SOLUTIONS TO

PROBABILITY AND STATISTICS WITH APPLICATIONS:

A PROBLEM SOLVING TEXT

SECOND EDITION

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PREFACE TO SOLUTION MANUAL

Our solution manual contains detailed solutions to the questions in our Probability and Statistics with Applications textbook. Frequently, multiple solution methods are illustrated.

Our goal is to help you learn and utilize calculus-based probability. Your level of understanding will reflect the level of thought and the connections that you make while solving problems (practicing), not just the solutions written by others (reading). When you are attempting to solve problems, we recommend that you be careful and take your time. Always define variables that you introduce. If you do not know how to start, rewrite the question. Then write down facts that you know that might help you solve the problem. Then think. The time invested in thinking through solutions to problems may seem excessive, especially to busy students juggling many activities. However, the investment will pay strong dividends down the road in terms of your comprehension and your ability to make those critical connections to by now familiar routines.

Look at the written solution as your last resort. When looking at a solution, try to minimize your reliance on this crutch. Specifically, once you have seen enough to get unstuck, try to finish your solution on your own. Finally, go back and read the written solution carefully to see if there are insights or shortcuts that will help you in the future.

The most ***ineffective*** method to attempt to understand any mathematics is by initially looking at a written solution. Learning mathematics is about your journey, not the punch-line. Every year we hear students say "I understood in class and I understood while doing homework, but I did not understand on the exam." What they mean to say is "I followed when the teacher solved a problem in class and I followed reading a written solution, but the first time I was responsible to solve a problem on my own, I could not."

For students wishing to master this material, a useful exercise is to think about ways to modify a question that you solved into a new question and/or to think of alternate solutions.

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CHAPTER 1: COMBINATORIAL PROBABILITY

Section 1.1 The Probability Model

1-1 (a) We systematically list the sample space.

Main Course	Side	Beverage
hamburger	potato chips	soda
hamburger	potato chips	water
hamburger	potato chips	milk
hamburger	coleslaw	soda
hamburger	coleslaw	water
hamburger	coleslaw	milk
chicken sandwich	potato chips	soda
chicken sandwich	potato chips	water
chicken sandwich	potato chips	milk
chicken sandwich	coleslaw	soda
chicken sandwich	coleslaw	water
chicken sandwich	coleslaw	milk

(b) {hamburger & potato chips & soda, hamburger & coleslaw & soda, chicken sandwich & potato chips & soda, and chicken sandwich & coleslaw & soda}.

(c) $\Pr(\text{soda with meal}) = \dfrac{\text{number of meals with a soda}}{\text{total number of possible meals}} = \dfrac{4}{12}$.

Note:

Reducing $\dfrac{4}{12} = \dfrac{1}{3}$ has the interpretation that one of the three beverages is a soda.

1-2 (a) The event that doubles are rolled is the set of outcomes $\{11, 22, 33, 44, 55, 66\}$. There are 6 ways to roll doubles.

(b) $\Pr(\text{sum is prime}) = \Pr(\text{sum equals 2, 3, 5, 7, or 11}) = \dfrac{15}{36}$.

1-3 (a) Legend

P_1	First plain M&M® package
P_2	Second plain M&M package
P_3	Third plain M&M package
N_1	First peanut M&M package
N_2	Second peanut M&M package
B	Peanut butter M&M package
D	Dark chocolate M&M package
A	Almond M&M package

There are two ways to list the sample space. If you grab two packages at the same time, then the outcome of DA represents one package of dark M&M's and one package of almond M&M's. In this case DA is the same outcome as AD (order does not matter), and therefore only needs to be listed once in the sample space. Then there are 28 outcomes:

$\{P_1 P_2,\ P_1 P_3,\ P_1 N_1,\ P_1 N2,\ P_1 B,\ P_1 D,\ P_1 A,\ P_2 P_3,\ P_2 N_1,\ P_2 N_2,\ P_2 B,\ P_2 D,\ P_2 A,\ P_3 N_1,\ P_3 N_2,\ P_3 B,\ P_3 D,\ P_3 A,\ N_1 N_2,\ N_1\mathrm{B},\ N_1\mathrm{D},\ N_1\mathrm{A},\ N_2\mathrm{B},\ N_2\mathrm{D},\ N_2\mathrm{A},\ \mathrm{BD},\ \mathrm{BA},\ \mathrm{DA}\}$.

If you envision the experiment as grabbing one package and then grabbing a second package (in other words, order matters), there will be 56 elements in the sample space.

(b) If order doesn't matter the answer is $\frac{15}{28}$. If order matters then the answer is $\frac{30}{56}$. Of course $\frac{15}{28} = \frac{30}{56}$, so both perspectives lead to the same answer so long as you are consistent in your treatment of the sample space.

(c) $\frac{18}{28} = \frac{36}{56}$.

Section 1.2 Finite Discrete Models with Equally Likely Outcomes

1-4 There are $3 \cdot 2 \cdot 6 = 36$ current outfits

3	2	6
Shoes	Pants	Shirts

Adding shoes will allow $4 \cdot 2 \cdot 6 = 48$ different outfits, adding a shirt will allow $3 \cdot 2 \cdot 7 = 42$. You increase the total number of outfits most by adding to the item that you have the least. The problem mentions shoes or shirts only, so the correct answer is "shoes."

1-5 (a) 6^3

$\boxed{6}\ \boxed{6}\ \boxed{6}$

Red White Blue

(b) 6 There are six choices for the blue die.

(c) $\Pr(\text{first die is 3 and second die is 4}) = \dfrac{1 \cdot 1 \cdot 6}{6^3} = \dfrac{1}{36}$.

(d) No, tree diagrams are not so useful if tracking more than 50 or so outcomes.

1-6 10^9

$\square\ \square\ \square \;-\; \square\ \square \;-\; \square\ \square\ \square\ \square$

1-7 $10 \cdot 23 \cdot 26 \cdot 23 \cdot 10 \cdot 10 \cdot 10$

1-8 One could try to use a tree diagram to systematically list all possible way to rob four banks. The thief could rob any bank first. He has only five choices of bank to rob second (he does not rob the bank he robbed first). Similarly, the thief has five choices of bank to rob third (it just cannot be the bank he robbed second, but it could be the bank he robbed first). The total is $6 \cdot 5 \cdot 5 \cdot 5 = 750$.

1-9 $8 \cdot 10 \cdot 10 \cdot 9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

1-10 (a) $\dfrac{7!}{(7-6)!} = \dfrac{7!}{1!} = 7!$

(b) $\dfrac{13!}{1!} = 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 13!$

(c) $\dfrac{7!}{3!} = 7 \cdot 6 \cdot 5 \cdot 4 = 840$

(d) 20

(e) $4! = 24$

(f) Undefined

(g) $23 \cdot 22 \cdot 21$

(h) $\frac{3!}{3!} = 1$

(i) $5! = 120$

(j) 10

(k) $\frac{7!}{3! \cdot 4!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35$

(l) 1

1-11 (a) $17!$

(b) $16!$ Kendra gets her own exam, the other 16 exams can be redistributed to any of the remaining 16 students.

(c) $\frac{16!}{17!} = \frac{1}{17}$

1-12 $\frac{1}{{}_8P_3}$

1-13 ${}_6P_3 = 6 \cdot 5 \cdot 4$

1-14 ${}_{12}P_4$

1-15 $r = 23$. Solve $1 - \frac{{}_{365}P_r}{365^r} \geq .5$ by trial and error.

1-16 $r = 4$. Solve $1 - \frac{{}_7P_r}{7^r} \geq .4$

1-17 $\Pr[\text{people get off on unique floors}] = \frac{{}_{11}P_7}{11^7}$

1-18 $\frac{{}_{35}P_{12}}{35^{12}}$

1-19 $n = 5$. Solve $1 - \frac{{}_{10}P_n}{10^n} \geq .5$ by trial and error for the minimum n.

1-20 (a) ${}_7C_6 = \frac{7!}{1! \cdot 6!} = 7$

(b) ${}_{13}C_{12} = \frac{13!}{12! \cdot 1!} = 13$

(c) ${}_{13}C_1 = \frac{13!}{1! \cdot 12!} = 13$

(d) ${}_5C_2 = \frac{5!}{3! \cdot 2!} = 10$

(e) ${}_4C_4 = \frac{4!}{4! \cdot 0!} = 1$ Remember that $0! = 1$.

(f) Undefined.

(g) ${}_5C_3 = \frac{5!}{2! \cdot 3!} = 10$

(h) ${}_4C_0 = \frac{4!}{0! \cdot 4!} = 1$

(i) ${}_5C_5 = \frac{5!}{5! \cdot 0!} = 1$

(j) ${}_{5867}C_1 = \frac{5867!}{5866! \cdot 1!} = 5867$

(k) $\frac{{}_7P_4}{4!} = \frac{7!}{3! \cdot 4!} = {}_7C_3 = 35$

(l) ${}_0C_0 = \frac{0!}{0! \cdot 0!} = 1$

1-21 Both expressions equal 220.

1-22 ${}_nC_r = \frac{n!}{r! \cdot (n-r)!} = \frac{n!}{(n-r)! \cdot r!} = {}_nC_{n-r}$.

Placing r toppings on a pizza is equivalent to keeping $n - r$ off the pizza.

1-23 $_{6}C_{4}$

1-24 The multiplication principle says that the total number of possible pizzas is
Number of ways to select toppings × ways to select size × ways to select crust

(a) $_{13}C_{2} \cdot {_3C_1} \cdot {_2C_1} = 468$
(b) $_{7}C_{1} \cdot {_6C_1} \cdot {_3C_1} \cdot {_2C_1} = 252$
(c) $_{7}C_{4} \cdot {_3C_1} \cdot {_2C_1} = 210$

1-25 (a) $_{15}C_{5}$
(b) $_{4}C_{1} \cdot {_5C_2} \cdot {_6C_2}$

1-26 $_{11}C_{2} \cdot {_3C_1} \cdot {_5C_1}$

1-27 $_{14}C_{3} \cdot {_3C_1} \cdot {_7C_2} \cdot {_5C_1}$

1-28 (a) $_{10}C_{2} \cdot {_{20}C_2} = 8550$

(b) To match exactly one white ball, your lottery ticket must match one of the two "good" white balls. There are $_{2}C_{1} = 2$ ways to do this. The second white ball could be any of the remaining eight "bad" balls.

$$\left(\underbrace{{_2C_1}}_{\text{match 1 good white}} \cdot \underbrace{{_8C_1}}_{\text{match 1 bad white}} \cdot \underbrace{{_2C_2}}_{\text{match both blue balls}}\right) \div 8550 = \frac{16}{8550}$$

1-29 (a) $\dfrac{\left({_{10}C_1}\right)^6}{_{60}C_6} = .01997$

(b) $$\frac{\overbrace{{_6C_5}}^{\text{choose 5 varieties}} \cdot \overbrace{{_5C_1}}^{\text{choose one variety w/2 pieces}} \cdot \overbrace{\left({_{10}C_1}\right)^4}^{\text{choose 1 piece from 4 of the chosen varieties}} \cdot \overbrace{{_{10}C_2}}^{\text{choose 2 pieces for variety w/2 pieces}}}{_{60}C_6} = .2697$$

1-30 $$\underbrace{{_4C_3}}_{\text{centers}} \cdot \underbrace{{_5C_3}}_{\text{forwards}} \cdot \underbrace{{_6C_4}}_{\text{guards}} = 4 \cdot 10 \cdot 15 = \underbrace{\underbrace{{_4C_1}}_{\text{centers}} \cdot \underbrace{{_5C_2}}_{\text{forwards}} \cdot \underbrace{{_6C_2}}_{\text{guards}}}_{\text{starters}}$$

1-31 $\binom{15}{5,3,7} = \dfrac{15!}{5!\cdot 3!\cdot 7!} = \underbrace{{}_{15}C_5}_{\text{start 5 of the 15}} \cdot \underbrace{{}_{10}C_3}_{\substack{\text{3 of remaining}\\ \text{10 are on the bench}}} \cdot \underbrace{{}_{7}C_7}_{\substack{\text{7 of the final}\\ \text{7 do not dress}}}$

1-32 The number of words is $\binom{4}{2,1,1} = \dfrac{4!}{2!\cdot 1!\cdot 1!} = 12$. The sample space of possible words is

{hoop, hopo, hpoo, pooh, poho, phoo, ohop, ohpo, opho, opoh, ooph, oohp}

$\text{Pr(chosing HOOP)} = \dfrac{1}{12}$

1-33 $\binom{5}{2,2,1} = \dfrac{5!}{2!\cdot 2!\cdot 1!} = \binom{5}{2}\cdot\binom{3}{2}\cdot\binom{1}{1} = 30$

1-34 $\binom{12}{\underbrace{1}_{P\text{'s}},\underbrace{1}_{e\text{'s}},\underbrace{3}_{n\text{'s}},\underbrace{1}_{s\text{'s}},\underbrace{1}_{y\text{'s}},\underbrace{1}_{l\text{'s}},\underbrace{1}_{v\text{'s}},\underbrace{2}_{a\text{'s}},\underbrace{1}_{i\text{'s}}} = \dfrac{12!}{1!\cdot 1!\cdot 3!\cdot 1!\cdot 1!\cdot 1!\cdot 1!\cdot 2!\cdot 1!}$

1-35 (a) Deal 5 cards to the first poker player. There are ${}_{52}C_5$ ways to do this. Next, deal 5 of the remaining 47 cards to the second player and continue in this fashion. The total number of ways to distribute six 5-card poker hands is

$${}_{52}C_5 \cdot {}_{47}C_5 \cdot {}_{42}C_5 \cdot {}_{37}C_5 \cdot {}_{32}C_5 \cdot {}_{27}C_5 = \frac{52!}{5!\cdot 5!\cdot 5!\cdot 5!\cdot 5!\cdot 5!\cdot 22!}$$

(b) The 52 playing cards need to partitioned into 6 piles of 5 and 1 pile of 22.

There are

$\binom{52}{5,5,5,5,5,5,22} = \dfrac{52!}{5!\cdot 5!\cdot 5!\cdot 5!\cdot 5!\cdot 5!\cdot 22!}$ ways to do this.

1-36 (a) $\underbrace{{}_{25}C_5}_{\text{frat party}} \cdot \underbrace{{}_{20}C_5}_{\text{soup kitchen}} \cdot \overbrace{\underbrace{{}_{15}C_{15}}_{\text{stay home}}}^{\text{this equals 1}} = \dfrac{25!}{20!\cdot 5!}\cdot\dfrac{20!}{15!\cdot 5!}\cdot\dfrac{15!}{15!} = \dfrac{25!}{5!\cdot 5!\cdot 15!}$

In terms of a multinomial coefficient, $\binom{25}{5,5,15}$.

(b) This is similar to part (a), but we need to take into account that the 5 person teams are essentially the same. That is, suppose the hoop game was team blue versus team green. If the teams are {Amy, Bree, Candie, Diane, Edna} versus {Zoe, Yolanda, Xena, Willaminia, Venus}, then it does NOT matter which is team green and which group is team blue.

$$\frac{{}_{25}C_5 \cdot {}_{20}C_5}{2}$$

1-37 ${}_{30}C_7 \cdot {}_{23}C_7 \div 2$

1-38 There are $\binom{9}{2,4,3} = \dfrac{9!}{2!\cdot 4!\cdot 3!} = {}_9C_2 \cdot {}_7C_4 \cdot {}_3C_3$ total ways to assign the jobs.

$$\text{Pr(women get both bad jobs)} \frac{\overbrace{{}_2C_2}^{\substack{\text{assign 2 women}\\ \text{both bad jobs}}} \cdot \overbrace{{}_7C_4}^{\substack{\text{assign 4 of the 7}\\ \text{men the average jobs}}} \cdot \overbrace{{}_3C_3}^{\substack{\text{assign the remaining}\\ \text{3 men the good jobs}}}}{{}_9C_2 \cdot {}_7C_4 \cdot {}_3C_3} = \frac{1}{36}.$$

Note: This can also be solved as $\dfrac{\binom{2}{2}\cdot\binom{7}{7}}{\binom{9}{2}} = \dfrac{1}{36}$ since the problem doesn't involve the distinction between average and good jobs.

Section 1.3 Sampling and Distribution

1-39 There are 16 different side dishes. The customer must select different sides in a specific order: ${}_{16}P_3 = \underbrace{16}_{\text{first side}} \cdot \underbrace{15}_{\text{second side}} \cdot \underbrace{14}_{\text{third side}} = 3360$

1-40 Order does not matter. So, for example, the following 3-side choices are equivalent:
{(1) corn & dressing & mac-N-cheese,
(2) corn & mac-N-cheese & dressing,
(3) mac-N-cheese & corn & dressing,
(4) mac-N-cheese & dressing & corn,
(5) dressing & mac-N-cheese & corn,
(6) dressing & corn & mac-N-cheese}

There are ${}_{16}C_3 = \dfrac{3360}{6} = 560$ such orders of 3 side dishes

1-41 $16^3 = 4096$

1-42 This is equivalent to distributing 3 indistinguishable balls (selections) into 16 fixed urns (side-dishes) without exclusion. ${}_{16+3-1}C_3 = {}_{18}C_3 = 816$

1-43 Ordered samples without replacement. See Exercise 1-39 and solution.

1-44 Unordered samples with replacement. See Exercise 1-42 and solution.

1-45 $\Pr(\text{order 3 servings of the same side}) = \dfrac{16}{816}$

1-46 (a) ${}_{3+5-1}C_5 = {}_7C_5$

(b) The parent gives each child one dollar bill. She is left with two dollar bills which she can distribute to her 3 children: ${}_4C_2$.

1-47 (a) The fundamental theorem of counting implies the solution equals:
Number of ways to distribute the Snickers × Number of ways to distribute the gum ${}_{13}C_{10} \cdot {}_{10}C_7$

(b) After giving each trick-or-treater a Snickers and a pack of gum, she has 6 Snickers bars and 3 packs of gum to distribute in any way she likes.
$${}_{6+4-1}C_6 \cdot {}_{3+4-1}C_3 = {}_9C_6 \cdot {}_6C_3$$

1-48 Each child could receive multiple pickled beets. There are ${}_{10+5-1}C_{10} = {}_{14}C_{10}$ ways to give beets.

1-49 A bridge hand consists of 13 cards dealt from a standard deck of 52 different cards.
${}_{52}C_{13}$

1-50 ${}_{52}C_5$

1-51 It depends upon the number of chicken sandwiches. If you do not order a chicken sandwich, then you have 5 dollars to distribute to 5 different \$1 items. There are ${}_9C_5$ ways to do this. If you order exactly one chicken sandwich, then you are left with 3 dollars with which to buy \$1 items. You either get zero, one, or two chicken sandwiches.
$${}_9C_5 + {}_7C_3 + {}_5C_1$$

1-52 ${}_{36}C_{21} \cdot {}_{45}C_{21} = 2.101 \times 10^{22}$

Section 1.4 More Applications

1-53 (a) $$\begin{aligned}(x+3)^4 &= 1x^4 + 4\cdot x^3\cdot 3^1 + 6\cdot x^2\cdot 3^2 + 4\cdot x\cdot 3^3 + 1\cdot 3^4\\ &= x^4 + 12x^3 + 54x^2 + 108x + 81\end{aligned}$$

(b) $$\begin{aligned}(2x+y)^5 &= 1\cdot(2x)^5 + 5\cdot(2x)^4\cdot y + 10\cdot(2x)^3\cdot y^2\\ &\qquad + 10\cdot(2x)^2\cdot y^3 + 5\cdot(2x)\cdot y^4 + 1\cdot y^5\\ &= 32x^5 + 80x^4y + 80x^3y^2 + 40x^2y^3 + 10xy^4 + y^5\end{aligned}$$

(c) $$\begin{aligned}(4x-5y)^3 &= ((4x)+(-5y))^3\\ &= 1\cdot(4x)^3 + 3\cdot(4x)^2\cdot(-5y)^1 + 3\cdot(4x)^1\cdot(-5y)^2 + 1\cdot(-5y)^3\\ &= 64x^3 - 240x^2y + 300xy^2 - 125y^3\end{aligned}$$

1-54 The key here is to write out $\binom{n-1}{r-1}$ and $\binom{n-1}{r}$, and then combine using a common denominator.

$$\begin{aligned}\binom{n-1}{r-1}+\binom{n-1}{r} &= \frac{(n-1)!}{(n-r)!\cdot(r-1)!}+\frac{(n-1)!}{(n-r-1)!\cdot r!}\\ &= \frac{(n-1)!\cdot r}{(n-r)!\cdot r\cdot(r-1)!}+\frac{(n-1)!\cdot(n-r)}{(n-r)\cdot(n-r-1)!\cdot r!}\\ &= \frac{(n-1)!\cdot r}{(n-r)!\cdot r!}+\frac{(n-1)!\cdot(n-r)}{(n-r)!\cdot r!}\\ &= \frac{(n-1)!\cdot r+(n-1)!\cdot(n-r)}{(n-r)!\cdot r!}=\frac{n!}{(n-r)!\cdot r!}=\binom{n}{r}.\end{aligned}$$

1-55 Let $x=1$ and $y=1$. $(1+1)^n = 2^n = {}_nC_0 + {}_nC_1 + {}_nC_2 + \cdots + {}_nC_n$.

1-56 Selecting r items from n possible toppings to put on the pie is equivalent to selecting $n-r$ to keep off of the pie.

1-57 Since $\binom{n}{4}+\binom{n}{5} = \binom{n+1}{5}$, the coefficient of

$$x^5y^{n-4} = x^5y^{(n+1)-5} \text{ is } \binom{n+1}{5} = 3{,}876+11{,}628 = 15{,}504.$$

Alternatively, $\dfrac{\binom{n}{5}}{\binom{n}{4}} = \dfrac{n-4}{5} = \dfrac{11{,}628}{3{,}876} = 3$. Thus $n=19$ and the coefficient of

$x^5y^{n-4} = x^5y^{15}$ in the expansion of $(x+y)^{19+1} = (x+y)^{20}$ is $\binom{20}{5} = 15{,}504$.

1-58 (a) $$\begin{aligned}(x-2y+5z)^3 &= 1\cdot x^3 + 3\cdot x^2\cdot(-2y) + 3\cdot x^2\cdot(5z)\\ &\quad +3\cdot x\cdot(-2y)^2 + 3\cdot x\cdot(5y)^2\\ &\quad + 6\cdot x\cdot(-2y)\cdot(5z) + 1\cdot(-2y)^3 + 3\cdot(-2y)^2\cdot(5z)\\ &\quad + 3\cdot(-2y)\cdot(5z)^2 + 1\cdot(5z)^3\\ &= x^3 - 6x^2y + 12xy^2 - 8y^3 + 15x^2z - 60xyz\\ &\quad + 60y^2z + 75xz^2 - 150yz^2 + 125z^3\end{aligned}$$

(b) $$\begin{aligned}(w+x-y+2z)^2 &= 1\cdot w^2 + 1\cdot x^2 + 1\cdot(-y)^2 + 1\cdot(2z)^2\\ &\quad + 2\cdot w\cdot x + 2\cdot w\cdot(-y) + 2\cdot w\cdot(2z)\\ &\quad + 2\cdot x\cdot(-y) + 2\cdot x\cdot(2z) + 2\cdot(-y)\cdot(2z)\end{aligned}$$

1-59 Detailed calculations follow the exercise in the text.

1-60 Each of the five dice has 6 possible outcomes. So the total number of ways to roll five dice is 6^5.

(a) $\Pr(\text{Five dice-of-a-kind}) = \dfrac{{}_6C_1\cdot{}_5C_5}{6^5} = .00077.$

(b) $$\Pr(\text{Four dice-of-a-kind}) = \frac{\overbrace{{}_6C_1}^{\text{4 of what kind?}}\cdot\overbrace{{}_5C_4}^{\text{ways to select 4 of the 5 die to match}}\cdot\overbrace{{}_5C_1}^{\text{the remaining die must be different}}}{6^5} = \frac{5^2}{6^4} = .01929.$$

(c) There are 2 possible straights, 1-2-3-4-5 and 2-3-4-5-6. It remains to count how many ways these straights can arise in a roll of 5 dice.

$$\Pr(\text{straight}) = \frac{2\cdot\binom{5}{1}\cdot\binom{4}{1}\cdot\binom{3}{1}\cdot\binom{2}{1}\cdot\binom{1}{1}}{6^5} = \frac{2\cdot 5!}{6^5} = .03086$$

(d) A full house would have three dice be of one value (1, 2, 3, 4, 5, or 6) and two dice of a second value.

$$\Pr(\text{Full house}) = \frac{\binom{6}{1}\cdot\binom{5}{3}\cdot\binom{5}{1}\cdot\binom{2}{2}}{6^5} = \frac{6\cdot 10\cdot 5\cdot 1}{6^5} = .03858$$

(e) The only way to get nothing is to have dice of the form:

1-2-3-4-6, 1-2-3-5-6, 1-2-4-5-6, or 1-3-4-5-6.

It remains to count the ways these can sequence can be arranged on 5 dice.

$$\Pr(\text{nothing}) = \frac{4\cdot 5!}{6^5} = \frac{480}{6^5} = .06173$$

(f) $$\Pr(\text{Three dice-of-a-kind}) = \frac{\binom{6}{1}\cdot\binom{5}{3}\cdot\binom{5}{2}\cdot\binom{2}{1}\cdot\binom{1}{1}}{6^5} = \frac{6\cdot 10\cdot 10\cdot 2}{6^5} = .15432$$

(g) $$\Pr(\text{Two pair}) = \frac{\binom{6}{2}\cdot\binom{5}{2}\cdot\binom{3}{2}\cdot\binom{4}{1}\cdot\binom{1}{1}}{6^5} = \frac{15\cdot 10\cdot 3\cdot 4}{6^5} = .23148$$

(h) $$\Pr(\text{one pair}) = \frac{\binom{6}{1}\cdot\binom{5}{2}\cdot\binom{5}{3}\cdot\binom{3}{1}\cdot\binom{2}{1}\cdot\binom{1}{1}}{6^5} = \frac{6\cdot 10\cdot 10\cdot 3\cdot 2}{6^5} = .46296$$

It is neat that you are more likely to get 3 dice-of a kind than you are to roll nothing. Did you check that the sum of these probabilities is one?

1-61 Solutions tabulated in the text.

1-62 $$\text{Expected Winnings} = 49{,}999{,}999\cdot\frac{1}{120{,}526{,}770} + 99{,}999\cdot\frac{41}{120{,}526{,}770} + \cdots + (-1)\cdot\frac{117{,}184{,}724}{120{,}526{,}770}$$
$$= \frac{-49{,}638{,}178}{120{,}526{,}770} = -.4118.$$

That is, the expected winning is negative 41 cents per play.

Section 1.5 Chapter 1 Sample Examination

1. Order does not matter. Both solutions are ${}_{26}C_5 = 65,780$.

2. $799 \cdot 10^4$

3. Order matters here. ${}_{15}P_3$

4. $$\underbrace{{}_{15}C_2}_{\substack{\text{give slinky's}\\ \text{away to 2 of}\\ \text{the 15 students}}} \cdot \underbrace{{}_{13}C_1}_{\substack{\text{give bop-on-nose}\\ \text{to 1 of the}\\ \text{remaining 13}}} = 1365 = \underbrace{{}_{15}C_1}_{\substack{\text{give bop-on-nose}\\ \text{away first}}} \cdot \underbrace{{}_{14}C_2}_{\substack{\text{give slinky's away}\\ \text{to 2 of the remaining}\\ \text{14 students}}}$$

5. (a) ${}_{51}C_{17}$

(b) $$\frac{\overbrace{{}_{25}C_2 \cdot {}_2C_2 \cdot {}_2C_2 \cdot {}_{23}C_{13} \cdot \left({}_2C_1\right)^{13}}^{\text{2 pair and 13 other cards without the old maid}} + \overbrace{{}_{25}C_2 \cdot {}_2C_2 \cdot {}_2C_2 \cdot {}_1C_1 \cdot {}_2C_2 \cdot {}_{23}C_{12} \cdot \left({}_2C_1\right)^{12}}^{\text{two pair, the old maid, and 12 other cards}}}{{}_{51}C_{17}}$$

6. (a) ${}_{52}C_5$

(b) $$\underbrace{{}_4C_1}_{\text{which suit?}} \cdot \underbrace{{}_{13}C_4}_{\substack{\text{ways to select 4}\\ \text{cards from the}\\ \text{selected suit}}} \cdot \underbrace{{}_{39}C_1}_{\substack{\text{the remaining}\\ \text{card}}}$$

7. (a) ${}_7C_3 \cdot {}_6C_2$

(b) $\Pr(\text{5 kids selected included 3 soccer and 2 football}) = \dfrac{{}_7C_3 \cdot {}_6C_2}{{}_{13}C_5} = .408.$

8. Using the multiplication rule gives $3 \cdot 2 \cdot 3 = 18$.

9. ${}_{49}C_5 \cdot {}_{42}C_1 = 80{,}089{,}128$. The Jackpots were not getting large enough, so there were more balls added to decrease the chances of winning and therefore increasing cumulative Grand Prizes.

10. $(2x - y)^5 = 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$

11. There are 15 terms in the multinomial expansion.

$$\left(x-2y+5z\right)^4 = 1\cdot x^4+4\cdot x^3\cdot(-2y)+4\cdot x^3\cdot(5z)+6\cdot x^2\cdot(-2y)^2+\cdots+1\cdot(5z)^4$$

12. $$\Pr(\text{all 3 cards have faces}) = \frac{\overbrace{{}_{12}C_3}^{\text{12 cards have faces}}}{{}_{52}C_3} = .00995.$$

13. $$\Pr(\text{matching socks}) = \frac{\overbrace{{}_{8}C_2}^{\text{ways to match white socks}} + \overbrace{{}_{4}C_2}^{\text{ways to match black socks}}}{{}_{12}C_2} = \frac{34}{66}$$

$$= 1-\Pr(\text{one white and one black sock}) = 1-\frac{8\cdot 4}{66}.$$

14. There are a total of 100 birds, therefore ${}_{100}C_6$ ways to take six birds.

Joe could take 2 Canada geese (and one from each of the other prey), or 2 ducks, or 2 eagles, or 2 cranes, or 2 flamingos. This can be done in

$$\begin{aligned}
&{}_{20}C_2\cdot{}_{25}C_1\cdot{}_{40}C_1\cdot{}_{10}C_1\cdot{}_{5}C_1\\
&\quad+{}_{20}C_1\cdot{}_{25}C_2\cdot{}_{40}C_1\cdot{}_{10}C_1\cdot{}_{5}C_1\\
&\quad\quad+{}_{20}C_1\cdot{}_{25}C_1\cdot{}_{40}C_2\cdot{}_{10}C_1\cdot{}_{5}C_1\\
&\quad\quad\quad+{}_{20}C_1\cdot{}_{25}C_1\cdot{}_{40}C_1\cdot{}_{10}C_2\cdot{}_{5}C_1\\
&\quad\quad\quad\quad+{}_{20}C_1\cdot{}_{25}C_1\cdot{}_{40}C_1\cdot{}_{10}C_1\cdot{}_{5}C_2 = 47{,}500{,}000 \text{ ways.}
\end{aligned}$$

$$\Pr(\text{at least one bird of each variety}) = \frac{47{,}500{,}000}{1{,}192{,}052{,}400} = .03985.$$

15. $$\Pr(\text{passing on first attempt}) = \frac{5}{13}.$$

16. Use the multiplication rule with the following steps:

Step 1. Choose any 3 rows (order immaterial) - # ways $= {}_6C_3 = 20$.
List the rows in ascending order for definiteness.
Step 2. Choose a column for the first listed row - # ways $= 5$.
Step 3. Choose a column for the 2nd listed row - # ways $= 4$.
Step 4. Choose a column for the 3rd listed row - # ways $= 3$.

Answer $= 20\times5\times4\times3 = 1200$ (C)

CHAPTER 2: GENERAL RULES OF PROBABILITY

Section 2.2 Set Theory Survival Kit

2-1 (a) $O = \{1,3,5,7,9\}$ and $P = \{2,3,5,7\}$.

(b) The odd primes less than 10 are $O \cap P = \{3,5,7\}$.

(c) The set of odd numbers or prime numbers less than 10 is $O \cup P = \{1,2,3,5,7,9\}$.

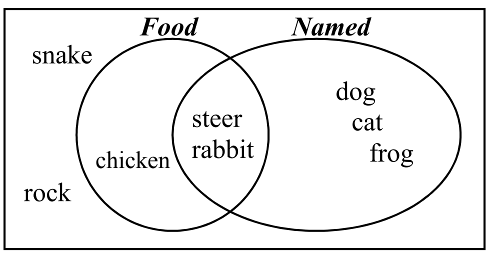
(d) The odd numbers less than 10 that are not prime is the set $O - P = \{1,9\}$.

2-2 (a) $A \cap B = \{1,2\}$.

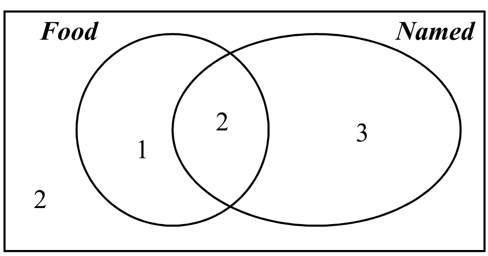
(b) $B' = \{\pi, \text{water}\}$, so $N(B') = 2$.

(c) $A \cup B = \{1,2,\pi,\text{Jamaal},\text{gum}\}$, so $(A \cup B)' = \{\text{water}\}$.

2-3 (a) The Luisi family pets can be summarized as:

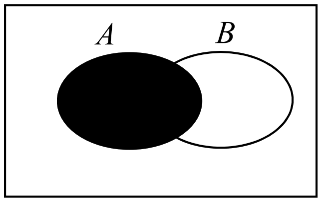


(b) The number of pets of each type is summarized as:

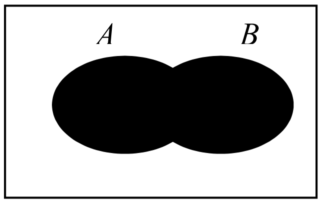


2-4 The shaded section denotes the indicated set.

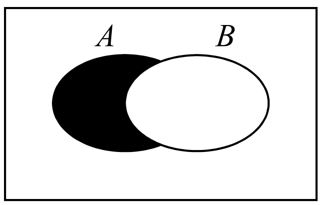
A



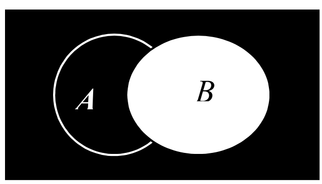
$A \cup B$



$A - B$



B'



2-5 From Subsection 1.4.3, we already have the probability of the winning prizes.

$$\Pr(\text{losing}) = 1 - \Pr(\text{winning something})$$

$$\Pr(\text{losing}) = 1 - \underbrace{\frac{1}{120{,}576{,}770}}_{\substack{\text{probability of winning} \\ \text{the grand prize}}} - \cdots - \underbrace{\frac{1{,}712{,}304}{120{,}576{,}770}}_{\substack{\text{probabilty of matching} \\ \text{only the powerball}}} = \frac{117{,}184{,}724}{120{,}576{,}770}.$$

2-6 Doubles $=\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$.

Sum is divisible by 3 $=\{(1,2),(1,5),(2,1),(2,4),(3,3),(3,6),$
$(4,2),(4,5),(5,1),(5,4),(6,3),(6,6)\}$.

Doubles $\cap$ Sum is divisible by 3 $=\{(3,3),(6,6)\}$

Doubles $\cup$ Sum is divisible by 3 $=\{(1,1),(1,2),(1,5),(2,1),(2,2),(2,4),(3,3),(3,6),$
$(4,2),(4,4),(4,5),(5,1),(5,4),(5,5),(6,3),(6,6)\}$.

Pr(Doubles $\cup$ Sum is divisible by 3) $=\dfrac{16}{36}=\dfrac{6}{36}+\dfrac{12}{36}-\dfrac{2}{36}$

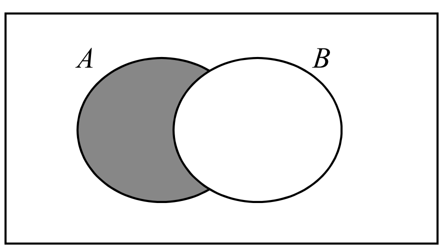
2-7 Let K equal the percent of king size mattresses sold, Q equal the percent of queen size mattresses sold, and T equal the percent of twin size mattresses sold.

We are given: $K+Q+T = 100\%$, $Q = \dfrac{1}{4}(K+T)$, and $K=3T$.
Solving for the variables, $K=60\%$, $Q=20\%$, and $T=20\%$.

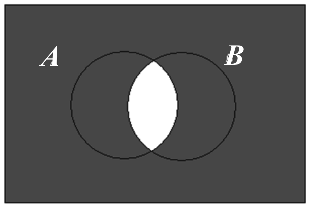
The probability that the next mattress sold is king or queen size is $K+Q = 80\%$ (C).

2-8 $A'=\{2,4,6,8,10\}$
$B'=\{1,4,6,8,9,10\}$
$A\cup B=\{1,2,3,5,7,9\}$
$A\cap B=\{3,5,7)$
$A'\cup B'=\{1,2,4,6,8,9,10\}$
$A'\cap B'=\{4,6,8,10\}$
$\left(A\cup B\right)'=\{4,6,8,10\}$

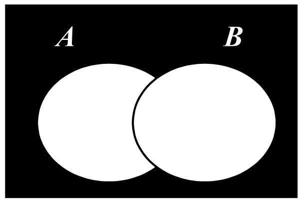
2-9 $A-B = A\cap B'$ is represented by the shaded area.



2-10 $(A \cap B)' = A' \cup B'$

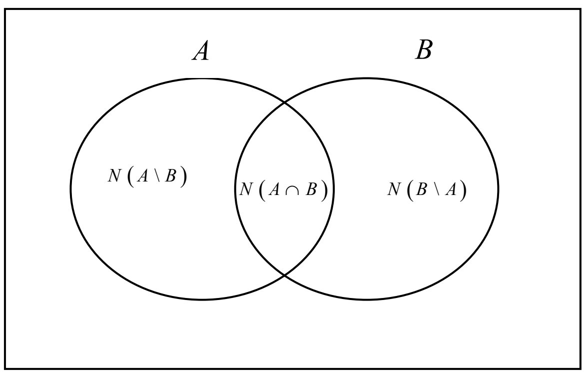


$(A \cup B)' = A' \cap B'$



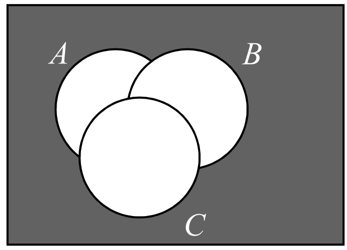
2-11 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

2-12

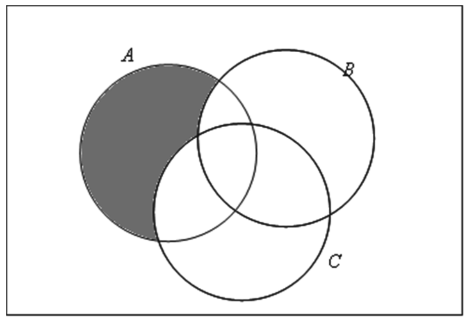


$$\begin{aligned} N(A \cup B) &= N(A \setminus B) + N(A \cap B) + N(B \setminus A) \\ &= \left[N(A \setminus B) + N(A \cap B)\right] + \left[N(B \setminus A) + N(A \cap B)\right] - N(A \cap B) \\ &= N(A) + N(B) - N(A \cap B) \end{aligned}$$

2-13 $\left(A \cup B \cup C\right)'$ is the shaded area



$A \cap \left(B \cup C\right)'$.

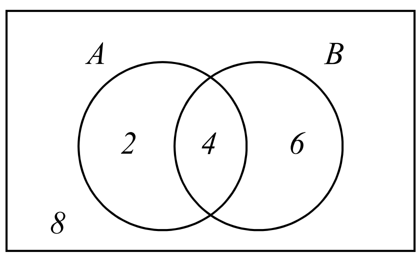


2-14 Method 1: Using the Axioms

$N(A) \;=\; 6 \;=\; N(U) - N(A')$.
$N(B) \;=\; 10 \;=\; N(U) - N(B')$.

From the inclusion-exclusion relationship, $N(A \cup B) + N(A \cap B) \;=\; N(A) + N(B)$.
$N(A \cap B) = 6 + 10 - 12 = 4$.

Method 2: Venn Diagram



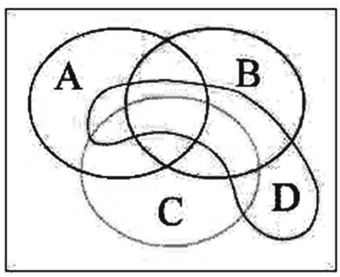
Method 3: Venn Box diagram (Subsection 2.2.4)

	A	A'	
B	4	6	10
B'	2	8	10
	6	14	20

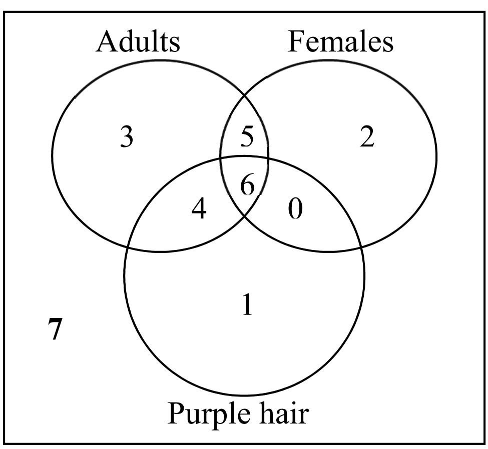
2-16 $$\begin{aligned} N(A \cup B \cup C \cup D) = {} & N(A) + N(B) + N(C) + N(D) \\ & - N(A \cap B) - N(A \cap C) \\ & - N(A \cap D) - N(B \cap C) \\ & - N(B \cap D) - N(C \cap D) \\ & + N(A \cap B \cap C) + N(A \cap B \cap D) \\ & + N(A \cap C \cap D) + N(B \cap C \cap D) \\ & - N(A \cap B \cap C \cap D) \end{aligned}$$

There are ${}_4C_1 = 4$ ways to select one of the four regions.
There are ${}_4C_2 = 6$ intersections of two of the four regions.
There are ${}_4C_3 = 4$ intersections of three of the four regions.

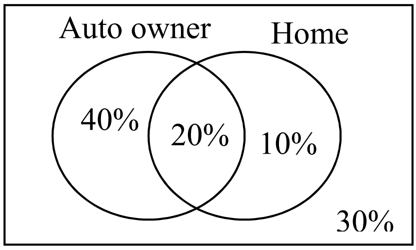
2-17 A four region Venn diagram could be drawn as:



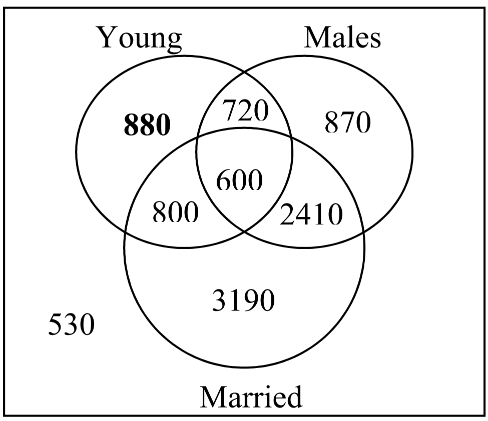
2-18 There are seven male children without purple hair in this family.



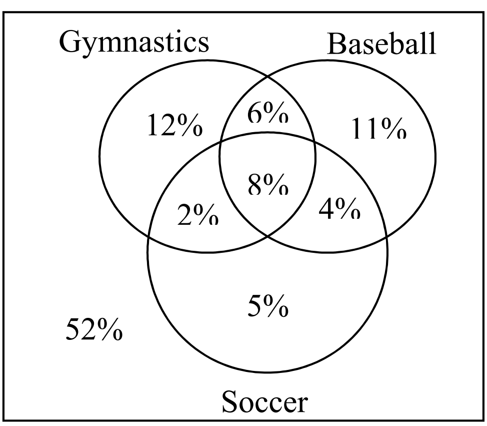
2-19 (B) $50\% = 40\% + 10\%$ of the population owns exactly one of auto and home.



2-20 (D) There are 880 young married females.



2-21 (D) 52% of these viewers watch none of the three sports.

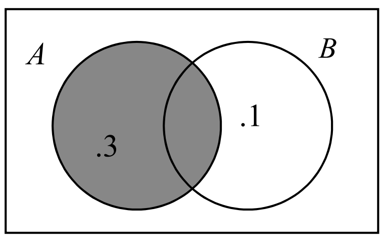


2-22 (A) 5% of the patients need both lab work and are referred to a specialist. The percentages in bold are given in the problem. Try this just using a Venn diagram.

	Lab work	No lab work	
Referred to Specialist	5%	**25%**	**30%**
Not Referred to Specialist	**35%**	35%	70%
	40%	60%	100%

2-23 (D) $\Pr(A) = .60$.

The key here is to note that $\Pr(A \cup B) = 0.7$ implies that $\Pr(A' \cap B') = 0.3$. Similarly, $\Pr(A \cup B') = 0.9$ implies $\Pr(A' \cap B) = 0.1$. These are versions of De Morgan's laws. We do not have enough information to find $\Pr(A \cap B)$ or $\Pr(A \cap B')$, but fortunately we are just asked to find $\Pr(A) = 1 - (.1 + .3) = .6$



2-24 (D) We have two equations with two unknown variables x and y (see box diagram below). The first comes from the fact that the sum of the probability of four disjoint outcomes must equal 1. The second equation comes from the statement of the problem.

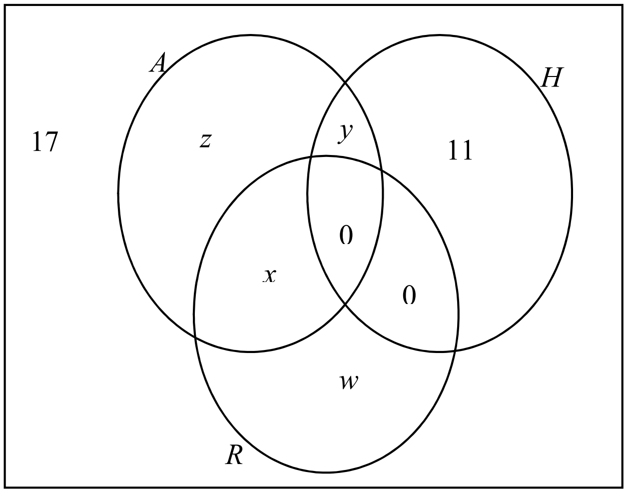
$$22\% + x + y + 12\% = 100\% \quad \text{implies that} \quad x + y = 66\%.$$

$$\underbrace{22\% + y}_{\text{probabilty visit chiropractor}} = \underbrace{22\% + x}_{\text{probabilty visit P.T.}} + \underbrace{14\%}_{\text{exceeds by 14\%}} \quad \text{implies that} \quad y = x + 14\%.$$

$x = 26\%$ and $y = 40\%$. $\Pr(\text{visit P.T.}) = 22\% + 26\% = 48\%$.

	Visit P.T	No P.T	
Chiropractor	22%	y	$22\% + y$
No chiropractor	x	12%	$x + 12\%$
	$22\% + x$	$y + 12\%$	100%

2-25 Draw the Mickey Mouse diagram, with Auto, Homeowners and Renters labeled as shown below. Since *H* and *R* are mutually exclusive the subsets comprising their intersection have cardinality zero. Statement (i) gives $N\left[(A \cup B \cup C)'\right] = 17$. Statement (v) gives $N(H \setminus A) = 11$. The remaining unknown subsets are labeled x, y, z, w.



From (ii) $x + y + z = 64$. Also,

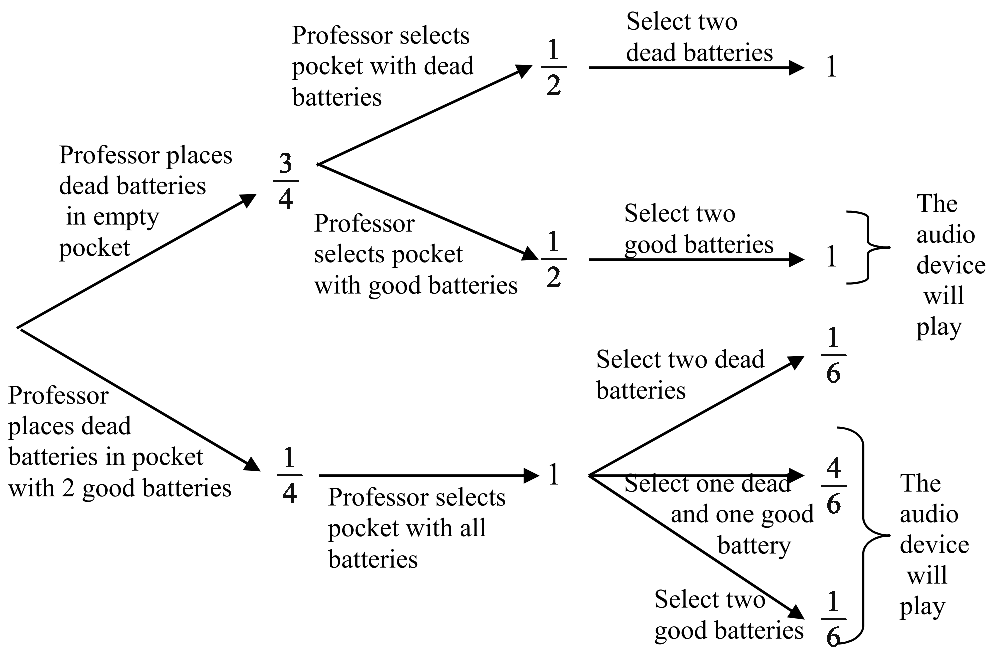
$$100 = 17 + 11 + (x + y + z) + w = 17 + 11 + 64 + w \Rightarrow w = 8.$$

From (iii), $y + 11 = 2(x+8)$, and,

From (iv), $x + y = 35$. Solving the last two equations simultaneously gives $x = N(A \cap R) = 10$ (B)

Section 2.3 Conditional Probability

2-26 There are 3 levels of choices in the following tree. The first branch concerns whether the professor put the batteries in an empty pocket or in the pocket with the two new batteries. The second level concerns which pocket the professor withdraws batteries from. In the third level, if the four batteries are in the same pocket, we will use counting techniques from chapter 1 to determine the likelihood of grabbing either two batteries that do not work, two batteries that do work, or one battery that works along with one battery that does not work.



(a) Pr(get at least one good battery) $= \frac{3}{4}\cdot\frac{1}{2}\cdot 1 + \frac{1}{4}\cdot 1\cdot\frac{5}{6} = \frac{14}{24}$.

(b) Pr(two good batteries | at least one good battery)

$$= \frac{\text{Pr(two good} \cap \text{at least one good battery)}}{\text{Pr(at least one good battery)}}$$

$$= \frac{\text{Pr(two good)}}{\text{Pr(at least one good battery)}}$$

$$= \frac{\frac{3}{4}\cdot\frac{1}{2}\cdot 1 + \frac{1}{4}\cdot 1\cdot\frac{1}{6}}{\frac{14}{24}} = \frac{10}{14}.$$

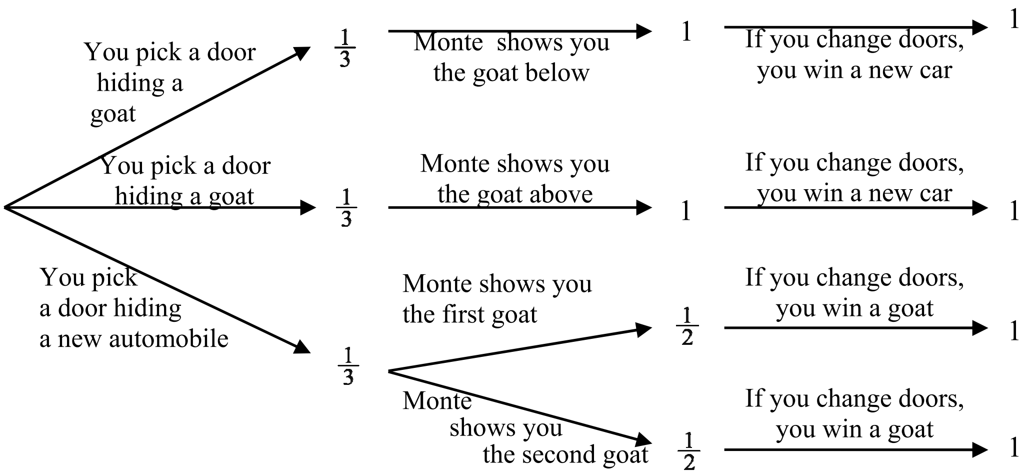
(c) Pr(batteries in same pocket | at least one good battery)

$$= \frac{\text{Pr(batteries in same pocket} \cap \text{at least one good battery)}}{\text{Pr(at least one good battery)}}$$

$$= \frac{\frac{1}{4}\cdot 1\cdot\frac{5}{6}}{\frac{14}{24}} = \frac{5}{14}.$$

2-27 (a) This is the scenario where we will keep our original door.
You will win the car if and only if you selected the correct door originally (it just doesn't matter what Monte does). There is a one in 3 chance of this. $\Pr(\text{win auto}\,|\,\text{keep original door}) = \frac{1}{3}$.

(b) If you originally select the door with the new car, then Monte can select from either of two doors to show you a goat. If you select a door concealing a goat, Monte only has one door that he can open. Consider the following diagram.



$$\Pr(\text{win auto}\,|\,\text{change doors}) = \tfrac{1}{3}\cdot 1\cdot 1 + \tfrac{1}{3}\cdot 1\cdot 1 = \tfrac{2}{3}.$$

2-28 Consider the following Box Diagram with the information in (i), (ii) and (iii) filled in:

	High	Low	Normal	
Regular				
Irregular	a		c	15
	14	22	b	100

$$b = 100 - 14 - 22 = 64$$

From (iv),
$a = \Pr[\text{Irregular} \cap \text{High}] = \Pr[\text{High}|\text{Irregular}]\cdot\Pr[\text{Irregular}] = \frac{1}{3}\cdot 15\% = 5\%$

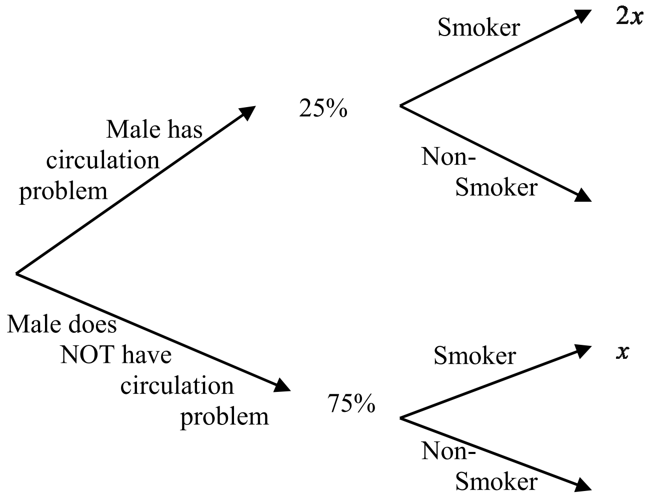
Similarly, from (v), $c = \frac{1}{8}\cdot 64\% = 8\%$. Completing the Box Diagram gives,

	High	Low	Normal	Row Total
Regular	9	20	56	85
Irregular	5	2	8	15
Column Total	14	22	64	100

$\Pr[\text{Regular} \cap \text{Low}] = 20\%$ (E)

2-29 (C) Let x equal the probability that the male is a smoker, given that he does NOT have a circulation problem.

$$\Pr(\text{circulation problem} \mid \text{smoker}) = \frac{\Pr(\text{circulation problem} \cap \text{smoker})}{\Pr(\text{smoker})}$$
$$= \frac{.25 \cdot 2x}{.25 \cdot 2x + .75 \cdot x} = \frac{.5}{1.25} = \frac{2}{5}.$$



2-30 (C) $\Pr(\text{smoker} \mid \text{died}) = \dfrac{\Pr(\text{smoker} \cap \text{died})}{\Pr(\text{died})} = \dfrac{.10 \cdot .05}{.10 \cdot .05 + .90 \cdot .01} = \dfrac{.005}{.014} = 36\%.$

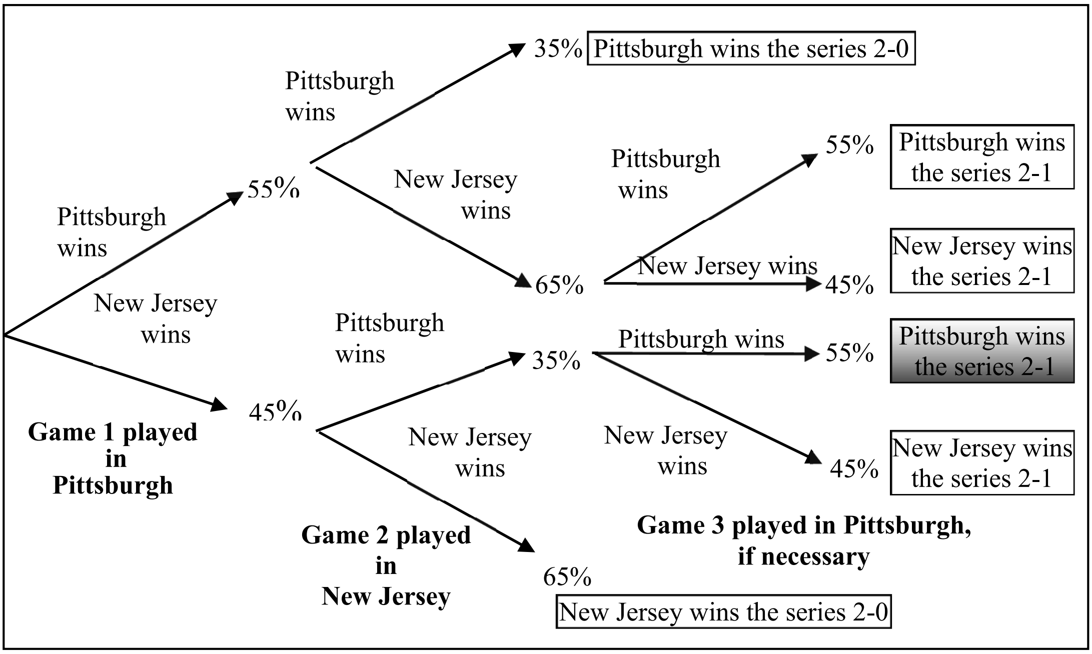
2-31 (B)

$$\Pr(N \geq 1 \mid N \leq 4) = \frac{\Pr(1 \leq N \leq 4)}{\Pr(N \leq 4)}$$
$$= \frac{\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}}{\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30}}$$
$$= \frac{2}{5}.$$

N	$\Pr(N = n)$
0	$\frac{1}{1 \cdot 2} = \frac{1}{2}$
1	$\frac{1}{2 \cdot 3} = \frac{1}{6}$
2	$\frac{1}{3 \cdot 4} = \frac{1}{12}$
3	$\frac{1}{4 \cdot 5} = \frac{1}{20}$
4	$\frac{1}{5 \cdot 6} = \frac{1}{30}$
5	$\frac{1}{6 \cdot 7} = \frac{1}{42}$
…	…

2-32 (a) Pr(Penguins win series) $= .55\cdot.35 + .55\cdot.65\cdot.55 + .45\cdot.35\cdot.55 = .47575$.

(b) Pr(Devils win at least one game) $= 1 - .55\cdot.35 = .8075$.



2-33 (B) This is a standard Bayes' formula problem, easily solved by drawing the appropriate tree diagram. The data may also be displayed in a Box diagram similar to the one in Exercise 2-28:

	Critical	Serious	Stable	Row Total
Survive				
Die	a	b	c	
Column Total	10	30	d	100

$d = 100 - 10 - 30 = 60$

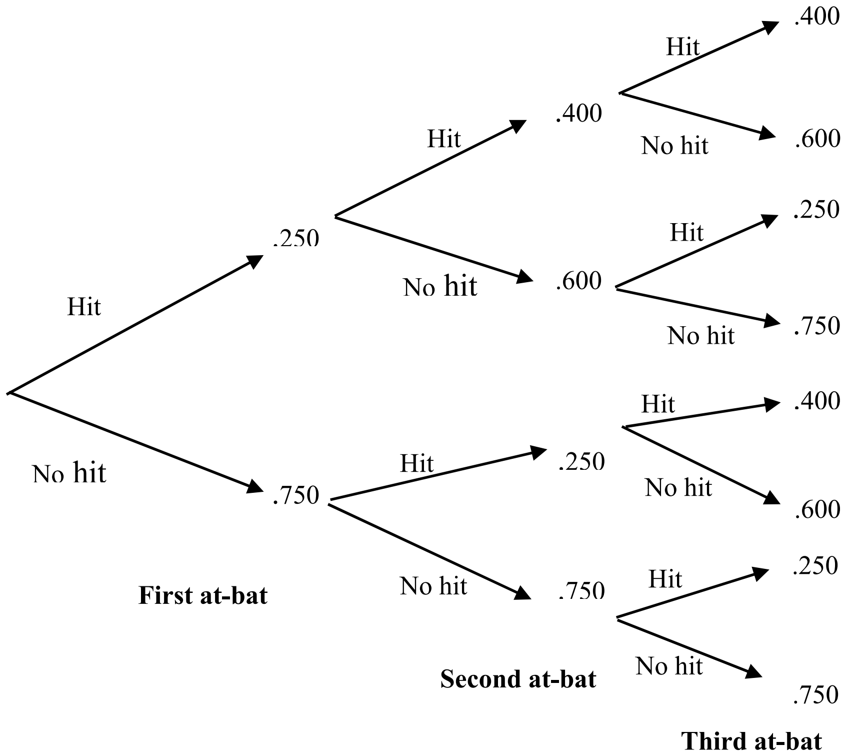
$a = \Pr[\text{Die} \cap \text{Critical}] = \Pr[\text{Die} \mid \text{Critical}]\Pr[\text{Critical}] = (40\%)\cdot(10\%) = 4\%$.

Similarly, $b = (10\%)\cdot(30\%) = 3\%$, and $c = (1\%)\cdot(60\%) = .6\%$. Completing the box gives,

	Critical	Serious	Stable	Row Total
Survive	6	27	59.4	92.4
Die	4	3	.6	7.6
Column Total	10	30	60	100

$$\Pr\left[\text{serious} \middle| \text{survive}\right] = \frac{\Pr\left[\text{serious} \cap \text{survive}\right]}{\Pr\left[\text{survive}\right]} = \frac{27}{92.4} = 29.2\% \quad \text{(B)}$$

2-34 Pr(exactly 1 hit in 3 at-bats) $= .25 \cdot .6 \cdot .75 + .75 \cdot .25 \cdot .6 + .75 \cdot .75 \cdot .25 = .365625$.



2-35 $$\Pr(\text{call Packy "dad"} \mid \text{Packy is your father}) = \frac{\Pr(\text{call "dad"} \cap \text{Packy is your father})}{\Pr(\text{Packy is your father})} = \frac{.01 \cdot .90}{.01 \cdot .90 + .99 \cdot .05} = .1538.$$

2-36

	One Car	Multiple Cars	Row Total
Sports Car		a	20
No Sports Car			
Column Total		64	100

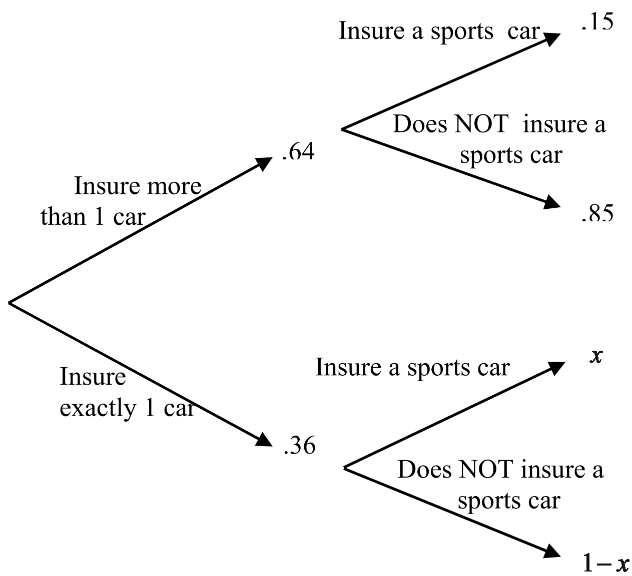
Using (iv),
$$\begin{aligned} a &= \Pr[\text{Sports Car} \cap \text{Multiple Cars}] \\ &= \Pr[\text{Sports Car} \mid \text{Multiple Cars}]\Pr[\text{Multiple Cars}] \\ &= (15\%) \cdot (64\%) = 9.6\% \end{aligned}$$

The complete Box Diagram is now easily calculated:

	One Car	Multiple Cars	Row Total
Sports Car	10.4	9.6	20
No Sports Car	25.6	54.4	80
Column Total	36	64	100

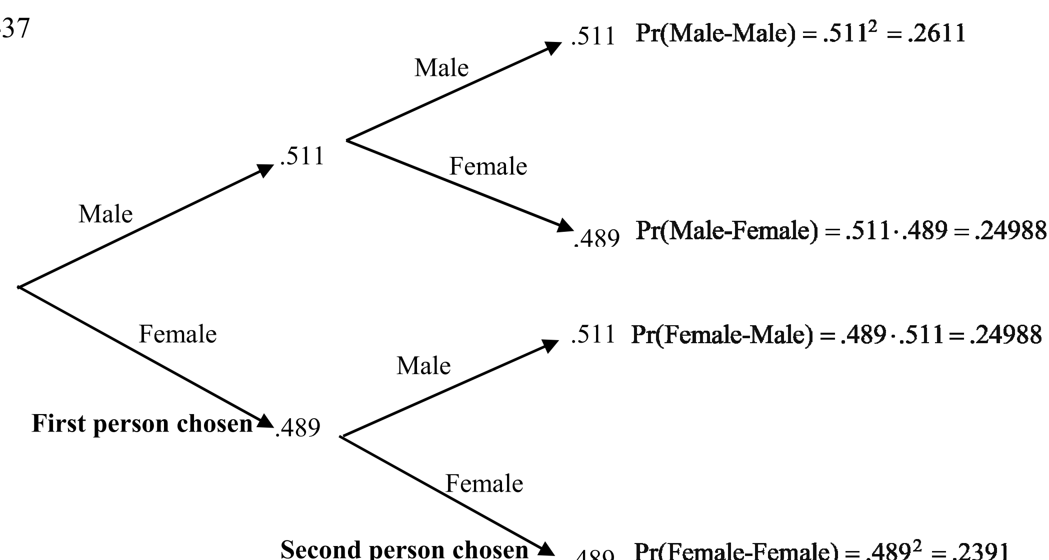
Thus, $\Pr\left[\text{One Car} \cap \text{No Sports Car}\right] = 25.6\%$ (C)

Note: One may also display the information in a tree diagram:

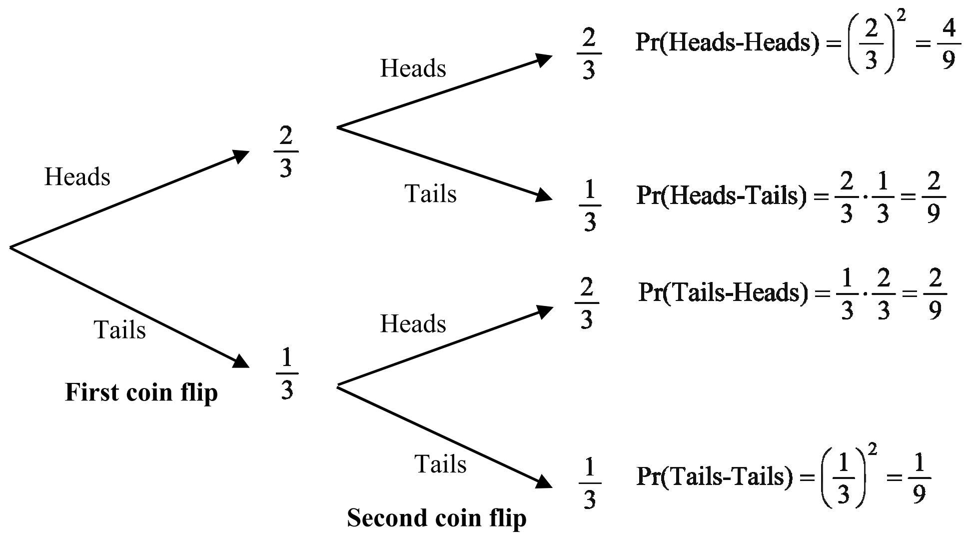


Section 2.4 Independence

2-37

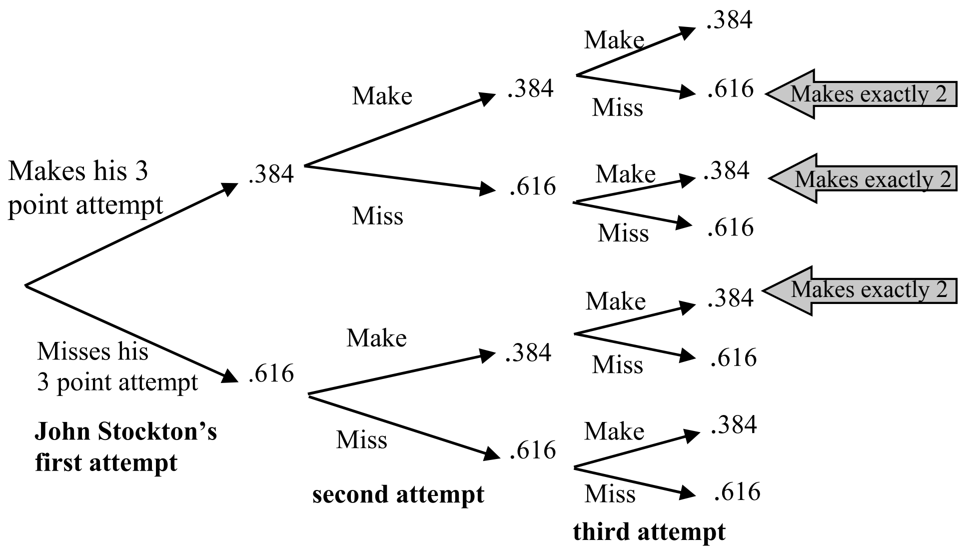


2-38 You are encouraged to observe that this is structurally the same as problem 2-37.



Number of Heads	0	1	2
Probability	$\frac{1}{9}$	$\underbrace{2}_{\text{paths through the tree that have one head and one tail}} \cdot \underbrace{\frac{1}{3}\cdot\frac{2}{3}}_{\text{probability}} = \frac{4}{9}$	$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

2-39



Pr(Stockton makes exactly two 3-pointer)

$$= \underbrace{3}_{\text{ways to make exactly two 3-pointer on three attepts}} \cdot \underbrace{.384}_{\text{probability of making a 3-pointer}} \cdot \underbrace{.384}_{\text{probability of making a 3-pointer}} \cdot \underbrace{.616}_{\text{probability of missing a 3-pointer}} = .2725.$$

2-40 $\Pr(B \mid A) = \dfrac{12}{51}$. $\Pr(B) = \dfrac{13}{52}$. The events A and B are dependent.

2-41 (a) Since $\Pr(B) = b$, we know $\Pr(B') = 1 - b$. To show that A and B' are independent, we observe $\Pr(A \cap B') = \Pr(A) \cdot \Pr(B') = a \cdot (1{-}b)$.

	B	B'	
A	ab	$a(1-b)$	$\boldsymbol{a}$
A'	$b(1-a)$	$(1-a)(1-b)$	$1-a$
	b	$1-b$	1

2-42 Not necessarily.

2-43 (a) $U = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$.

(b) $A = \{HHH, HHT, TTH, TTT\}$, $B = \{HHH, HTH, THT, TTT\}$, and $C = \{HHH, HTT, THH, TTT\}$.

Thus, $\Pr(A) = \Pr(B) = \Pr(C) = \frac{1}{2}$.

(c) $\Pr(A \cap B) = \Pr(HHH, TTT) = \frac{2}{8} = \frac{1}{4}$. Similarly,

$\Pr[A \cap C] = \Pr[B \cap C] = \Pr[HHH, TTT] = \frac{1}{4}$.

(d) Yes, because for example, $\Pr(A \cap B) = \frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2} = \Pr(A) \cdot \Pr(B)$.

(e) $\Pr(A \cap B \cap C) = \Pr[HHH, TTT] = \frac{1}{4}$.

(f) No, since $\Pr(A \cap B \cap C) = \frac{1}{4} \neq \frac{1}{8} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \Pr[A] \cdot \Pr[B] \cdot \Pr[C]$.

2-44 Let $x = \Pr(A \cap B)$. Using independence, solve $(.2 + x) \cdot (.3 + x) = x$. There are two solutions, $x = .2$ or $x = .3$.

$$\Pr(A \cup B) = .7 \text{ or } .8.$$

2-45 (a) Since the coin is fair (all outcomes are equally likely) and we do not need to track probabilities, listing the sample space will be sufficient.

$U = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$.

A = first coin is tails $= \{THH, THT, TTH, TTT\}$.

B = the third flip is heads $= \{HHH, HTH, THH, TTH\}$.

(b) $\Pr(B \mid A) = \frac{2}{4} = \frac{1}{2}$.

(c) Yes, the events are independent.

2-46 (a) Now is when tracking probabilities through a tree diagram is critical.

(b) $\Pr(B \mid A) = \dfrac{\Pr(A \cap B)}{\Pr(A)} = \dfrac{.4 \cdot .6 \cdot .6 + .4 \cdot .4 \cdot .6}{.4} = 60\%.$

(c) Yes, the events are independent. You can see this by computing $\Pr(B) = \Pr(\text{third flip is heads}) = .60.$

2-47 Let O be the event of using orthodontic work, F fillings and E extractions. Using (i) and (ii) we have:

	O	O'	Row Total
F			
F'		a	c
Column Total	$1/2$	b	1

$a = \Pr\left[F' \cap O'\right] = 1 - \Pr\left[F \cup O\right] = 1 - 2/3 = 1/3$

$b = 1 - 1/2 = 1/2$. By independence, $a = bc \Rightarrow c = a/b = 2/3$.
Completing the O and F box gives:

	O	O'	Row Total
F	$1/6$	$1/6$	$1/3$
F'	$1/3$	$1/3$	$2/3$
Column Total	$1/2$	$1/2$	1

Similarly, using (i) and (iii) and the independence of O and E gives:

	O	O'	Row Total
E	$1/4$	$1/4$	$1/2$
E'	$1/4$	$1/4$	$1/2$
Column Total	$1/2$	$1/2$	1

We now use $\Pr\left[F\right] = 1/3$ and $\Pr\left[E\right] = 1/2$, plus (iv) to fill out the F and E box:

	F	F'	Row Total
E	$1/8$	$3/8$	$1/2$
E'	$5/24$	$7/24$	$1/2$
Column Total	$1/3$	$2/3$	1

Note: F and E are not independent.

$\Pr\left[E \cup F\right] = 1 - \Pr\left[E' \cap F'\right] = 1 - 7/24 = 17/24$ (D)

2-48 We take the statement that a "different brand of pregnancy test was purchased" to imply that these test results are independent.

$$\Pr(\text{both tests are incorrect}) = .01 \cdot .02 = 0.0002.$$

2-49 First we fill in the missing information.

Major	Grades					
	A	B	C	D	E	Totals
Business	2	3	3	5	1	**14**
Science	7	7	1	0	2	**17**
Totals	**9**	**10**	**4**	**5**	**3**	**31**

(a) $\Pr(\text{student assigned 'B'}) = \dfrac{10}{31}.$

(b) $\Pr(\text{science major}) = \dfrac{\text{number of science majors}}{\text{number of students}} = \dfrac{17}{31} = \dfrac{\overbrace{31-14}^{\text{students - business majors}}}{31}$

(c) $\Pr(\text{science major} \cap \text{assigned 'B'}) = \dfrac{7}{31}.$

(d) This is most efficiently calculated by looking at the table. We are told that the student is a business major. Therefore the student lives in the shaded row of the table.

$$\Pr(\text{assigned 'C'} \mid \text{business major}) = \frac{3}{14}.$$

(e) $\Pr(\text{business major} \mid \text{assigned 'A'}) = \dfrac{2}{9}.$

(f) No, because $\Pr(\text{assigned 'C'} \mid \text{business major}) \neq \Pr(\text{assigned 'C'})$.

2-50 (a) We will use counting techniques (combinations) that we studied in chapter 1. The number of ways to select 2 batteries from a flashlight containing 5 batteries is ${}_5C_2 = 10.$

$$\Pr(\text{both batteries lost their charge}) = \frac{{}_2C_2}{{}_5C_2} = \frac{1}{10}.$$

(b) $\Pr(\text{exactly one battery lost its charge}) = \dfrac{{}_3C_1 \cdot {}_2C_1}{{}_5C_2} = \dfrac{6}{10}.$

(c) Draw a tree diagram with appropriate probabilities.

$$\Pr(\text{both batteries tested lost their charge}) = \left(\frac{2}{5}\right)^2 = \frac{4}{25} = 16\%.$$

$$\Pr(\text{exactly one tested battery lost its charge}) = 2 \cdot \frac{2}{5} \cdot \frac{3}{5} = \frac{12}{25} = 48\%.$$

2-51 Let Pr(purchase disability coverage) $= \mathrm{P}[D] = x$, then
Pr(purchase collision coverage) $= \Pr[C] = 2x$.
$0.15 = 2x^2$ implies that $x = .274$ and $2x = .548$. The completed Box Diagram is:

	C	C'	Row Total
D	.15	.124	.274
D'	.398	.328	.726
Column Total	.548	.452	1

Pr(select neither insurance) $= \Pr\left[C' \cap D'\right] = .328.$ (B)

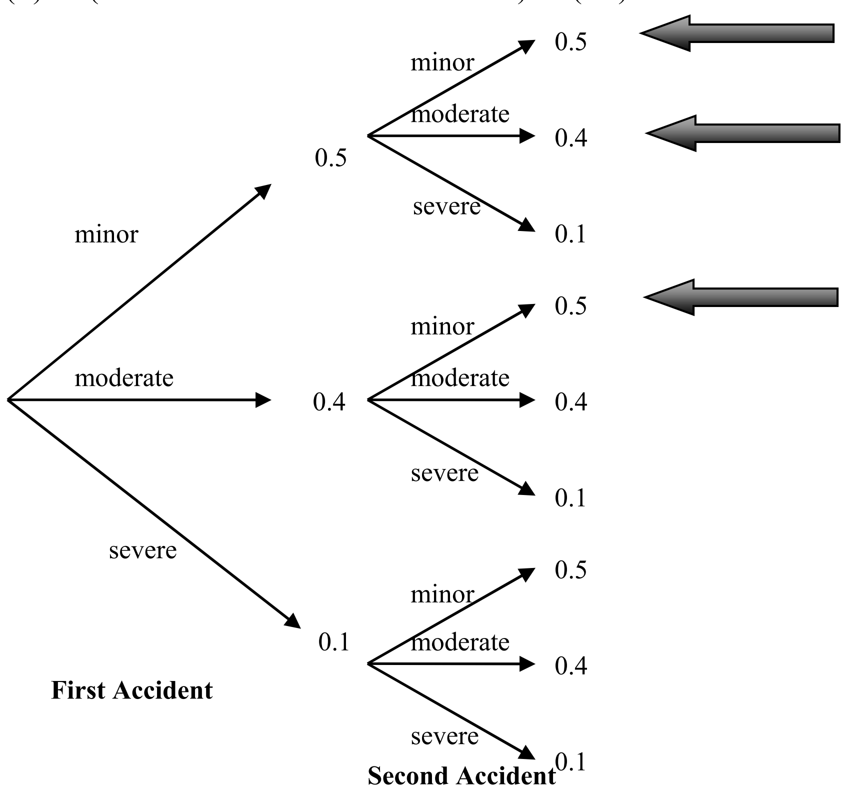
2-52 Pr(select two consecutive face cards) $= \frac{12}{52} \cdot \frac{11}{51}$, or using combinations
Pr(select two consecutive face cards)

$$= \frac{\text{number of ways to deal 2 face cards}}{\text{number of ways to deal 2 cards}} = \frac{{}_{12}C_2}{{}_{52}C_2}.$$

2-53 (a) Pr(next roulette spin is black) $= \frac{18}{38}$.

(b) Pr(next eight roulette spins are black) $= \left(\frac{18}{38}\right)^8 \doteq .00253.$

2-54 (E) Pr(no severe and at most one moderate) $= (0.5)^2 + 0.5 \cdot 0.4 + 0.4 \cdot 0.5 = .65.$

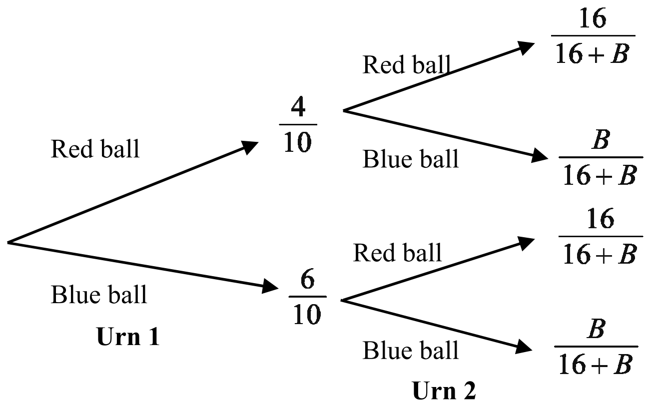


2-55 (A) Let B denote the number of blue balls in urn 2.

$$\Pr(\text{both ball same color}) = .44 = (0.4)\cdot\frac{16}{16+B} + (0.6)\cdot\frac{B}{16+B} = \frac{6.4+.6B}{16+B}.$$

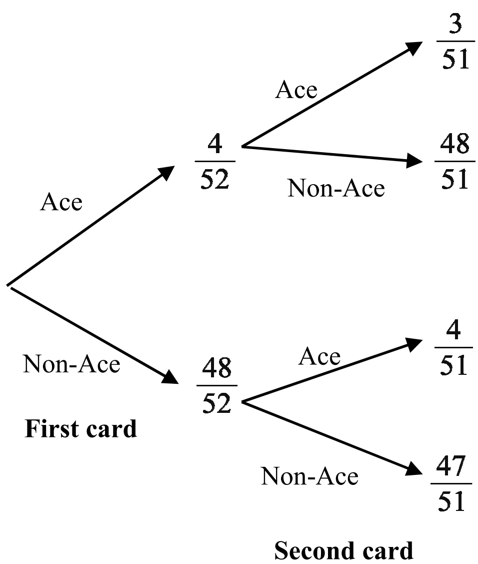
Solve for B to find there are four blue balls in urn 2.

$$\frac{16}{16+B}$$



Section 2.5 Bayes' Theorem

2-56 Draw the standard tree diagram to track probabilities.



(a) $\Pr(\text{first card is an ace}) = \dfrac{\text{number of aces}}{\text{number of cards}} = \dfrac{4}{52}.$

(b) $\Pr(\text{second card is an ace} \mid \text{first card not ace}) = \dfrac{4}{51}.$

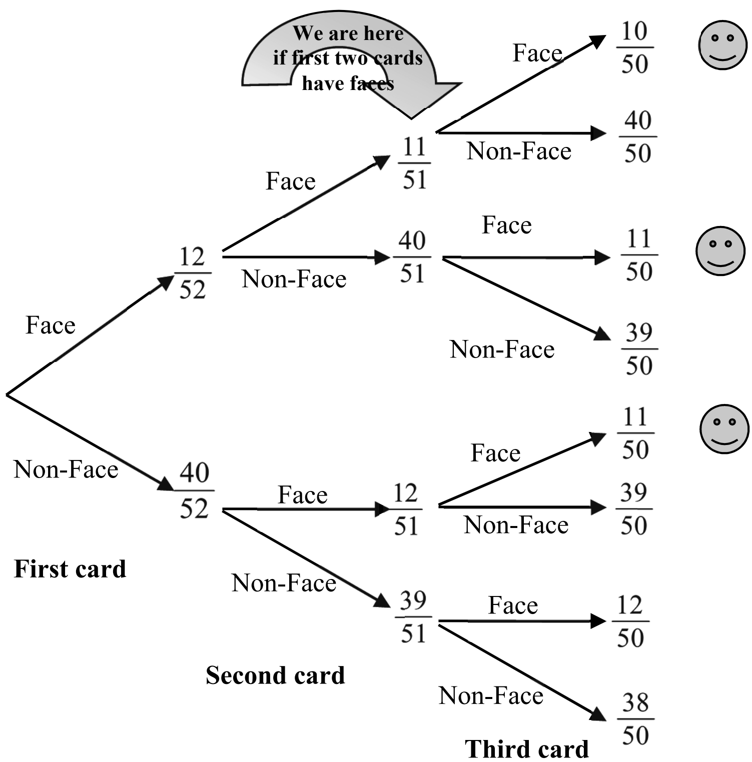
(c) Pr(first card is an ace | second card not ace)

$$= \frac{\text{Pr(first card is an ace} \cap \text{second card not ace)}}{\text{Pr(second card not ace)}}$$

$$= \frac{\frac{4}{52}\cdot\frac{48}{51}}{\frac{4}{52}\cdot\frac{48}{51}+\frac{48}{52}\cdot\frac{47}{51}}$$

$$= \frac{4}{51}.$$

2-57



(a)

Number of Face Cards	0	1	2	3
Probability	$\frac{9,880}{22,100}$	$\frac{9,360}{22,100}$	$\frac{2,640}{22,100}$	$\frac{220}{22,100}$

(b) Pr(at least 2 face cards) $= \frac{2,860}{22,100}$.

(c) Pr(3rd card is face | first two cards are face cards)

= Pr(all three face cards | first two cards are face cards) $= \frac{10}{50}$.

(d) Pr(all three face cards | at least two face cards)

$$= \frac{\text{Pr(all three face cards)}}{\text{Pr(at least two face cards)}} = \frac{\frac{220}{22{,}100}}{\frac{2{,}860}{22{,}100}} = \frac{220}{2{,}860}.$$

(e) Solution Method 1: Tree diagram and Bayes' Theorem

Pr(at least 2 face cards | last card has face)

$$= \frac{\text{Pr(at least 2 face cards } \cap \text{ last card has face)}}{\text{Pr(last card has face)}}$$

$$= \frac{\overbrace{\frac{12}{52}\cdot\frac{11}{51}\cdot\frac{10}{50}+\frac{12}{52}\cdot\frac{40}{51}\cdot\frac{11}{50}}^{\text{this is the probability that the first and third cards have faces}}+\frac{40}{52}\cdot\frac{12}{51}\cdot\frac{11}{50}}{\underbrace{\frac{12}{52}\cdot\frac{11}{51}\cdot\frac{10}{50}+\frac{12}{52}\cdot\frac{40}{51}\cdot\frac{11}{50}+\frac{40}{52}\cdot\frac{12}{51}\cdot\frac{11}{50}+\frac{40}{52}\cdot\frac{39}{51}\cdot\frac{12}{50}}_{\text{This equals } \frac{12}{52}.\text{ Does it make sense to you?}}}$$

$$= \frac{11880}{30600} = .3882.$$

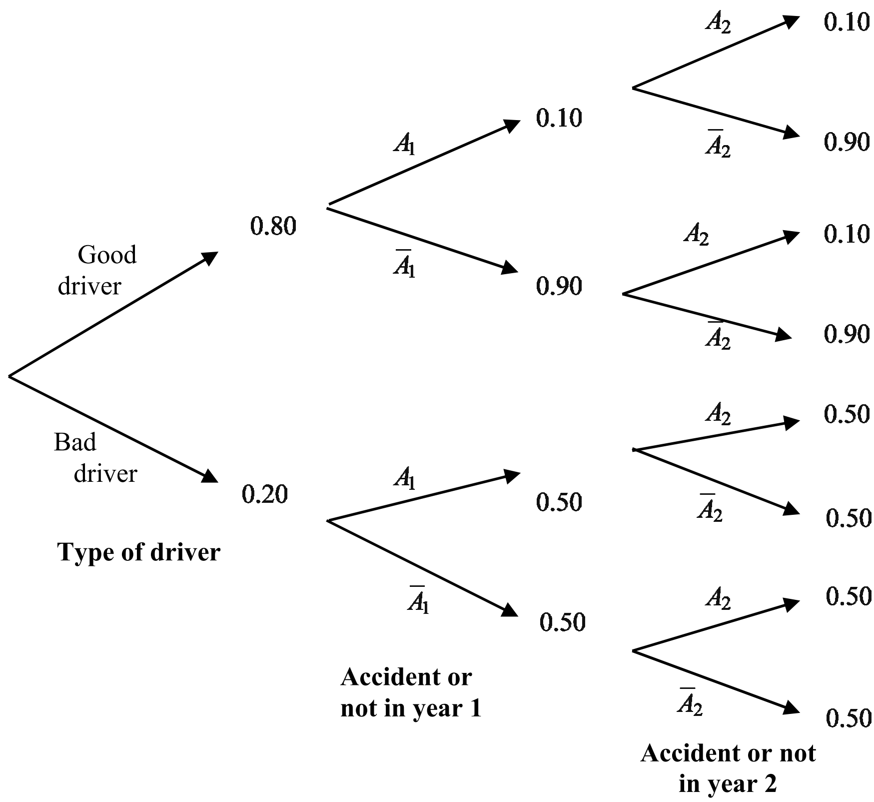
Solution Method 2: Using Combinations

Imagine that you know that the third card is a face card. That leaves 11 face cards in a deck of 51 total cards. Now imagine dealing the first two cards from this reduced deck. The question asks us to find the probability that at least one of these cards is a face card.

$$\text{Pr(at least one face)} = 1 - \text{Pr(two non-face cards)} = 1 - \frac{{}_{40}C_2}{{}_{51}C_2} = .3882.$$

Section 2.6 Credibility

2-58



(a) $\Pr(A_1 \cap A_2) = .8 \cdot (.1)^2 + .2 \cdot (.5)^2 = .058.$

(b) $\Pr(A_2 \mid A_1) = \dfrac{\Pr(A_2 \cap A_1)}{\Pr(A_1)} = \dfrac{.8 \cdot (.1)^2 + .2 \cdot (.5)^2}{.8 \cdot (.1) + .2 \cdot (.5)} = \dfrac{.058}{.18} = \dfrac{29}{90}.$

(c) $\Pr(G \mid A_1 \cap A_2) = \dfrac{\Pr(G \cap A_1 \cap A_2)}{\Pr(A_1 \cap A_2)} = \dfrac{.8 \cdot (.1)^2}{.8 \cdot (.1)^2 + .2 \cdot (.5)^2} = \dfrac{.008}{.058} = \dfrac{4}{29}.$

(d) $\Pr(B \mid \text{At least one accident}) = \dfrac{.2 \cdot (1 - .5^2)}{.2 \cdot (1 - .5^2) + .8 \cdot (1 - .9^2)} = \dfrac{.15}{.302} = .497.$

(e) $\Pr(G \mid \overline{A}_1 \cap \overline{A}_2) = \dfrac{\Pr(G \cap \overline{A}_1 \cap \overline{A}_2)}{\Pr(\overline{A}_1 \cap \overline{A}_2)} = \dfrac{.8 \cdot (.9)^2}{.8 \cdot (.9)^2 + .2 \cdot (.5)^2} = \dfrac{.648}{.698} = .9284.$

2-59 (D) $\Pr(1997 \mid 1997 \text{ or } 1998 \text{ or } 1999) = \dfrac{.16 \cdot (.05)}{.16 \cdot (.05) + .18 \cdot (.02) + .20 \cdot (.03)} = .455.$

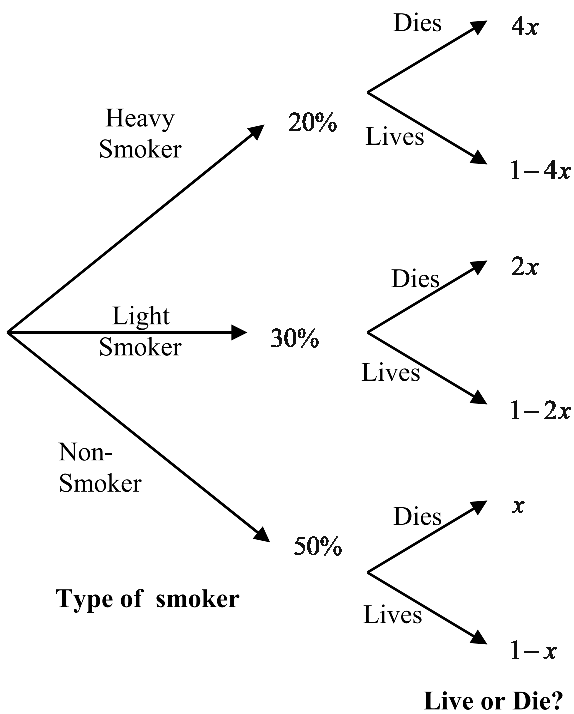
2-60 (D) Draw a tree diagram.

$$\Pr(\text{ultra-preferred} \mid \text{dies}) = \frac{(0.10) \cdot (.001)}{(0.10) \cdot (.001) + (0.40) \cdot (.005) + (0.50) \cdot (.01)} = .0141.$$

2-61 (D) Draw a tree diagram.

$$\Pr(\text{young adult} \mid \text{collision}) = \frac{(.16)(.08)}{(.08)(.15)+(.16)(.08)+(.45)(.04)+(.31)(.05)} = .22.$$

2-62 (D) Let x denote the probability that a non-smoker dies.

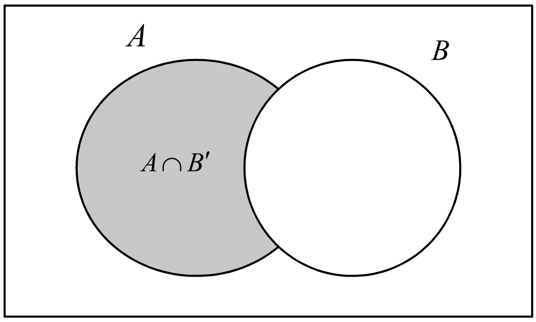


$$\Pr(\text{heavy smoker} \mid \text{died}) = \frac{(.2)(4x)}{(.2)(4x)+(.3)(2x)+(.5)x} = \frac{8}{19} = .42.$$

2-63 (B) $$\Pr(16-20 \mid \text{accident}) = \frac{(.08)(.06)}{(.08)(.06)+(.15)(.03)+(.49)(.02)+(.28)(.04)} = .158$$

Section 2.7 Chapter 2 Sample Examination

1.



2. $$\Pr(\text{dealt a full house}) = \frac{\text{number of full houses}}{\text{number of 5 card hands}} = \frac{{}_{13}C_1 \cdot {}_4C_3 \cdot {}_{12}C_1 \cdot {}_4C_2}{{}_{52}C_5} = \frac{3744}{2598960}.$$

3. You may wish to create a Venn diagram. The key is to note that that you must have some insurance to be a client.

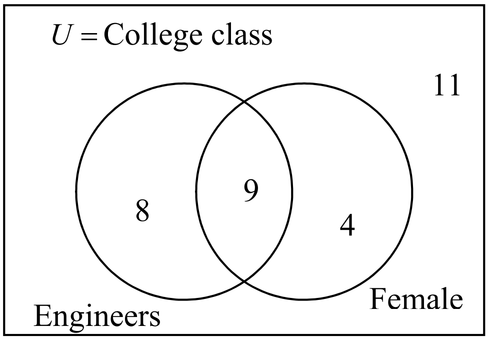
	Dismemberment Insurance	No Dismemberment	
Auto Insurance	17	45	62
No Auto Insurance	20	0	20
	37	45	82

(a) $N(\text{both auto and dismemberment}) = 17.$

(b) $N(\text{exactly one kind of insurance}) = 65 = 20 + 45.$

(c) No, because $\Pr(\text{auto} \cap \text{dismember}) = \frac{17}{82} \neq \frac{62}{82} \cdot \frac{37}{82} = \Pr(\text{auto}) \cdot \Pr(\text{dismember}).$

4. (a) Venn Diagram



Venn Box Diagram

	Engineer	Non-Engineer	
Female	9	4	13
Male	8	11	19
	17	15	32

(b) $\Pr(\text{female} \cap \text{engineer}) = \dfrac{9}{32}$.

(c) $\Pr(\text{female} \cup \text{engineer}) = \dfrac{21}{32} = 1 - \dfrac{11}{32}$.

(d) $\Pr(\text{female} \mid \text{non-engineer}) = \dfrac{4}{15}$.

(e) No, since $\Pr(\text{male} \cap \text{engineer}) \neq \Pr(\text{male}) \cdot \Pr(\text{engineer})$.

5. (a) $\Pr(\text{dealt 3 Kings}) = \dfrac{\text{number of ways to be dealt 3 Kings}}{\text{number of ways to be dealt 3 cards}} = \dfrac{{}_4C_3}{{}_{52}C_3} = \dfrac{4}{22100}$.

(b) $\Pr(\text{NOT dealt 3 Kings}) = 1 - \dfrac{{}_4C_3}{{}_{52}C_3} = \dfrac{22096}{22100}$.

(c) $\Pr(3^{\text{rd}} \text{ card King} \mid 1^{\text{st}} \text{ and } 2^{\text{nd}} \text{ card King}) = \dfrac{2}{50}$.

(d) $\Pr(\text{3 Kings} \mid \text{at least 2 Kings}) = \dfrac{{}_4C_3}{{}_4C_3 + {}_4C_2 \cdot {}_{48}C_1} = \dfrac{4}{292}$.

6. (a) There are 16 elements in the set. The **bold numbers** denote a sum of 6.

$$U = \{22, 23, \mathbf{24}, 25, 32, \mathbf{33}, 34, 35, \mathbf{42}, 43, 44, 45, 52, 53, 54, 55\}.$$

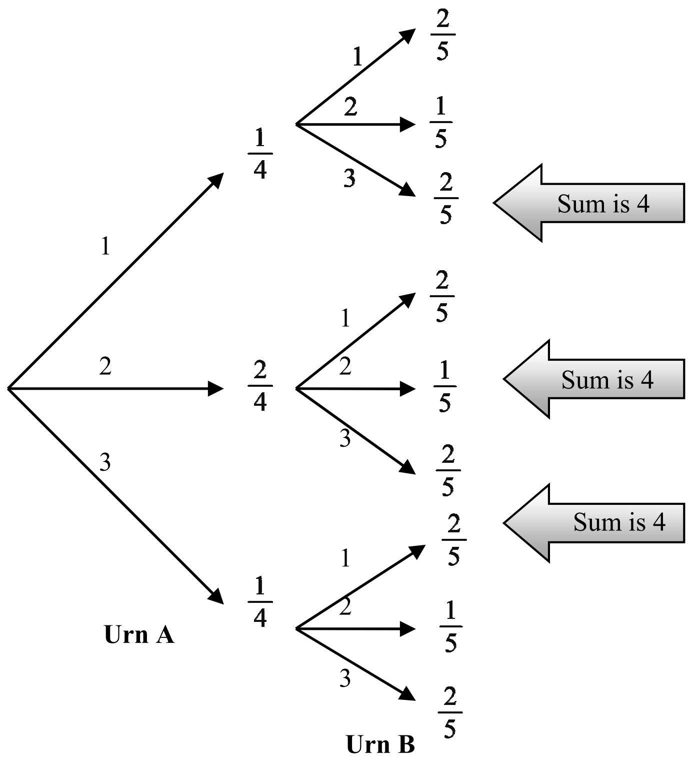
(b) $\Pr(\text{sum is nine}) = \dfrac{2}{16}$.

(c) $\Pr(\text{at least one die is 5}) = \dfrac{7}{16}$

(d) $\Pr(\text{sum is 6} \mid \text{at least one 5}) = \dfrac{0}{9}$.

(e) $\Pr(\text{at least one 5} \mid \text{sum is 6}) = \dfrac{0}{3}$.

7. (a)



(b) $\Pr(\text{sum is 4}) = \dfrac{1}{4}\cdot\dfrac{2}{5}+\dfrac{2}{4}\cdot\dfrac{1}{5}+\dfrac{1}{4}\cdot\dfrac{2}{5} = \dfrac{6}{20}$.

(c) $\Pr(\text{at least one 2}) = \dfrac{1}{4}\cdot\dfrac{1}{5}+\dfrac{2}{4}+\dfrac{1}{4}\cdot\dfrac{1}{5} = \dfrac{12}{20}$.

(d) $\Pr(\text{sum is 4} \mid \text{ball from Urn } A \text{ is 2}) = \Pr(\text{ball from Urn } B \text{ is 2}) = \dfrac{1}{5}$.

(e) $\Pr(\text{ball from Urn } B \text{ is 2} \mid \text{sum is 4}) = \dfrac{\frac{2}{4}\cdot\frac{1}{5}}{\frac{6}{20}} = \dfrac{1}{3}$.

8. (a)

Gender	Favorite Drink					
	Coke	Diet Pepsi	Ripple	Water	Red Bull	
Female	2	3	8	4	1	**18**
Male	5	1	2	0	4	**12**
	7	**4**	**10**	**4**	**5**	**30**

(b) $\Pr(\text{Red Bull}) = \dfrac{5}{30}$.

(c) $\Pr(\text{female} | \text{milk}) = \dfrac{N(\text{female} \cap \text{milk})}{N(\text{milk})} = \dfrac{8}{10}$.

d) $\Pr(\text{male} \cup \text{milk}) = \dfrac{20}{30}$.

9. (a) $\Pr(A) = 0.4$.

(b) $\Pr(A') = 0.6$.

(c) $\Pr(A \cap B) = .25$.

(d) $\Pr(A | B) = \dfrac{\Pr(A \cap B)}{\Pr(B)} = \dfrac{.25}{.65}$.

(e) $\Pr(A \cup B) = 0.8$.

(f) $\Pr(B | A) = \dfrac{\Pr(B \cap A)}{\Pr(A)} = \dfrac{.25}{.40}$.

(g) No, they are dependent. $\Pr(A \cap B) \neq \Pr(A) \cdot \Pr(B)$.

10. Assume for definiteness that card with 5 credits is in the left (L) pocket and the 4 credit card is in the right (R) pocket. She could either spend all 5 credits in L, or spend one from L and 4 from R, with the last one coming from R. These actions can be displayed as the following 5 letter words using L and R:

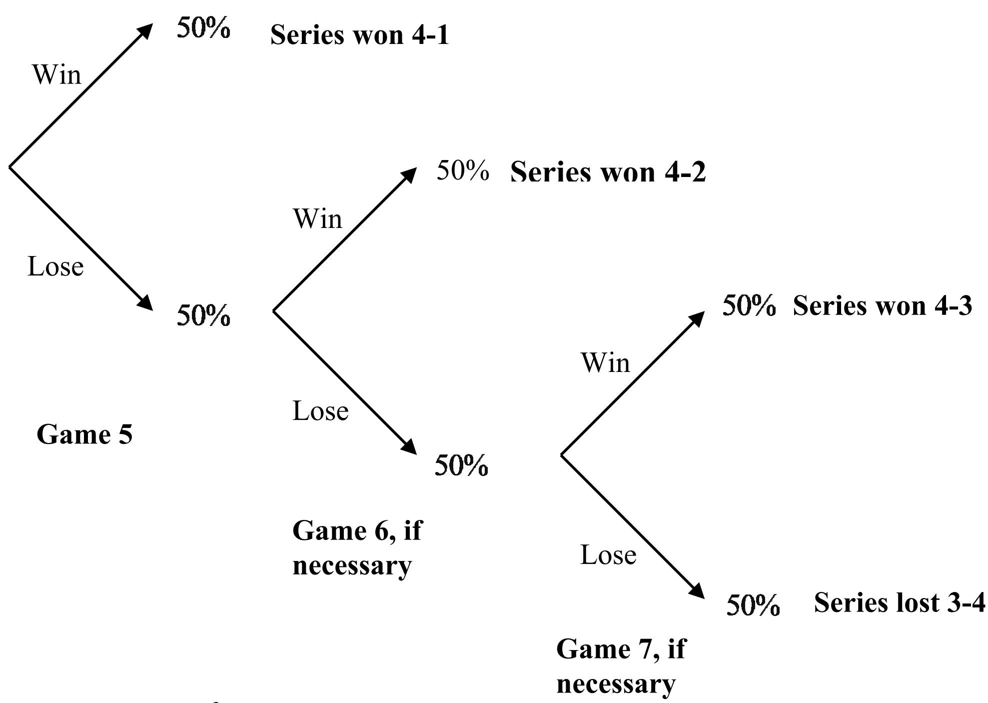
LLLLL, LRRRR, RLRRR, RRLRR, RRRLR.

Each word has probability $\left(\dfrac{1}{2}\right)^5$, and so the answer is $5 \cdot \left(\dfrac{1}{2}\right)^5 = \dfrac{5}{32}$.

11. Assume first that the left pocket is depleted leaving 3 credits in the right pocket. This means a total of 6 credits are spent, 1 from the right pocket, 5 from left, and the last one used is from the left pocket. This can be portrayed using 6 letter words with 1 *R* and 5 *L*'s, ending in *L*. That means the first 5 positions in the word consist of 1 *R* and 4 *L*'s. There are ${}_5C_1 = 5$ such words.

Next, assume that the right pocket is depleted leaving 3 credits in the left pocket. This means we have 6 letter words ending in *R* with 2 *L*'s and 3 *R*'s in the first 5 positions. There are ${}_5C_2 = 10$ such words. Thus, the total probability is $15 \cdot \left(\frac{1}{2}\right)^6 = \frac{15}{64}$.

12. Consider what can happen to the team that is leading 3 games to 1.



$\Pr(\text{win}) = 1 - .5^3 = .875.$

13. $\Pr(\text{win}) = 1 - .4^3 = .936.$

14.

	A	A'	Row Total
B	a		b
B'			$1/3$
Column Total	$2/3$		1

$b = 1 - 1/3 = 2/3$ and $a = \Pr\left[A \cap B\right] = \Pr\left[A \mid B\right]\Pr\left[B\right] = \frac{7}{10} \cdot \frac{2}{3} = \frac{7}{15}$. Complete the Box Diagram to give:

	A	A'	Row Total
B	$7/15$	$1/5$	$2/3$
B'	$1/5$	$2/15$	$1/3$
Column Total	$2/3$	$1/3$	1

Then $\Pr(A \cup B) = 1 - 2/15 = 13/15$.

15. Begin by writing out the sample space of possible dance couples.

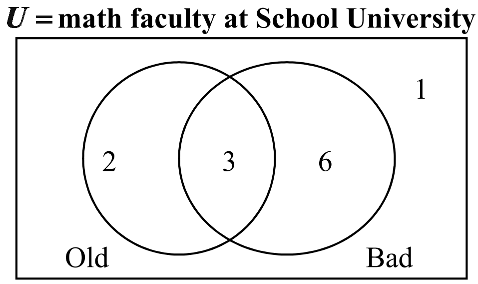
Ann's Dance Partner		Betty's Dance Partner		Danielle's Dance Partner	
Ann	**Andy**	Betty	Boris	Danielle	Dan
Ann	**Andy**	Betty	Dan	Danielle	Boris
Ann	**Boris**	Betty	Andy	Danielle	Dan
Ann	***Boris***	*Betty*	*Dan*	*Danielle*	*Andy*
Ann	**Dan**	Betty	Andy	Danielle	Boris
Ann	***Dan***	*Betty*	*Boris*	*Danielle*	*Andy*

$\Pr(\text{3 wives dance with non-spouse}) = \frac{2}{6}$.

16. $U = \{abcd, abdc, acbd, acdb, adbc, adcb, bacd, \mathbf{badc}, bcad, \mathbf{bcda}, \mathbf{bdac}, bdca$
$cabd, \mathbf{cadb}, cbad, cbda, \mathbf{cdab}, \mathbf{cdba}, \mathbf{dabc}, dacb, dbac, dbca, \mathbf{dcab}, \mathbf{dcba}\}$.

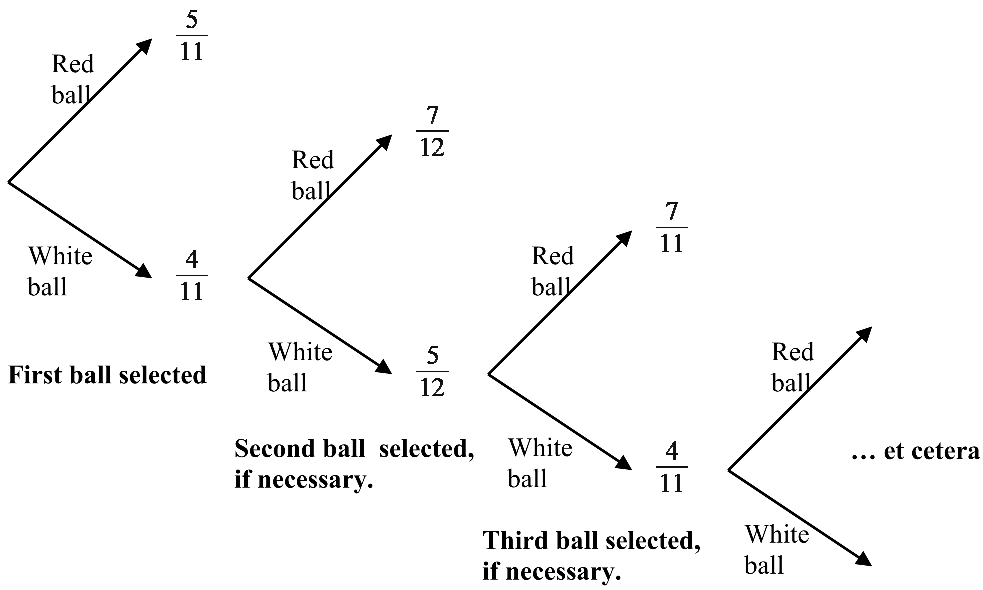
$\Pr(\text{all wives dances with non-spouse}) = \frac{9}{24}$

17. There is one good young teacher.



18. Pr(first person wins)

$$= \frac{7}{13} + \frac{6}{13}\cdot\frac{5}{12}\cdot\frac{7}{11} + \frac{6}{13}\cdot\frac{5}{12}\cdot\frac{4}{11}\cdot\frac{3}{10}\cdot\frac{7}{9} + \frac{6}{13}\cdot\frac{5}{12}\cdot\frac{4}{11}\cdot\frac{3}{10}\cdot\frac{2}{9}\cdot\frac{1}{8}\cdot\frac{7}{7}.$$

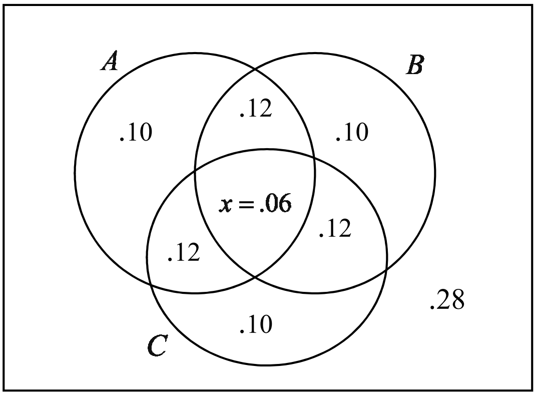


19. (a) $\Pr(1^{st}\text{ person wins}) = \frac{4}{6}\cdot\frac{2}{6} + \left(\frac{4}{6}\right)^3\cdot\frac{2}{6} + \left(\frac{4}{6}\right)^5\cdot\frac{2}{6} + \ldots = 40\%$

Use geometric series.

(b) $\Pr(1^{st}\text{ person wins}) = \frac{4}{6}\cdot\frac{3}{6} + \frac{4}{6}\cdot\frac{3}{6}\cdot\frac{2}{6}\cdot\frac{5}{6} = 42.59\%.$

20.



To solve for x, we know:

$$\frac{1}{3} = \Pr(A \cap B \cap C \mid A \cap B) = \frac{\Pr(A \cap B \cap C)}{\Pr(A \cap B)} = \frac{x}{x + .12} \Rightarrow x = .06.$$

$\Pr(\text{no risk factor} \mid A') = \dfrac{.28}{.60} = .4\overline{6}.$ (C)

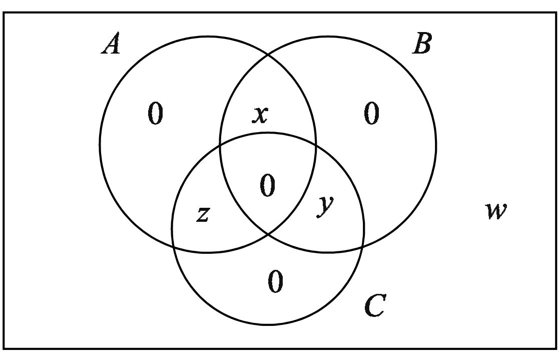
21. (C) We have four equations with four unknown variables:

$$x + z = \frac{1}{4}, \quad x + y = \frac{1}{3}, \quad y + z = \frac{5}{12}, \text{ and } w + x + y + z = 1.$$

Adding the first three equations together, we see

$$2x + 2y + 2z = 1 \Rightarrow (x+y+z) = \frac{1}{2}.$$

$\Pr(\text{no coverage}) = w = 1 - (x + y + z) = 1 - \dfrac{1}{2} = \dfrac{1}{2}.$



22. Let A represent the event of heart disease. Let B represent the event of at least one parent with heart disease. Then:

	A	A'	Row Total
B	**102**		**312**
B'	108		625
Column Total	**210**		**937**

$$\Pr(A|B') = \frac{108}{625} = .1728. \quad \text{(B)}$$

23.

	One Car	Multiple Cars	Row Total
Sports Car		a	20
No Sports Car			
Column Total		70	100

$$\begin{aligned} a &= \Pr\left[\text{Sports Car} \cap \text{Multiple Cars}\right] \\ &= \Pr\left[\text{Sports Car} \mid \text{Multiple Cars}\right]\Pr\left[\text{Multiple Cars}\right] \\ &= (15\%)(70\%) \\ &= 10.5\%. \end{aligned}$$

The completed Box Diagram is:

	One Car	Multiple Cars	Row Total
Sports Car	9.5	10.5	20
No Sports Car	20.5	59.5	80
Column Total	30	70	100

Pr(one car $\cap$ no sports car) = 20.5% (B)

24. The Auto/Homeowner Box Diagram is:

	A	A'	Row Total
H	15	50	65
H'	35	0	35
Column Total	50	50	100

Thus, 35% have auto insurance only, 50% have homeowners only and 15% have both. Then

$$\Pr(\text{renew}) = \underbrace{(.50)}_{\text{auto only}} \cdot \underbrace{(.40)}_{\substack{\text{renew given} \\ \text{insure only auto}}} + (.35)\cdot(.60) + (.15)\cdot(.80) = 53\%. \quad \text{(D)}$$

25. Begin with the Emergency/Operating room Box Diagram.
$\Pr\left[E' \cap O'\right] = 1 - \Pr\left[E \cup O\right] = 1 - 85\% = 15\%$.

	E	E'	Row Total
O			
O'		15	a
Column Total		25	100

Also, by independence, $15\% = 25\% \cdot a \Rightarrow a = 60\%$.

The complete Box Diagram is:

	E	E'	Row Total
O	30	10	40
O'	45	15	60
Column Total	75	25	100

$\Pr\left[O\right] = 40\%$ (D)

26. (D) A tree diagram would be nice, but would have infinitely many branches. The key is to figure out how many ways there can be a total of seven accidents in two weeks. There could be 0 accidents the first week and 7 accidents the second week. Or there could be 1 accident the first week and 6 accidents the second week, and so forth. Let (a,b) be the pair representing a accidents the first week and b accidents the second week.

Pr(exactly 7 accidents)

$$= \Pr\left[(0,7)\right] + \Pr\left[(1,6)\right] + \cdots + \Pr\left[(7,0)\right]$$

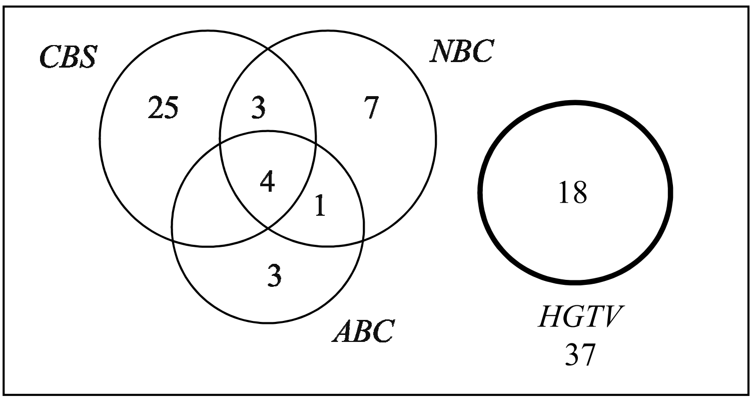
$$= \frac{1}{2^1} \cdot \frac{1}{2^8} + \underbrace{\frac{1}{2^2}}_{\substack{\text{prob of 1} \\ \text{accidents in} \\ \text{first week}}} \cdot \underbrace{\frac{1}{2^7}}_{\substack{\text{prob of 6} \\ \text{accidents in} \\ \text{second week}}} + \cdots + \frac{1}{2^8} \cdot \frac{1}{2^1} = \frac{8}{2^9} = \frac{1}{64}.$$

27.

	L	L'	Row Total
P	0.03	0.07	**0.10**
P'	**0.01**	0.89	0.90
Column Total	**0.04**	0.96	1.00

$\Rightarrow \Pr[L' \cap P'] = 0.89.$ (E)

28. Note that (vii) shows that HGTV is disjoint from the other 3 channels. Thus, we can use the Mickey Mouse diagram plus an extra circle for HGTV:



The number not watching any of the channels is 37. (B)

One may also use the inclusion-exclusion formula for 4 sets:

$$n(\text{CBS} \cup \text{NBC} \cup \text{ABC} \cup \text{HGTV}) = n(\text{CBS}) + n(\text{NBC}) + n(\text{ABC}) + n(\text{HGTV})$$

$$-n(\text{CBS} \cap \text{NBC}) - n(\text{CBS} \cap \text{ABC}) - n(\text{CBS} \cap \text{HGTV})$$

$$-n(\text{NBC} \cap \text{ABC}) - n(\text{NBC} \cap \text{HGTV}) - n(\text{ABC} \cap \text{HGTV})$$
$$+n(\text{CBS} \cap \text{NBC} \cap \text{ABC}) + n(\text{CBS} \cap \text{NBC} \cap \text{HGTV})$$
$$+ n(\text{CBS} \cap \text{ABC} \cap \text{HGTV}) + n(\text{NBC} \cap \text{ABC} \cap \text{HGTV})$$
$$- n(\text{CBS} \cap \text{NBC} \cap \text{ABC} \cap \text{HGTV})$$
$$= 34 + 15 + 10 + 18 - 7 - 6 - 0 - 5 - 0 - 0 + 4 + 0 + 0 + 0 - 0$$
$$= 63.$$

$$n([\text{CBS} \cup \text{NBC} \cup \text{ABC} \cup \text{HGTV}]') = 100 - 63 = 37.$$