$. Her profit at time .25 is the future value of the invested \$2 + the sum of the payoffs of the options in the portfolio.

$2e^{.02}+2(52-40)-6(52-50)+4(0)=14.04$

Answer E

1-3. We first use the relationship $C_{\$}\left(x_0, K, T\right) = x_0 K P_{euro}\left(\frac{1}{x_0}, \frac{1}{K}, T\right)$.

For this problem we have

$$.076 = .92(.90)P_{euro}\left(\frac{1}{.92}, \frac{1}{.90}, .5\right) \rightarrow P_{euro}\left(\frac{1}{.92}, \frac{1}{.90}, .5\right) = .09179$$

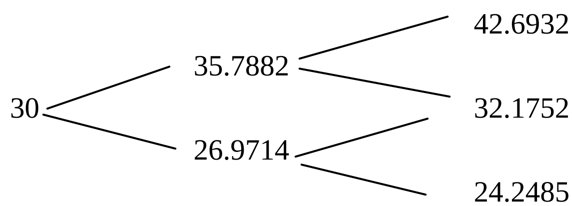
Then we can find the value of the euro-denominated call using parity.

$$C_{euro}\left(\frac{1}{.92}, \frac{1}{.90}, .5\right) - .09179 = \frac{1}{.92}e^{-.04(.5)} - \frac{1}{.90}e^{-.035(.5)}$$

$$C_{euro}\left(\frac{1}{.92}, \frac{1}{.90}, .5\right) = .06539$$

Answer A

1-4. It is best to use the p^* method here. The tree of stock prices and the calculated stock prices for the nodes are given below.



p^* 0.4647

Year 2	S_{uu}	S_{ud}	S_{dd}
	42.6932	32.1752	24.2485
	C_{uu}	C_{ud}	C_{dd}
	11.6932	1.1752	0
Year 1	S_u		S_d
	35.788		26.971372
	C_u		C_d
p^* Calculation	5.8253		0.5247
Year 0	S		
	30		
	Call value		
p^* Calculation	2.8707		

Answer A

1-5. Note that we can find p^* using the relation

$$C_d = e^{-rh}\left(p^* C_{ud} + (1-p^*)C_{dd}\right)$$

$$.1017 = e^{-.05(.5)}\left(p^*\left(.2243\right) + (1-p^*)0\right) \rightarrow p^* = .4649$$

Next we find C_u.

$$C_u = e^{-rh}\left(p^* C_{uu} + (1-p^*)C_{du}\right)$$

$$C_u = e^{-.025}\left(.4649\left(10.4314\right) + .5351\left(.2243\right)\right) \rightarrow C_u = 4.8469$$

Finally we find C_0

$$C_0 = e^{-rh}\left(p^* C_u + (1-p^*)C_d\right)$$

$$C_0 = e^{-.025}\left(.4649\left(4.8469\right) + .5351\left(.1017\right)\right) \rightarrow C_0 = 2.25$$

Answer C

1-6. We first need to find the correct put price using a one period binomial tree. This price is 1.40. The calculations are shown below.

δ	0	r	0.04		
K	34	h	0.5		
S_0	35				
u	1.142857	S_u	40	P_u	0
d	0.885714	S_d	31	P_d	3
Δ	-0.33333		B	13.069316	
Price	1.402649				

Since you can sell a 34-strike put for 1.45 in the market, you can offset that sale with a synthetic 34-strike put purchase for 1.40.

To do this you sell short .3333 shares of the stock and lend 13.0693. You will make a profit of 1.45 – 1.40 = .05 at time 0, and synthetic put purchase will offset the actual put sale in one year.

Answer B

1-7. The results of the put price calculations are given below.

Stock Price *S*	32.00
Exercise *K*	32.00
Sigma	0.25
r	0.035
T	0.25
Div Yield δ	0.01
*d*1	0.11
*d*2	-0.01
N(*d*1)	0.5438
N(*d*2)	0.4960
N(-*d*1)	0.4562
N(-*d*2)	0.5040
Put	1.425061

Answer B

1-8. The calculation results are given below.

x_0	0.810
Exercise *K*	0.800
Sigma	0.120
r	0.040
T	0.500
r_f	0.045
*d*1	0.16
*d*2	0.07
N(*d*1)	0.5636
N(*d*2)	0.5279
N(-*d*1)	0.4364
N(-*d*2)	0.4721
Put	0.024582

Answer E

1-9. Recall that the Black-Scholes call price can also be written as
$C = \Delta S - Ke^{-rT}N\left(d_2\right)$.

Thus
$$C = .5567\left(32\right) - 32e^{-.04(.25)}N\left(.0175\right)$$
$$= 17.81 - 31.68N\left(.0175\right)$$
$$= 17.81 - 31.68\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{.0175} e^{-x^2/2}dx$$
$$= 17.81 - 12.64\int_{-\infty}^{.0175} e^{-x^2/2}dx$$

Answer A

1-10. Since $d_1 = .12$, $N\left(d_1\right) = .5478$. Recall that $\Delta = e^{-\delta T}N\left(d_1\right)$. Thus
$\Delta = .5464 = e^{-.25\delta}\left(.5478\right) \rightarrow \delta = .0102$

Answer B

1-11. The tree of stock values is

		138.0244
	114.509	
95		96.91913
	80.40692	
		68.0555

The arithmetic averages for the branches are

A_{uu}	126.2667
A_{ud}	105.7141
A_{du}	88.66302
A_{dd}	74.23121

The final period option values at exercise and the prior option values computed using p^* are given below.

				C_{uu}	11.75768
		C_u	5.280758		
				C_{ud}	0
C_0	4.359219				
				C_{du}	8.256105
		C_d	3.708088		
				C_{dd}	0

Answer C

1-12. The calculations are in the next table.

p^* 0.455921

				C_{uu}	38.02
		C_u	17.07605		
				C_{ud}	0
C_0	7.6694191				
				C_{du}	0
		C_d	0		
				C_{dd}	0

Answer B

1-13 In this case $a(S,t)=\lambda[\alpha-S]$ and $\sigma(S,t)=\sigma$. $F(S)=y=S^{3/2}$.

$$F_S=\frac{3}{2}S^{1/2} \qquad F_{SS}=\frac{3}{4}S^{-1/2} \qquad F_t=0$$

Applying Ito's lemma for $F(S)=S^{3/2}$

$$\begin{aligned}
dF(S,t)&=\left(F_S a(S,t)+\frac{F_{SS}\sigma(S,t)^2}{2}+F_t\right)dt+\sigma(S,t)F_S dZ\\
&=\left(\frac{3}{2}S^{1/2}a(S,t)+\frac{3}{8\sqrt{S}}\sigma(S,t)^2\right)dt+\frac{3}{2}S^{1/2}\sigma(S,t)dZ\\
&=\left(\frac{3}{2}S^{1/2}\lambda(\alpha-S)+\frac{3}{8\sqrt{S}}\sigma^2\right)dt+\frac{3}{2}S^{1/2}\sigma dZ
\end{aligned}$$

Answer A