Solutions Manual

for

PROBABILITY FOR RISK MANAGEMENT

Second Edition

by

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and

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INTRODUCTORY NOTE

This solutions manual complements a study of probability theory, based on the second edition of the textbook *Probability for Risk Management*, by Matthew Hassett and Donald Stewart.

The solutions given here for basic problems are brief, and are intended to give an outline of how to proceed. If the reader understands the examples in the text, these solutions should be adequate. The solutions to the sample exam questions are more detailed.

In most cases there will be more than one way to work a problem. The reader may expect to encounter solutions which differ from those given here.

CHAPTER 2

2-1. The red suits are hearts (H) and diamonds (D), and the face cards kings (K), queens (Q) and jacks (J). Thus the outcomes are KH, QH, JH, KD, QD and JD.

2-2. (a) The loss can be any positive rational number.

(b) The loss is any rational number in (1,000, 1,000,000).

2-3. (a) S consists of the positive integers from 1 to 25.

(b) E consists of the odd integers from 1 to 25.

2-4. S consists of all ordered pairs (r, g) where $r = 1, 2, 3, 4, 5,$ or 6 and $g = 1, 2, 3, 4, 5,$ or 6.

2-5. Count the number of ordered pairs with the desired sum in the list in the answer For Exercise 2-4. For example, the only two pairs which sum to 11 are $(5,6)$ and $(6,5)$, so the answer to part (c) is 2.

2-6. S consist of all sequences xyz where each of x, y and z is either B or G.

2-7. The outcomes in $\sim E$ are the even (not odd) integers between 1 and 25.

2-8. The outcomes in $A \cap B$ are the club face cards.

2-9. $A \cup B$ is all rational numbers in (1,000, 500,000).
$A \cap B$ is all rational numbers in (50,000, 100,000).

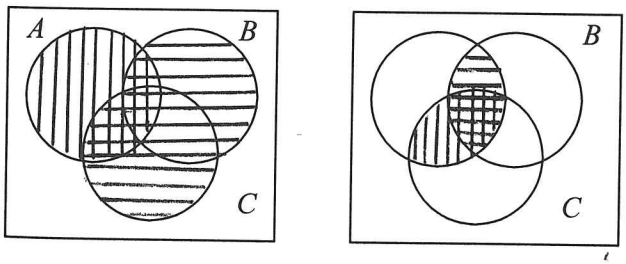
2-10. $A \cap B$ requires an outcome where the coin shows a head and the number on the die is greater than 2. It consists of the ordered pairs $(H,3)$, $(H,4)$, $(H,5)$, and $(H,6)$.

2-11. $E = \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$
$F = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$
$E \cup F = \{(1,5),(2,4),(3,3),(4,2),(5,1),(1,1),(2,2),(4,4),(5,5),(6,6)\}$
$E \cap F = \{(3,3)\}$

2-12. $E = \{GGG, GGB, GBG, GBB\}$
$F = \{GBG, GBB, BBG, BBB\}$
$E \cup F = \{GGG, GGB, GBG, GBB, BBG, BBB\}$
$E \cap F = \{GBG, GBB\}$

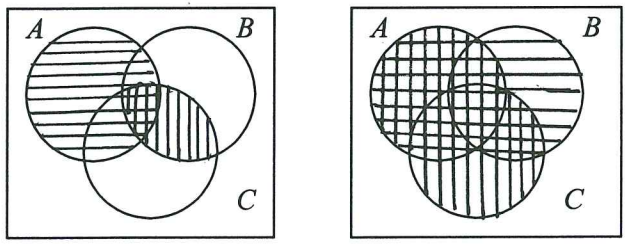
2-13. To verify (2.1) draw a Venn diagram for $A \cap (B \cup C)$ and a second one showing $(A \cap B)$ and $(A \cap C)$ and observe that the first is the union of the second and the third.

$A \cap (B \cup C)$ $(A \cap B) \cup (A \cap C)$



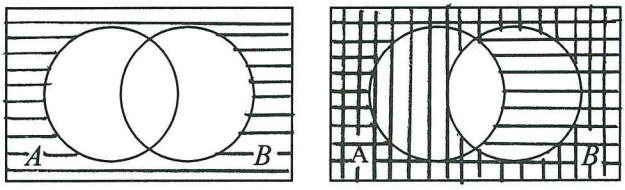
To verify (2.2) draw a Venn diagram for $A \cup (B \cap C)$ and a second one showing $(A \cup B)$ and $(A \cup C)$, and observe that first is the intersection of the second and the third.

$A \cup (B \cap C)$ $(A \cup B) \cap (A \cup C)$



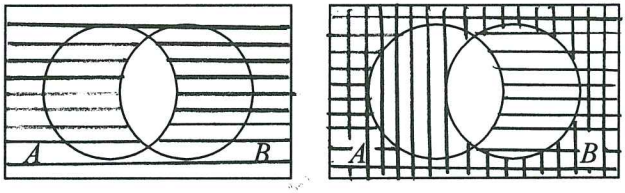
2-14. To verify (2.3) draw a Venn diagram for $\sim (A \cup B)$ and a second one showing $\sim A$ and $\sim B$, and verify that the first is the intersection of the second and the third.

$\sim (A \cup B)$ $\sim A \cap \sim B$



To verify (2.4) draw a Venn diagram for $\sim (A \cap B)$ and a second one showing $\sim A$ and $\sim B$, and verify that the first is the union of the second and the third.

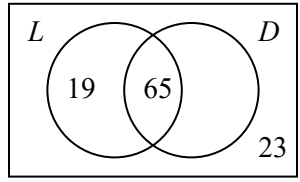
$\sim (A \cap B)$ $\sim A \cup \sim B$



2-15. See verbal statements in Answers to the Exercises.

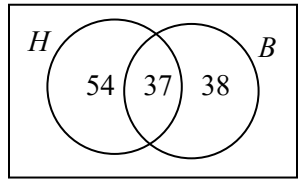
2-16. Let H be the set of those with health insurance and L be the set of those with life insurance. Then $(L \cup H) = 38 + 29 - 21 = 46.$

2-17. Let L be the set of those with the company more than 10 years and D be the set of those with college degrees. The given data can be used to fill in the Venn diagram below. Then $n(L \cup D) = 134 - 23 = 111$. Then $n(D) = n(L \cup D) - 19 = 111 - 19 = 92.$

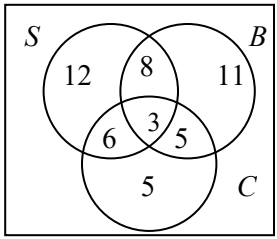


2-18 Let S be the set of those who own stocks and B be the set of those who own bonds. So $n(S \cap B) = 67 + 52 - 94 = 25.$

2-19. The given data can be used to fill in the Venn diagram below. The number of students taking exactly one of these courses is $54 + 38 = 92.$

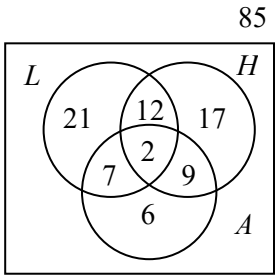


2-20. Using the given information we get the following Venn diagram.



The total number of clients is 61.

2-21. Using the given information we get the following Venn diagram.



(a) The number with no policies is $85 - 74 = 11$.
(b) The number with only health policies is 17.
(c) The number with exactly one policy is $21 + 17 + 6 = 44$.
(d) The number with life or health but not auto is $21 + 12 + 17 = 50$.

2-22. Number of outcomes is $2 \cdot 6 = 12$.

2-23. Number of ways to select car is $4 \cdot 15 \cdot 6 = 360$.

2-24. Number of ways to select classes is $7 \cdot 8 \cdot 4 \cdot 7 = 1568$.

2-25. Number of outcomes is $26 \cdot 2 + 26 \cdot 6 = 208$.

2-26. Each subset corresponds with exactly one sequence. For example the subset $\{x_1, x_2\}$ corresponds with the sequence $(1,1,0,0,...,0)$. There are 2^n sequences, so there are 2^n subsets.

2-27. If repetitions are allowed, the number of words is $6^4 = 1296$.

If repetitions are not allowed, the number is $P(6,4) = 360$.

2-28. If repetitions are allowed there are $8 \cdot 10^6$ possible numbers. If repetitions are not allowed there are

$$8 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 483{,}840$$

numbers.

2-29. The number of seating arrangements (in order) is

$$P(12,7) = 3{,}991{,}680.$$

2-30. The number of rankings (in order) is $P(10,4) = 5040$.

2-31. The number of ways to fill the offices is $P(30,3) = 24{,}360$.

2-32. The number of seating arrangements is $P(4,4)P(6,6) = 17{,}280$. (Arrange the left-handers in the first 4 seats, and then arrange the right-handers in the remaining 6 seats.)

2-33. First choose a pair of adjacent seats, which can be done in 7 ways, and the two people can be seated in those seats in 2 ways. Then arrange the 6 remaining people in remaining 6 chairs. This can be done in 6! ways, so the number of seating arrangements is $7 \cdot 2 \cdot 6! = 10,080.$

2-34. The number of possible committees is $C(30,3) = 4060.$ (Order is irrelevant.)

2-35. The number of hands is $C(52,5) = 2,598,960.$ (Order is irrelevant.)

2-36. (a) Number of hands with all hearts is $C(13,5) = 1287.$

(b) Number of hands with all same suit is $4C(13,5) = 5148.$ (Choose one of the 4 suits and then choose 5 cards.)

(c) Number of hands with (AAKKJ) is $C(4,2)C(4.2) \cdot 4 = 144.$ (Choose 2 aces and then 2 kings and then 1 jack.)

2-37. Number of ways to pick cast is $C(15,4)C(13,5) = 1,756,755.$ (Choose 4 boys and then 5 girls.)

2-38. Number of ways to select balls is $C(55,5) \cdot 42 = 146,107,962.$

2-39. Number of distinguishable arrangements of MISSISSIPPI is $\frac{11!}{4!4!2!} = 34,650.$ (There are 11 slots to fill with the 11 letters. First select 4 slots for the 4 I's, and then 4 slots for the 4 S's, and 2 slots for the 2 P's, leaving one slot left for the M.)

2-40. Number of ways to assign the actuaries is $\frac{12!}{5!3!4!} = 27,720,$ since this constitutes a partition of the group.

2-41. Number of ways to assign the analysts is $9!/(3!)^3 = 1680,$ since this constitutes a partition of the group.

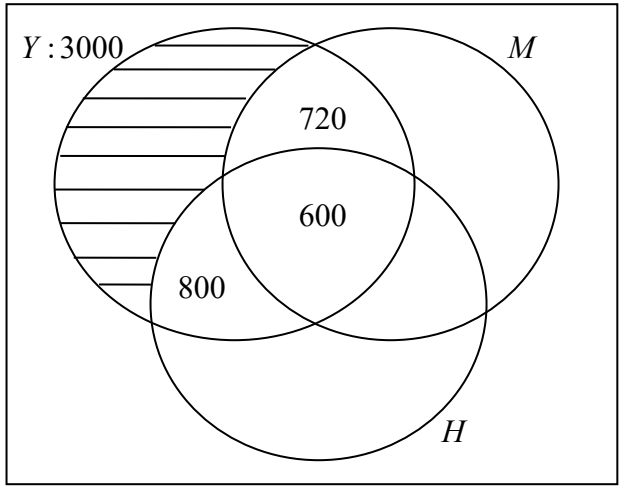
2-42. In this case the 3 teams are not distinguishable by task. If the 9 analysts are divided into 3 three-man teams, A, B and C, and have distinguishable tasks, these teams can be assigned to the 3 tasks in 3! = 6 ways. But if the tasks are all the same, those 6 partitions constitute a single division into teams. Hence the divisions into teams is the number of partitions divided by 6 which is $\frac{1680}{6} = 280.$

2-43. $(2s-t)^4 = (2s)^4 + 4(2s)^3(-t) + 6(2s)^2(-t)^2 + 4(2s)(-t)^3 + (-t)^4$

2-44. Term is $C(8,3)(2u)^5(-3v)^3,$ and coefficient is $-48{,}384.$

2-45. An $x^{n-k}y^k$ is obtained by selecting k of the $(x+y)$ factors from which to take y and taking x from the remaining $n-k$ factors. This can be done in $C(n,k)$ ways.

2-46. The number of subsets of size k from a set of size n is $C(n,k)$. Then $2^n = (1+1)^n = \Sigma C(n,k)$ equals the total number of subsets.

2-47. This is a standard counting problem from finite mathematics courses. It is usually visualized with a Venn diagram. Let Y denote the event that the policyholder is young, M the event that he is male and H the event that the policy holder is married. We need the count from the shaded region. The intersection of all three circles contains the 600 young married males, and $n(Y \cap H \cap M) = 600.$ Since there are 1320 young males, $n(Y \cap M) = 1320$ and the remaining segment of $Y \cap M$ will

contain 720 young single males. Similarly, since there are 1400 young married persons, $n(Y \cap H) = 1400$ and the remaining segment of $Y \cap M$ will contain 800 young married females. Then the shaded region contains $3000 - (800+600+720) = 880$ young single females.