A PROBLEM-SOLVING APPROACH TO PENSION FUNDING AND VALUATION

Second Edition

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ACTEX Publications
Winsted, Connecticut

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The ankh was the ancient Egyptian symbol for life.
Its use on the cover of this text symbolizes the payment of
pension income for life.

ISBN: 978-1-62542-841-7

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Preface

Over the past twenty-five years, several efforts have been made to produce a high-quality introductory textbook covering the mathematics of pension funding and valuation. Although all of the resulting texts have had consider-able merit, it does not appear that any one of them has been seen as totally satisfactory.

This work, first published in 1994, has been guided by the principle that many students, particularly at an introductory level, learn best by doing. The defining characteristic of this text is a large number of worked examples. Each unit is introduced with enough explanatory material to allow the reader to move quickly to problem solving. It is this basic feature that gives the text its name. The first mention of each technical term is indicated in bold type.

In addition to the explanatory material and worked examples, the text contains non-numerical discussion questions intended to provoke further thinking about the material. Finally, each major section contains several exercises for the reader to use as reinforcement of the learning attained in that section. Numerical answers to the exercises are given in the text. A separate solutions manual, containing answers to the discussion questions and detailed solutions to the exercises, is also available.

The text presumes a knowledge of basic actuarial mathematics, as contained in such textbooks as Parmenter [9], Jordan [6], or Bowers, *et al*. [5]. In particular, the reader is presumed to understand multiple-decrement theory, multiple life functions (including joint and last-survivor functions), and concepts of reserving.

The text is organized into eight chapters. The first chapter introduces the reader to pension plans themselves, and gives an overview of the remaining topics. Chapters 2-4 cover the standard cost methods (individual methods in Chapters 2 and 3, and aggregate methods in Chapter 4). Chapters 5-7 cover, respectively, gain and loss analysis, plan changes and ancillary benefits, and options and assets. Chapter 8 presents an analysis and summary of the earlier material intended to enhance understanding on a more conceptual level. The text concludes with answers to the exercises, a bibliography, an index to notation (giving the page on which each symbol first appears), and a subject index.

Many people have contributed to the completion of this project, and their assistance is gratefully acknowledged. Excellent contributions were made by Mary Weiler, Nandanee Basdeo, Hoque Sharif, and Ali Zaker-Shahrak. Credit and appreciation for the improved typesetting reflected in the second edition go to Marilyn Baleshiski, at ACTEX Publications.

Early drafts of the first edition were made available to candidates for the EA-lB exam of the Joint Board for the Enrolment of Actuaries in May 1993 and May 1994, and to candidates for the Course 210 exam of the Society of Actuaries in November 1993. We received many valuable comments from this exposure, and we wish to thank these candidates for their help.

The first edition was reviewed by an industry task force consisting of Harold Brownlee, FSA, Richard Daskais, FSA, Ralph Garfield, ASA, Ph.D., and Terry Vaughan, Ph.D., and chaired by Geoffrey Crofts, FSA, then of the University of Hartford. Additional review was provided by Professor Kelley McKeating, ASA, of the University of Iowa, and Professor Srinivasa Ramanujam, ASA, EA, Ph.D., of the College of Insurance.

Special thanks goes to Richard J. Barney, FSA, EA, of the West Patterson office of Milliman & Robertson, whose insights and suggestions over the past three years have strengthened the text in many ways.

Numerous improvements have been made in the second edition, with many of them arising from suggestions made by students and other readers. Several symbols have been changed to improve clarity, and certain unnecessary symbols were deleted altogether. Some practical observations, historical notes, summaries (*e.g.*, Section 4.6), and alternate solutions to examples have been added. There are also additional discussion questions and exercises. Thanks to Leslie N. Taylor, FCIA, FIA, for much of the improvement in Chapter 8.

Notwithstanding the review the text has received, there are no doubt some errors and inadequacies remaining. The author and publisher would appreciate hearing from readers in this regard.

We hope that you the reader will find this text of great value in your pursuit of an understanding of the world of pension mathematics.

Waterloo, Ontario, Canada William H. Aitken, FSA, FCIA, EA
April, 1996

William H. Aitken
1927 - 2000

William H. Aitken, Professor Emeritus at the University of Waterloo, passed away on July 30, 2000. Bill was a graduate of Mathematics and Physics at the University of Toronto and a Fellow of the Society of Actuaries. Bill's career spanned several jobs in the insurance and pensions industry. He was a respected Associate Professor at The University of Waterloo for two decades, and authored significant additions to the actuarial literature. After his retirement in 1992, he continued to work in the pensions consulting field. His funeral was well-attended by University of Waterloo colleagues and, especially, members of his bridge club and his church. He leaves a wife, Lillian, and two daughters.

ACTEX Publications thanks Keith P. Sharp, Ph.D., FSA, University of Waterloo, and Michael Barth, ASA for their contribution to this reprint.

To my wife, Lillian

Chapter 1

Pension Benefits

For a retired person, the prospect of no money coming in each month is not at all attractive. However, a *continuation* of the same financial standard of living after retirement as before represents an attractive and important objective. Attaining this objective requires planning, assumptions, funding, and continuous monitoring. Falling short of this objective suggests a decline in the standard of living of the retired person, whereas exceeding this objective suggests overly generous employer contributions and tax assistance.

1.1 Introduction

The rapid growth in pension plans since 1940 has resulted in pensions being a significant factor in the financial markets, where billions of dollars of pension assets are invested, and in personal financial security where, for many, the pension will become the largest asset. This rapid growth can be attributed to several factors, including the following:

- Employer participation
- Union demands
- A strong post-war economy
- Government legislation
- Tax assistance

Moreover, the need for personal retirement savings has increased due to factors such as the decline of the extended family, the change in attitude toward older workers, and increased competition for employees. Today we enter the work force at a later age. We leave earlier. We live longer. Therefore, we must accrue more retirement funds over a shorter period of time.

For most retirees there are three sources of retirement income, which include (a) retirement income from government sources, (b) individual income from individual savings, and (c) income from pension plans, generally employer sponsored. In turn, employer plans can be categorized as (a) single employer, (b) multiple employer, and (c) multi-employer.

The most common arrangement is the single employer plan. One employer creates, administers, and contributes to the operation of the plan for the employees. Multiple employer plans generally result when em-ployers have small numbers of employees. Since a minimum administration cost is unavoidable, a small number of employees will result in an administration cost per participant which is unmanageably large; the cost per participant can be reduced by employers banding together to share expenses. Multi-employer plans are usually negotiated by unions. This arrangement is effective for mobile employees who can move from job to job within an industry, without the concern of losing pension rights.

1.2 Design of Pension Plans

There are two basic types of pension plans, which are (a) defined contribution plans, and (b) defined benefit plans. The fundamental difference, as the names suggest, is the defined parameter.

Defined contribution plans define in advance what participants and plan sponsors will contribute each year. The benefit received by a participant at retirement is a function of the contributions made to his account and the fund's investment return during, and at the end of, the accrual period. The participant in a defined contribution plan bears the investment risk, for better or for worse.

Defined benefit plans use a formula to determine in advance the amount of the retirement benefit, although this may not be accurately known until retirement. Again, the participant's contributions, if any, are clearly defined. The plan sponsor's contributions vary with actual experience in order to accrue the amount of the defined benefit at retirement.

1.2.1 Defined Benefits

A common benefit is a **unit benefit per year of service**. One example of a retirement benefit would be \$40 per month at retirement for each year of service. This design is very popular with unions, due to ease of understanding the benefit and negotiating benefit improvements. When a union is bargaining, an improvement in the pension benefit can be effected by merely changing the unit amount from \$40 to, say, \$45 per month. This type of benefit improvement is usually retroactive, thereby providing protection to workers from preretirement inflation. Another example would be a pension benefit of 1.5% of compensation (or salary) for each year of service, where the compensation can be career average compensation or the average compensation over the last few years of service. Compensation generally increases with time due to merit and inflation. The closer the averaging period is to retirement, the more is the protection from preretirement inflation.

These several ways to define the pension benefit are illustrated in the following example.

EXAMPLE 1.1

Suppose a participant has 30 years of service, career average compensation of \$40,000, and final average compensation of \$100,000. Let B_r denote the annual pension benefit, payable monthly, from retirement age r. Find B_r if the defined pension benefit is (a) \$50 per month for each year of service, (b) 1.5% of career average compensation for each year of service, and (c) 1.5% of final average compensation for each year of service.

SOLUTION

(a) The annual pension benefit is $B_r = 50 \times 12 \times 30 = \$18{,}000$.
(b) Here the benefit is $B_r = .015 \times 40{,}000 \times 30 = \$18{,}000$.
(c) This time we have $B_r = .015 \times 100{,}000 \times 30 = \$45{,}000$. □

Two other basic types of defined benefit are flat and fixed. The **flat benefit** type is a function of compensation but not of service, such as 60% of compensation at retirement. The **fixed benefit** type is not a function of compensation *or* service, but rather a specified amount at retirement such as \$40,000 per year. Since these designs have an arbitrary accrual of benefit, they are often not acceptable to government or industry except for providing a minimum guaranteed benefit contingent on a minimum service period. This is illustrated in the following two examples.

EXAMPLE 1.2
Using the data from Example 1.1, find B_r if the annual pension benefit is to be \$10,000 or 1.5% of final average compensation for each year of service, whichever is greater.

SOLUTION The benefit is \$10,000 or, if greater,

$$(.015)(100{,}000)(30) = \$45{,}000.$$

In this case $B_r = \$45{,}000$. □

EXAMPLE 1.3
Rework Example 1.2 assuming the participant has only 5 years of service.

SOLUTION The benefit is \$10,000 or, if greater,

$$(.015)(100{,}000)(5) = \$7500.$$

In this case $B_r = \$10{,}000$. □

1.2.2 Defined Contributions and Defined Benefits

Plan designs differ slightly between Canada and the United States. In Canada, most pension plans are defined contribution plans, although they include only about 10% of the plan participants. Large pension plans, those with more than one thousand participants, are generally defined benefit plans. In the U.S., much more than 10% of plan participants are in defined contribution plans.

In a defined contribution plan an individual account is maintained for each plan participant. The amount in this account, including investment income, is then used to provide benefits at retirement. The employer makes a prearranged contribution to the account. No investment risk is borne by the employer.

Like defined benefit plans, defined contribution plans have different types of designs. The most significant structures in the marketplace are (a) money purchase plans, and (b) profit sharing plans. In Canada, about 95% of defined contribution plans are money purchase. Money purchase plans utilize a formula to predefine required employer contributions; the formula is either a function of the salary or the contributions made by the employee.

For example, the employer may contribute 6% of participant salaries or may match the employee contributions. Contributions may be reduced by any forfeitures on account of terminated participants with short periods of service. This design is popular with multi-employer plans.

Profit sharing plans comprise the remaining 5% of defined contribution plans. The contributions can vary within an allowable range and will be less in bad financial years and more in good years. As an example, the employer may contribute 10% of company profit before income tax to the pension plan, prorated by the salaries of the participants.

Canadian government legislation stipulates a minimum employer contribution of 1% of the earnings of plan participants. This requirement helps to protect participants from receiving very little pension upon retirement.

In the U.S., 401(k) plans, also referred to as cash-or-deferred arrangements, or salary reduction plans, allow a participant to contribute a portion of compensation to the plan on a before tax basis. The voluntary employee salary reduction contributions are not classified as income in the year of contribution, and the entire payout, including interest, is taxable income in the years in which the benefits are received. The employer may or may not make matching contributions. These plans are named after the section of the Internal Revenue Code under which they are regulated. More than half of all U.S. employees in defined contribution plans are in 401(k) plans. Their widespread acceptance is due mostly to allowing employees to make contributions on a before-tax basis. Tax treatment has a significant effect on the accumulation of pension funds, and 401(k) plans, along with Section 403(b) and Section 457 plans, are the principal private plans which allow employees to defer a portion of their salary on a before-tax basis. A typical 401(k) thrift savings plan might allow a participant to contribute up to 15% of compensation to the plan. The employer, in turn, might agree to make a matching contribution, but no more than 3% of compensation. Profit sharing plans are more common in the U.S. than in Canada.

The maximum dollar amount which may be deferred by an individual employee under a 401(k) plan is $8994 for 1993 and cost-of-living adjusted amounts for each subsequent year.

Defined pension benefits and contributions are often based on compensation as defined in the plan document. The measure of compensation may be either salary or wages and may or may not include overtime and bonuses. In this text we will use C to denote contribution and S to denote salary or compensation.

Pension plans always contain pension benefits and usually contain additional related benefits called **ancillary benefits**, payable in the event of total disability before retirement, death before retirement, or death after retirement.

Defined contribution arrangements inherently emphasize termination benefits over retirement benefits, whereas typical defined benefit plans emphasize retirement benefits over termination benefits. An employer wishing to attract younger, more mobile employees may feel that defined benefit plans should be abandoned in favor of money purchase plans. For certain employee groups this may indeed be a satisfactory approach. It may seem strange to adopt improved termination benefits as a hiring inducement, but to an employee assessing the merits of a particular plan, that may be a very significant factor and may outweigh the disadvantage of a less attractive retirement benefit that usually comes with a change to a defined contribution plan.

There are other alternatives to a pure defined contribution plan which retain the defined benefit concept and its inherent advantages in retirement and personnel planning, but at the same time incorporate some of the desirable features of defined contribution plans. According to The Mercer Bulletin [8], some of them are the following:

(1) A combination plan consists of a defined benefit base plan, typically non-contributory and with a low benefit level, in conjunction with a supplementary defined contribution plan. This type of arrangement can have significantly enhanced termination benefits if the supplementary portion requires employer contributions.

(2) A hybrid plan provides benefits based on the greater of a defined benefit formula and a defined contribution formula. In effect, such plans provide, on termination, money purchase benefits for employees terminating prior to a certain age and defined benefit pensions thereafter.

(3) A flexible pension plan consists of a non-contributory basic defined benefit pension plan supplemented by employee contributions which are notionally credited to the plan and used later at the employee's option to purchase ancillary benefits.

(4) Perhaps the simplest concept is to retain the basic defined benefit pension plan and then *enhance the commuted values* paid on termination. This can be done either by providing for assumed future salary increases up to normal retirement age, or by including the value of ancillary benefits, such as early retirement enhancements, which are now commonly excluded unless an employee meets minimum age and/or service requirements. Alternatively, deferred pensions could be indexed for inflation that may occur before their commencement date.

1.3 Plan Cost

In a defined benefit plan, an actuary is required to determine what contributions need to be made. The employer's year-to-year costs vary with the actuary's assumptions for investment income, mortality, terminations, and salary increases; the actual experience of the plan can also have a significant effect on costs. For an employer with a low profit margin this uncertainty may be undesirable. Pension benefit guarantees and funding standards exist, and actuarial reports must be completed periodically. Defined benefit plans are more complicated, leading to higher actuarial expenditures which are usually borne by the employer. Fortunately, there has been a substantial improvement in pension plan administration software and hardware.

In a defined contribution plan, contributions are set by a formula and, for the most part, are known, but legislation changes in the United States have made the filing requirements for defined contribution plans almost as complicated as for defined benefit plans. Furthermore, the advent of nondiscrimination, daily record-keeping, and Section 404(c) rules may make defined contribution plans nearly as expensive to administer as well.

Section 8.3 contains further comments on cost, contributions, and expenses.

1.3.1 Replacement Ratio

The replacement ratio is the ratio of retirement income just after retirement to compensation just before retirement. It is commonly felt that a ratio of 70% will, in most cases, maintain a satisfactory standard of living after retirement.

To be more sophisticated, we could assume mortgage payments and child-care expenses are spread over the preretirement period, we could recognize work-related expenses and pension contributions, and we could calculate preretirement and (lower) postretirement income taxes. Last but not least, we could estimate Social Security income. Then we could calculate a net replacement ratio taking these factors into account. A net ratio of 80% would represent a 20% drop in standard of living. With good forward planning it would be possible to achieve a net ratio of 100% and no financial diminution of the standard of living.

The defined benefit design, combined with Social Security, is more closely attuned to a satisfactory net replacement ratio than is the defined contribution design. With good investment performance, the defined contribution design could easily give a net ratio of over 130% and perhaps, in such a case, the tax relief on the pension fund investment income is excessive. With poor investment performance the defined contribution design could easily give a net ratio of under 70% and the pensioner would suffer.

1.3.2 The Effects of Age and Plan Design

Younger employees may be better served by a defined contribution plan. Generally they are more mobile and therefore need a pension plan which can move with them with no significant loss. In a defined benefit plan, the pension benefit is a usually a function of final average salary. If the participant changes jobs, the defined benefit reflects the present salary, not the salary at retirement age, and the participant usually suffers from the effects of inflation between termination and retirement.

In calculating the cost of a defined benefit, there is more discounting if the participant is young and less discounting for a participant near retirement; a participant close to retirement tends to receive a more expensive benefit from a defined benefit plan than from a defined contribution plan.

People closer to retirement are more security conscious and more protected in a defined benefit plan compared to a defined contribution plan. Another difference is investment risk; a fund manager for a large defined benefit plan may take a long-term view with more risk and reward; a small defined contribution fund is often more conservatively invested.

1.3.3 Anticipating Inflation

When performing an actuarial valuation for a pension plan, the actuary must choose an approximation to the future unknown rates of interest and salary increase. Either an explicit or implicit inflation component is normally

included in future interest rate and salary increase assumptions. A simple assumption for the interest rate might be 6%, of which 3% represents inflation and 3% represents the real interest rate. Similarly, a salary scale might include 3% for inflation and 2% for expected average annual merit increases, or 3% of current salary for inflation and 4% of the starting salary for the participant's salary class for average annual merit increases. Salary assumptions are discussed in Section 6.5.

The actuary will review the last ten or fifteen years, and will use this review of the past to assist in making assumptions for the future, recognizing current trends and being conservatively realistic without unwarranted complications. The assumptions should be understandable and exhibit common sense.

The mathematics of gains and losses will arise when actual experience is different from what was expected. If the expected inflation rate is lower than the actual inflation rate, there will often be experience losses from salary and experience gains from investment return. These gains and losses will partially offset each other; however, they must be quantified, disclosed, explained and amortized in each actuarial valuation. The mathematics of gains and losses and the concepts involved in developing actuarial assumptions are discussed in Sections 5.1, 5.5, 6.4 and 8.3.

1.3.4 Recognizing Inflation

The first two paragraphs of this section comment on the problem of protecting the participant's pension from inflation before retirement, and the remainder of this section deals with postretirement inflation protection.

Many flat dollar plans are issued to members of unions and are subject to renegotiation every three years. This may provide partial, full, or even more than full protection to the worker if the increase in the pension benefit is greater than the increase in the Consumer Price Index. For example, the benefit may change from $40 per month per year of service to $45 per month per year of service, an increase of 12.5%, while inflation may be only, say, 9% for the three-year period.

Final average compensation plans give reasonable protection against inflation occurring before retirement; career average compensation plans do not. This is because the compensation used to determine the benefit in a final average plan is a recent figure and presumably reflects past inflation, whereas that used in a career average plan can be many years old (perhaps fifteen years old for an employee with thirty years of service).

Participants begin receiving retirement benefits upon reaching retirement age, usually on the first day of each month. If these benefits are not indexed to inflation, then participants will receive the same amount of monthly pension throughout their entire retired lifetimes, but the purchasing power of the pension will decrease each year. As an example, inflation of 6% per year would leave pensioners with half of their original purchasing power after only 12 years.

Some plans provide *ad hoc* adjustments in which the plan sponsor decides each year, without prior commitment, if the benefit for the upcoming year will be increased. Other plans may provide partial indexing or even full indexing.

For a defined contribution plan participant to receive postretirement benefit increases, the participant may purchase an increasing annuity; the increases may or may not match inflation, and the initial income will be lower.

1.3.5 Benefit Payments

Pension payments are often made from the pension fund or from a retired life section of the pension fund. This is the usual case when pension pay ments are indexed for inflation.

On the other hand, pension payments are often purchased from a life insurance company at the date of retirement, and the purchase price may be less than or more than the liability for the participant which was held just before the date of retirement. The benefits are usually guaranteed for life by the insurance company, and the retired life usually does not receive indexation.

The payment of pension benefits in a lump sum at retirement is a feature of many pension plans. This feature gives flexibility and control to the participant; on the other hand, monthly pension benefits give the retired participant security and peace of mind, and the arbitrariness of actuarial equivalence is avoided. The actuarial mathematics of retirement is developed in Sections 5.4, 6.2, 7.1, and 7.2.

In addition to the payment of pension benefits, many pension plans also provide for the payment of death benefits and withdrawal benefits.

1.3.6 Surplus

Before 1975, many pension funds were in a deficit position because of initial unfunded liabilities and low investment returns. Since then, many of the

initial unfunded liabilities have been paid off and many pension plans have experienced favorable investment returns. As a result, surpluses have emerged in many defined benefit plans and have grown to such significant levels that unions, employers, employees, and government have been actively debating the question of who owns them.

Recent trends in the industry have been toward stating ownership rights in the plan document, using part of this surplus for reductions in employer contributions, and introducing partial indexation. Indexation can be an enormous expenditure, and in some cases these surpluses have been replaced by unfunded liabilities. When plan improvements are introduced to a pension plan (such as partial indexation) and unfunded liabilities result, legislation places maximum time restrictions on the amortization of the new liability. In the United States, the maximum time frame is sometimes 30 years and often less; in Canada, the maximum is often 15 years and often 5 years, depending on the type of liability being amortized. The concepts, but not the regulations, are discussed in Section 3.6.

DISCUSSION QUESTIONS

1-1 Using any reasonable assumptions, what is the ratio of a career average pension to a final average pension for a typical participant?

1-2 Describe in detail whether or not defined contribution plans provide full inflation protection before and after retirement.

1-3 Explain the differences between anticipating inflation and protecting against inflation.

1.4 Plan Documents and Funding

A pension plan can include various documents. The most important one is the **plan document** which describes (a) who gets a benefit, (b) under what circumstances it becomes payable, (c) how much it is to be, and (d) what are the terms of payment. A trust agreement or an insurance or annuity contract may be used as a vehicle to hold and protect the funds contributed to the plan, and the investment income thereon, until they are paid to the beneficiaries. Other agreements may be used to provide services such as investment advice, investment analysis, and administration.

The main content of this text is a description of the common **actuarial cost methods** for the funding of defined benefit pension plans, with accompanying examples (with solutions) and exercises. These cost methods have the common goal of the orderly build-up of the necessary funds over the working lifetimes of the participants (but only while the pension plan is operating) and the full (or almost full) funding by retirement of appropriate pension benefits.

The primary reason for the goal described above is *participant security*. Suppose a pension plan has been in force for seven years, and the funds or invested assets amount to 60 million dollars. Let us measure the participant security. Based on the commitments given in the plan document, it would be possible to measure the liability incurred for each participant; the total liability for all participants might amount to, say, 100 million dollars. In this case the participant security could be described as 60%. After another seven years the funds might have grown to, say, 270 million dollars, and the total liability to 300 million dollars; then the participant security would be 90% of full funding, and the unfunded portion would be 10% of the total liability. After another seven years, the funds might be 840 million dollars and the total liability 800 million dollars, in which case there would be full funding and, in addition, a surplus of 5% of the total liability. The plan document may give more security to pensioners than to active participants; in the example describing 60% security to all participants, the plan could provide 100% security to pensioners and hence less than 60% to active participants.

The funds arise from the contributions to the pension fund and the investment earnings on the pension fund assets (including capital gains), less benefit payments. The contributions are often equal to, but sometimes more or less than, the **normal cost**, which is a measure of suggested funding based on the actuarial cost method which has been adopted. All the cost methods are rational methods of funding, but some are faster (with larger funds) than average and some are slower (with smaller funds). The **actuarial liabilities**, as we shall see, are related to the normal costs, and both are based on the cost method. In general, however, the actuarial liability (for each participant separately and for all participants combined) grows from zero at entry to an amount which makes provision for all pension plan benefits at and after retirement. The reader familiar with life insurance terminology should note that normal costs and actuarial liabilities are the pension terms analogous to the insurance terms premiums and actuarial reserves.

A subtle but important point is that the actuarial liability based on the cost method and the actuarial liability for the benefits promised in the plan document are not necessarily the same. The former relates to the desired pace of funding, whereas the latter relates to the accrued legal entitlement of the participants.

A second reason for funding is to relate pension costs to the *working years* rather than the *retirement years*. Salaries are clearly a cost of production, and many actuaries consider pension costs to be a cost of production as well. Often pension costs are expressed as a percentage of payroll, and the funding proceeds at this pace; for example, the actuary may express pension costs as 13% of payroll, and the employer may remit 13% of payroll to the pension fund each month.

Pay-as-you-go funding, where the pension costs are charged to the retirement years as benefits are paid out, is not usually acceptable because, if adopted, the production costs would be understated in the working years and overstated in the retirement years, and participant security would be entirely insufficient. Terminal funding, where all the funding is done in the year of retirement, is usually not acceptable for the same reasons.

A third reason for funding is to achieve lower costs and lower taxes. In order to encourage the funding of pensions, the government, within reasonable limits, allows tax relief on the investment income earned on the tens of billions of dollars in pension funds. The result of this tax policy is huge pools of savings invested in equities, bonds, mortgages, and real estate, millions of pension accounts, and fewer retired lives dependent on the state. The government allows a choice of funding or cost methods; the faster the funding, the greater the tax relief.

Comments on matters related to funding are found in Chapter 8.

1.5 Actuarial Valuations

Actuarial valuations are performed periodically to inform those interested of the current condition of the pension plan. Valuations of pension plans are required every three years (or more frequently) in the United States and in Canada.

The normal cost (*NC*) for a year is the actuarial value of the part of the total pension benefit assigned to the year following the valuation date, assuming valuation at the beginning of the year.

The actuarial liability (AL) for a participant will vary among different cost methods and will increase with age. For individual cost methods the total actuarial liability is the sum of the individual actuarial liabilities. The actuarial liability may be determined by either of the following approaches:

(1) The **prospective approach**, whereby AL is the present value of future benefits (pvB) minus the present value of future normal costs ($pvNC$).

(2) The **retrospective approach**, in which AL is the accumulation of normal costs, with adjustments due to interest and benefit payments.

Pension plans have trust funds into which contributions are deposited and from which benefits are withdrawn. The amount in the **pension fund**, denoted by F, consists of the contributions made and the investment income earned, less benefit withdrawals and expenses if paid from the fund. At retirement, the accumulated value of the assets is approximately equal to the actuarial liability required for the payment of the pension benefits. Any excess of the actuarial liability over the fund is called the **unfunded actuarial liability** (UAL). The breakdown of the plan can be shown using a valuation balance sheet as follows:

Assets	**Liabilities**
F	AL
UAL	
Total	Total

In the balance sheet, AL is the estimated amount *owed* by the pension fund to the participants, F is the value of the invested assets *owned* by the fund (usually with a dampening of market value fluctuations), and UAL is the balancing item. Note, however, that UAL is often converted by the method of Section 3.6 into an amount owed to the fund by the plan sponsor. Assets are dicussed briefly in Section 7.3. If F exceeds AL, the excess, called the **surplus**, is shown on the right side of the balance sheet and there is no UAL. AL is also called the **accrued liability**, or simply the **liability**. It can always be expressed by the prospective approach. The retrospective approach is normally equivalent. For simplicity, we refer to the date of valuation as time 0 and one year later as time 1. Then at time 0, the balance sheet equation is normally $F_0 + UAL_0 = AL_0$.

In addition to calculating AL and UAL at time 0, the actuary will calculate the **expected unfunded actuarial liability** (${}^{exp}UAL$) at time 1. If the **actual unfunded actuarial liability** (${}^{act}UAL$) at time 1 turns out to be less than ${}^{exp}UAL$, then a gain will result. This is called the **total experience gain**, and it will be broken down in the valuation report into gains (or losses) due to mortality, investment earnings, termination, compensation, and early (or late) retirement.

Chapters 2, 3, and 4 discuss the various cost methods usually under the assumption that actual experience is the same as expected experience, so that no gain or loss results, and Chapter 5 discusses the gains and losses resulting from actual experience being different from expected experience under the various cost methods.

In addition to what has already been discussed, an actuarial valuation should also include a summary of the plan provisions, the actuarial assumptions, the components of the fund, and the actual experience. Actuarial valuations are discussed further in Section 8.2.

DISCUSSION QUESTIONS

1-4 Is investment income earned on normal costs, actuarial liabilities, funds, or assets?

1-5 The retrospective actuarial liability is normally the accumulation of past normal costs. Are there pitfalls in using the retrospective method?

1.6 Notation and Terminology

This text assumes that the reader is familiar with standard actuarial notation. (Those in need of a review of this notation should consult a standard actuarial text, such as Parmenter [9], Jordan [6], or Bowers, *et al*. [5].) Since pension calculations are generally made from a multiple-decrement service table, the formulas in this text will use standard multiple-decrement notation. (See Section 7.6 of Parmenter [9], Chapters 14 and 16 of Jordan [6], or Chapter 10 of Bowers, *et al*. [5].)

On occasion the pension actuary might assume that there are no preretirement decrements other than death. This means that the calculations can be made from a single-decrement table for mortality only. For notational consistency in this text, however, we will use multiple-decrement notation

(such as $\ell_x^{(\tau)}$, $p_x^{(\tau)}$, $q_x^{(d)}$, and so on), rather than the corresponding single-decrement notation (ℓ_x, p_x, q_x, and so on). In effect we are simply assuming that the service table entries of $d_x^{(i)}$ (and hence of $q_x^{(i)}$) are equal to zero for all decrements i other than death.

Going one step further, some pension actuarial calculations may be made assuming no preretirement decrements at all, which means that $q_x^{(d)} = 0$ for all ages prior to retirement. The effect of this simplifying assumption is that the present value calculations are done at interest only. The assumptions in this, and the previous, paragraph appear in many of the examples and exercises in this text.

1.6.1 Dates and Ages

For convenience, most of the dates used in this text are January 1; all dates are stated in the form month/day/year.

There are four key ages that arise in pension calculations.

(1) **Age *e***. This is the age at entry to a pension plan, or the age from which benefits accrue. The actuarial liability at age e is usually 0.

(2) **Age *a***. This is the age of a participant at plan inception, if the plan inception occurs later than the beginning of benefit accrual. This would occur if the plan is installed after an employee begins service, and benefit credit is given for the past service. Note that necessarily $a > e$. The actuarial liability at age a is based on the benefits, if any, which accrue from age e to age a.

(3) **Age *x***. This is the age at which an actuarial valuation is performed.

(4) **Age *r***. This is the age at normal retirement as specified in the plan document; it is commonly age 65.

To illustrate these four ages, consider a pension plan that was introduced on 1/1/80. At 1/1/93 there are two participants in the plan, and an actuarial valuation is being performed. Participant A was born on 1/1/40 and was hired on 1/1/70. Participant B was born on 1/1/50 and was hired on 1/1/85. The following is a time diagram for this plan.

Age of A	$e = 30$	$a = 40$		53	54	$r = 65$	
Age of B			$e = 35$	43	44		$r = 65$

Date (1/1)	1970	1980	1985	1993	1994	2005	2015
Time				0	1		
Age				x	$x+1$		

1.6.2 Definitions

The following terms are used frequently in this text:

- **Participant**. A person belonging to a pension plan (also called member or employee in some cases).
- **Deferred Vested**. A former participant who has terminated employment but has acquired a right to a future benefit. (See Example 2.6.)
- **Plan Sponsor**. The party establishing the pension plan (also called employer in some cases).

1.6.3 Pension Notation Specific to This Text

The symbol B_r was introduced in Section 1.2.1. It is the expected annual pension benefit, payable monthly, commencing at retirement. B_r *accrues* over the years from age e to age r. The *portion* of B_r that accrues or is attributed to the period from age e to age x is denoted B_x. The *present value* of B_r and of B_x will be needed, sometimes at age e and sometimes at age x. Thus we need to specify both the age at which the present value is taken and the age to which the benefit has accrued. We will use the symbols ${}_{e}v^{e}B_r$, ${}_{e}v^{e}B_x$, ${}_{x}v^{e}B_r$, and ${}_{x}v^{x}B_x$ for these four different present values. Note that the *expected* final salary and the *expected* B_r may change on each actuarial valuation date!

The concept of benefit payments was introduced in Section 1.3.5. We will often have occasion to quantify, for the year under consideration, both the benefits actually paid out and those expected to be paid out. The notation we will use includes *BEN* (for benefits in general), made up of *DBEN* (death benefits), *WBEN* (withdrawal benefits), *RBEN* (refund benefits on contributory plans), and *PBEN* (pension benefits, either monthly, annual or lump sum).

If the benefit is being accounted for at the end of the year (EOY), including interest to EOY, we will use the notation ${}^{i}BEN$, ${}^{i}DBEN$, and so on, where i is the interest *assumption* being used by the actuary in the pension valuation, not the actual earned rate. For example, if BEN is a value at EOY, then ${}^{i}BEN = BEN$, but if BEN is payable at the beginning of the year (BOY), then ${}^{i}BEN = BEN(1+i)$. Benefit payments are included in a few examples in Chapters 3 and 4, and in many examples in Chapters 5 and 6.

Additional notation is defined for presenting the subject matter of this text, such as the symbols NC, AL, ${}^{\exp}UAL$, and so on, already defined in Section 1.5. These symbols will be subscripted by age (*e.g.*, NC_x or AL_x) when there is only one participant or one entry age, or by time (*e.g.*, NC_0 or AL_1) when there are several different ages involved. Each example of such special notation will be carefully defined when first introduced. This notation, and the page of first mention, is listed in the Index of Notation beginning on page 397.

1.7 Cost Methods

The next three chapters describe the common cost methods; they are generally acceptable to the supervisory authorities for funding purposes.

Chapters 2 and 3 describe **individual methods**, wherein the total actuarial liability is equal to the sum of the individual actuarial liabilities. Chapter 2 describes two cost methods which are based on the pension benefits accrued to the date of valuation, as opposed to the total pension benefits that accrue to the date of retirement. The costs are low at the low ages with these methods, due to discounting from retirement back to these ages, but the costs usually *increase* with age. Chapter 3 describes four individual methods based on the pension benefits accrued to the date of retirement; the normal costs are *level* with age. The notation TNC and TAL will be used to distinguish the total normal cost and total actuarial liability, respectively, for the entire group from the corresponding values for a single participant.

Chapter 4 describes four **aggregate methods**, wherein the TNC value under each method is increased to the extent that there are liabilities for retired lives and decreased to the extent that there are funds on hand. Here too the TNC values are level with age.

Cost methods are discussed in Chapter 8 on a more conceptual basis.