

# EXAM GIADV

## Practice Problems for Advanced Topics in General Insurance

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#### Introduction to Practice Problems in Advanced Topics in General Insurance Gennady Stolyarov II, FSA, ACAS, MAAA, CPCU, ARe, ARC, API, AIS, AIE, AIAF

The purpose of this book is to help you pass Exam GIADV: Advanced Topics in General Insurance, the final exam within the Society of Actuaries' new General Insurance Track. Thus far, as of February 2022, fourteen sittings of this relatively new exam have been administered biannually since 2014. Cumulatively throughout these exam sittings, 75 candidates have passed the exam to date. I was among them, passing the Spring 2015 exam on my first attempt. I am hopeful that the existence of this book will greatly increase the number of candidates who register for the exam and pass it – including you.

While preparing for the exam, I noticed a distinct absence of available systematic study resources apart from the syllabus readings themselves. I therefore embarked on a project to craft my own study materials, in addition to assembling any useful practice problems I could find from previous exams. I found that a thorough, piece-by-piece consideration of key concepts within the syllabus readings could give rise to a breadth of exercises – both basic and challenging. Furthermore, in addition to past questions from sittings of Exam GIADV, I discovered questions relevant to each of the syllabus readings, scattered throughout past sittings of Casualty Actuarial Society (CAS) Exams 7, 8, and 9. Studying the relevant past CAS exam questions adds to the range of possible problems with which candidates can become familiar in order to facilitate greater mastery of the Exam GIADV syllabus.

This book of practice problems is the most comprehensive culmination of my efforts to date, and I am pleased to have the opportunity to work with ACTEX Publications to bring all of these resources to you in one convenient compilation so that you will spend less time gathering problems from many separate sources. The edition of this book includes relevant problems and solutions from each of the past Exam GIADV sittings, relevant recent CAS exam sittings, and original problems that I developed. This book is structured to align precisely with the five syllabus topics and eight syllabus papers – each of which has a section of problems devoted to it. The following is a summary breakdown of what you will find:

Section (and Syllabus Paper)	Original Problems	SOA Problems	CAS Problems	Total Problems
1 (Mack)	21	17	5	43
2 (Venter)	22	16	5	43
3 (Clark LDF)	60	16	6	82
4 (Marshall et al.)	103	16	4	123
5 (Lee)	44	12	12	68
6 (Clark Reinsurance)	139	32	9	180
7 (D'Arcy / Dyer)	99	16	6	121
8 (Mango)	43	16	2	61
TOTAL	531	141	49	721

Each section presents all of the problems in succession, followed by the solutions at the end. You are encouraged to attempt each problem on your own and write down or type your solution, and then look at the answer key for step-by-step explanation and/or calculations. As this book is a learning tool, I have provided relevant citations from the syllabus readings for many of the practice problems. Also, I am not an advocate of leaving any problems as unexplained "exercises to the reader." While *each* of these

problems is intended to be an exercise for you, this book's purpose is to show you how they can be solved as well – so give each of them your best attempt, but know that detailed answers are available for you to check your work and fill in any gaps that may have prevented you from solving a problem yourself.

It is important to emphasize that the exam is always based on the syllabus readings and not primarily on any external study materials. As such, you are strongly encouraged to read and re-read the syllabus papers and internalize their contents. This book should be viewed as a companion and supplement to, *not* a substitute for, the syllabus readings. Here is a suggested approach for how to use this book in conjunction with the syllabus papers.

**Step 1.** Read a particular syllabus paper from start to finish, as you would an article or book. This helps you gain a familiarity with the contents and the structure of the paper, as well as where to find particular concepts and methods.

**Step 2.** Perform a second, closer reading of the syllabus paper, this time in conjunction with this book. The original exercises in this book were structured to align with the sequence of each syllabus paper's content. Look at the citations within each exercise to see where you will find the corresponding discussion within the syllabus paper. Once you have visited the relevant portions of the syllabus paper, attempt the exercise, and check your answer. This process will facilitate active reading of each paper. At this stage, you should be engaged with the material in detail and check your understanding at every step of the way.

**Step 3.** Create flashcards from the conceptual questions in this book and review them daily so as to internalize key ideas, methods, formulas, and even calculation shortcuts that may help if deployed properly during the exam. Making your own flashcards helps you actively engage with the material further. You have many options regarding how to create them – from the traditional pen-and-notecard approach, to often-free online and mobile applications such as Anki, StudyDroid, or StudyBlue. Even you as move on to subsequent syllabus topics, you should be regularly reviewing flashcards from previous papers and topics to keep these materials fresh in your mind.

**Step 4.** Once you have completed all of the exercises in this book, re-read each of the syllabus papers once more and focus on any areas that may still require additional work for you to understand and recall. Think about how else any particular idea might be tested. I encourage you to extend your practice by developing your own original problems as well. Nothing helps you learn the material as much as trying to teach it in a stepwise manner, even to yourself.

#### **Other Study Recommendations**

The key for success on any actuarial exam is to set ambitious but flexible study goals that require a regular exertion of effort but can also adapt to changing circumstances without sacrificing other priorities in life. My greatest successes on exams came during sittings for which I studied using a self-developed point system, assigning a certain number of points for every page I read, every practice problem I solved or created, and every flashcard I reviewed. The point assignment could vary based on the type of activity and its difficulty level. For each day, I would set a point goal and try to exceed it, ideally raising my all-day average of points every day. Of course, my point system is not scientific and does not precisely match the difficulty level of each studying activity, but the existence of a point goal is a subjective motivator for continual effort while also giving one an eventual sense of satisfaction with what one has done on any given day. If one does need to attend to other priorities during the day, one can tailor one's activities to match (for instance, reviewing electronic flashcards during a trip, or reading a syllabus paper on a tablet during an elliptical-trainer run) while still meeting the point goal. It is also important to deploy one's available energy and resources wisely, always being heedful of the

Aristotelian "golden mean" – a useful principle to follow with regard to any physical or mental exertion. Avoiding excessive stress and burnout is vital for any candidate who seeks to make steady exam progress. Try to keep your mind fresh and find ways to build buffers of time into your schedule to enable you to swiftly react to the inevitable changes of circumstance. Remember that this endeavor is an ultramarathon, not a sprint.

Use a variety of study techniques to keep the information fresh in your mind. Simple memorization creates anchors in your mind that can render the application of a skill more instantaneous. You should also be solving practice problems on a daily basis, if possible. The more different problem types and approaches for solving them that you are able to internalize, the more capable you will be when facing an unfamiliar problem. With enough practice you might, indeed, be able to recognize some seemingly completely new problems as variations on familiar themes.

Exams are time-limited, and it is important to pace yourself appropriately. During the 15-minute reading period, make a mental note of the problems that you know how to approach right away, and do those first. At the end, you should strive to give yourself a sufficient time buffer to think through the problems you find more challenging and unusual. Try, as much as possible, to always keep moving forward somewhere. If you hit a block on one problem, shift to another and work through it; perhaps an insight on the first problem will arrive later.

If you are preparing to take Exam GIADV, you have already come far. Hopefully, this book will assist you in mastering the exam syllabus and achieving another milestone on your journey to Fellowship along the SOA's General Insurance Track.

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#### Section 2: Testing the Assumptions of Age-to-Age Factors

#### **Topic 1: Basic Stochastic Reserving**

**Syllabus Learning Objective Addressed:** The candidate will understand how to use basic loss-development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes Addressed: The candidate will be able to

- (a) Identify the assumptions underlying the chain-ladder estimation method.
- (b) Test for the validity of these assumptions.
- (c) Identify alternative models that should be considered depending on the results of the tests.
- (d) Estimate the standard deviation of a chain-ladder estimator of unpaid claims.

#### References

• Venter, Gary G., "Testing the Assumptions of Age-to-Age Factors," PCAS LXXXV, 1998, pp. 807-847. Available at http://www.casact.org/library/studynotes/Venter\_Testing\_Assumptions.pdf. Accessed 18 November 2015. Society of Actuaries. Past Exams and Solutions. <u>https://www.soa.org/education/exam-reg/syllabus-study-materials/edu-multiple-choice-exam/</u> • Society of Actuaries. Exam GIADV - Spring 2014. Available at https://soa.org/Files/Edu/edu-2014-05-giadv-exam-0098uT.pdf. Society of Actuaries. GIADV Spring 2014 Solutions. Available at <a href="http://www.soa.org/Files/Edu/edu-2014-05-giadv-exam-sol-m98uTq.pdf">http://www.soa.org/Files/Edu/edu-2014-05-giadv-exam-sol-m98uTq.pdf</a>. • Society of Actuaries. Exam GIADV - Fall 2014. Available at https://soa.org/Files/Edu/edu-2014-10-giadv-exam-08845a.pdf. Society of Actuaries. GIADV Fall 2014 Solutions. Available at <a href="https://soa.org/files/edu/edu-2014-10-giadv-exam-sol-hH445a.pdf">https://soa.org/files/edu/edu-2014-10-giadv-exam-sol-hH445a.pdf</a> • Society of Actuaries. Exam GIADV - Spring 2015. Available at https://soa.org/Files/Edu/2015/spring-exams/giadv-exam.pdf. Society of Actuaries. GIADV Spring 2015 Solutions. Available at https://soa.org/Files/Edu/2015/spring-exams/giadv-exam-solution.pdf. • Society of Actuaries. Exam GIADV - Fall 2015. Available at https://soa.org/Files/Edu/2015/fall-exams/edu-2015-10-giadv-exam.pdf. • Society of Actuaries. GIADV Fall 2015 Solutions. Available at https://www.soa.org/Files/Edu/2015/fall-exams/edu-2015-10-giadv-exam-solutions.pdf. • Society of Actuaries. Exam GIADV – Spring 2016. Available at https://www.soa.org/Files/Edu/2016/spring/edu-2016-05-gi-adv-exam.pdf. · Society of Actuaries. GIADV Spring 2016 Solutions. Available at https://www.soa.org/Files/Edu/2016/spring/edu-2016-08-gi-adv-exam-solutions.pdf. • Society of Actuaries. Exam GIADV - Fall 2016. Available at https://www.soa.org/Files/Edu/2016/fall/edu-2016-11-giadv-exam.pdf. • Society of Actuaries. GIADV Fall 2016 Solutions. Available at https://www.soa.org/Files/Edu/2016/fall/edu-2016-11-gi-adv-exam-solutions.pdf • Society of Actuaries. Exam GIADV - Spring 2017. Available at https://www.soa.org/Files/Edu/2017/spring/edu-2017-04-gi-adv-exam.pdf. • Society of Actuaries. GIADV Spring 2017 Solutions. Available at https://www.soa.org/Files/Edu/2017/spring/edu-2017-spring-gi-adv-exam-solutions.pdf • Society of Actuaries. Exam GIADV - Fall 2017. Available at https://www.soa.org/Files/Edu/2017/fall/edu-giadv-exam.pdf. · Society of Actuaries. GIADV Fall 2017 Solutions. Available at https://www.soa.org/Files/Edu/2017/fall/edu-2017-fall-gi-adv-exam-solutions.pdf. • Society of Actuaries. Exam GIADV - Spring 2018. Available at https://www.soa.org/Files/Edu/2018/spring/edu-2018-04-gi-adv-exam.pdf. • Society of Actuaries. GIADV Spring 2018 Solutions. Available at https://www.soa.org/Files/Edu/2018/spring/edu-2018-spring-gi-adv-exam-solutions.pdf. • Society of Actuaries. Exam GIADV - Fall 2018. Available at https://www.soa.org/Files/Edu/2018/fall/edu-giadv-exam.pdf. · Society of Actuaries. GIADV Fall 2018 Solutions. Available at https://www.soa.org/globalassets/assets/files/edu/2018/fall/edu-2018-fall-gi-adv-exam-solutions.pdf • Society of Actuaries. Exam GIADV - Spring 2019. Available at https://www.soa.org/globalassets/assets/files/edu/2019/spring/exams/edu-2019-05-giadv-exam-am.pdf • Society of Actuaries. GIADV Spring 2019 Solutions. Available at https://www.soa.org/globalassets/assets/files/edu/2019/spring/solutions/edu-2019-spring-gi-adv-exam-solutions.pdf Society of Actuaries. Exam GIADV - Fall 2019. Available at <a href="https://www.soa.org/globalassets/assets/files/edu/2019/exams/edu-2019-fall-giadv-exam.pdf">https://www.soa.org/globalassets/assets/files/edu/2019/exams/edu-2019-fall-giadv-exam.pdf</a> • Society of Actuaries. GIADV Fall 2019 Solutions. Available at https://www.soa.org/globalassets/assets/files/edu/2019/fall/solutions/edu-2019-fall-giadv-exam-solutions.pdf Casualty Actuarial Society. Spring 2011 Exam 7 and Sample Solutions. http://www.casact.org/admissions/studytools/exam7/11-7.pdf

• Casualty Actuarial Society. Spring 2014 Exam 7 and Examiners' Report with Sample Solutions. Available at

http://www.casact.org/admissions/studytools/exam7/14-7.pdf.

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### Venter's Notation (Venter, p. 808)

c(w,d): cumulative loss from accident year w as of age d  $c(w,\infty)$ : total loss from accident year w when end of triangle reached q(w,d): incremental loss for accident year w from (d - 1) to d f(d): factor applied to c(w,d) to estimate q(w, d+1)F(d): factor applied to c(w,d) to estimate  $c(w,\infty)$ 

#### **Practice Problems**

**Problem 2-1.** State Mack's three assumptions pertaining to the chain-ladder method in terms of Venter's notation (Venter, p. 809).

**Problem 2-2.** Under Mack's three assumptions, what sort of estimator of future emergence does the chain-ladder method produce? (Venter, p. 810)

**Problem 2-3.** What did Mack assume about the function a(\_, \_) as used by Venter in expressing Mack's third assumption pertaining to the chain-ladder method? (Venter, p. 810)

**Problem 2-4.** What is the expression generally minimized in order to find the minimum-variance unbiased estimator of f(d)? (Venter, p. 810)

**Problem 2-5.** Venter (pp. 812-813) lists six testable implications of the hypothesis that the chain-ladder method is appropriate in a given situation. What are these implications?

**Problem 2-6.** Venter (p. 812) lists three possible alternative emergence patterns to Mack's first assumption that E[q(w, d+1) | data to (w+d)] = f(d)\*c(w,d). Describe these three alternative assumptions mathematically.

**Problem 2-7.** (a) Fill in the blanks (Venter, p. 813): A usual test is that the absolute value of a factor is required to be at least \_\_\_\_\_ (how many?) times its standard deviation in order for the factor to be regarded as significantly different from zero. If a development triangle fails this test, then the chain-ladder method \_\_\_\_\_ (is or is not?) optimal for those triangles.

(b) Fill in the blanks (Venter, p. 813): For a Normal distribution, the requirement in (a) provides that there is only a \_\_\_\_% probability of getting a factor of this absolute value or greater if the true probability is zero. Many analysts have sufficient comfort with a factor with absolute value \_\_\_\_ (how many?) times its standard deviation, which could happen about \_\_\_\_% of the time by chance alone.

(c) Fill in the blank (Venter, p. 814): If a factor is not significant under a Normal distribution, then it is probably \_\_\_\_\_ (more or less?) significant under other distributions.

**Problem 2-8.** (a) Mathematically describe three adjustments suggested by Venter (pp. 814-815) to the sum of squared errors (SSE), using the notation SSE = sum of squared errors, n = number of observations, and p = number of parameters.

(b) Compute the relevant measures using each of those adjustments for a model where the SSE is 5520, the number of observations is 43, and the number of parameters is 5.

Problem 2-9. (a) Fill in the blanks (Venter, pp. 815-816): If one tests for the linear-with-constant alternative emergence pattern and finds that the constant term is more statistically significant than the development factor (or the development factor is not significant at all), it is important to \_\_\_\_\_\_ the development triangle as much as possible. One way to do this is by \_\_\_\_\_\_. For these adjustments, use a \_\_\_\_\_\_ method instead of a purely multiplicative method.

(b) Fill in the blanks (Venter, pp. 815-816): For the linear-with-constant alternative emergence pattern, the constant is often significant in the age range of \_\_\_\_\_, especially for \_\_\_\_\_ lines of business such as \_\_\_\_\_.

**Problem 2-10. (a)** What method is an example of the assumption of the alternative emergence pattern of a factor times a parameter? (Venter, p. 816)

(b) For a complete development triangle with n accident years, the method in (a) has how many parameters? How does this number compare to the number of parameters involved in the chain-ladder method? (Venter, p. 817)

(c) Fill in the blank: The method in (a) has to produce much \_\_\_\_\_ (higher or lower?) fit errors than the chain-ladder method in order to give a better fit statistic.

(d) What method is, according to Venter, an "important special case" of the parameterized method in part (a)? How is it described using Venter's notation with regard to the parameter h(w)?

**Problem 2-11. (a)** According to Venter (pp. 818-819), what action could improve the predictions of the Bornhuetter-Ferguson model, especially for the most recent years?

(**b**) How might the Cape Cod model be considered a reduced-parameter version of the Bornhuetter-Ferguson model? (Venter, p. 819)

(c) What two other approaches does Venter (p. 819) describe for reducing the number of parameters in a Bornhuetter-Ferguson model?

**Problem 2-12.** (a) For attempting to fit data to a BF model, what is the expression (using Venter's notation) for the individual residual, the sum of whose squares it is the goal to minimize? (Venter, p. 822)

(**b**) Venter discusses an iterative regression procedure. What is the starting point of the procedure? (Venter, p. 822)

(c) What is the standard linear regression formula that gives the best values of h for the initial set of values of f? (Venter, p. 822)

(d) After the set of best values of h has been found through the formula in (c), what is the regression formula that gives the best values of f for those values of h? (Venter, p. 822)

(e) For the Cape Cod method, what is the regression formula to determine h? (p. 825)

(f) What other method gives the same fit accuracy as the Cape Cod method? Why? (Venter, p. 826) **Problem 2-13.** The chain-ladder method assumes that the incremental losses at each age are a linear function of previous cumulative losses. What pattern in a plot of residuals against previous cumulative losses be used to identify a departure from this assumption? (Venter, pp. 829-830)

**Problem 2-14. (a)** If a plot of residuals over time does *not* show the residuals to exhibit a stable pattern, what two corrections are possible in order for a chain-ladder method to still be usable? (Venter, pp. 830-831)

(**b**) If the plot of residuals shows a lot of variability around a fixed level of data, how should the data be adjusted (e.g., what data elements should be used)? (Venter, p. 832)

**Problem 2-15.** (a) Let r be the same correlation coefficient. Let n be the number of observations. What is formula for the test statistic T, as given by Venter on page 833?

(**b**) Fill in the blanks: According to Venter (p. 833), if we assume the T statistic to follow a \_\_\_\_\_\_ distribution with \_\_\_\_\_ degrees of freedom, then if T > the \_\_\_\_\_ statistic for \_\_\_\_, then \_\_\_\_\_ is statistically significant at the 10% level, which would \_\_\_\_\_\_ (support or undermine?) the chain-ladder hypothesis.

## Problem 2-16. (a) Briefly describe Mack's high-low diagonal test.

(**b**) Briefly describe the conceptual essence of the high-low diagonal test discussed by Venter. (Venter, p. 834)

### Problem 2-17. SOA Fall 2015 Exam GIADV, Question 4(f).

In the paper "Testing the Assumptions of Age-to-Age Factors," Venter closes with the following statement: "An actuary might advise: 'If the chain ladder doesn't work, try Bornhuetter-Ferguson.'" Explain what Venter means when using the terms "doesn't work" and "try."

**Problem 2-18. Based on SOA Spring 2014 Exam GIADV, Question 3(e).** For a loss-development triangle with 6 accident years, you are given four models to evaluate the chain-ladder assumption and alternatives.

You are using ideas from "Testing the Assumptions of Age-to-Age Factors" by Venter, with the following notation:

c(w,k) = cumulative claims for accident year w at development year k.

q(w,k) = incremental claims for accident year w at development year k.

Model	<b>Model Description</b>	Sum of Squared Errors (SSE)
1	q(w,k+1) = c(w,k)*f(k)	2,889,833
2	q(w,k+1) = c(w,k)*f(k) + g(k)	2,754,418
3	q(w,k+1) = g(k)	8,463,380
4	q(w,k+1) = f(k+1)*h(w)	943,083

The results for each model are shown in the table below.

Rank the four models from best-fitting to worst-fitting using one of the three methods Venter suggests for accounting for the number of estimated parameters when comparing sums of squared errors.

Indicate if your results support Mack's expected-value assumption for the chain-ladder method.

**Problem 2-19. SOA Spring 2014 Exam GIADV, Questions 3(e) and (f).** You now wish to check the chain-ladder assumption for a particular loss-development triangle with 10 accident years, using ideas from "Testing the Assumptions of Age-to-Age Factors" by Venter. Your assistant has analyzed four models that describe the development pattern. For each model he determined the parameter estimates along with the estimated ultimate values (both not shown) and the sum of squared errors. The notation used in the models is:

c(w,k) = cumulative claims for accident year w at development year k. q(w,k) = incremental claims for accident year w at development year k.

The results are in the following table.

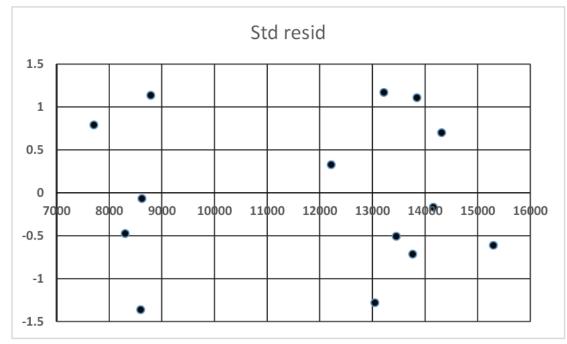
Model	Model Description	Sum of Squared Errors (SSE)
1	q(w,k+1) = c(w,k)*f(k)	1,869,591
2	q(w,k+1) = c(w,k)*f(k) + g(k)	1,120,615
3	q(w,k+1) = g(k)	1,696,523
4	q(w,k+1) = f(k+1)*h(w)	1,029,484

(e) Rank the four models from best fitting to worst fitting using one of the three methods Venter suggests for accounting for the number of estimated parameters when comparing sums of squared errors. Indicate if your results support Mack's assumption.

(f) Describe two other tests Venter recommends for determining the viability of using the chain ladder method.

#### Problem 2-20. SOA Fall 2014 Exam GIADV, Questions 4(f) and (g).

(f) Venter suggests that a test of the assumptions underlying Mack's method is to plot the residuals of the estimated versus actual increments. The following plot has the residuals standardized by dividing by the standard errors.



Indicate whether this plot supports the Mack assumptions. Justify your answer.

(g) Another suggestion of Venter is to calculate the correlation between the ratios of the incremental claims to the cumulative claims for successive development years. For this data, the estimated correlation coefficient of the ratios from development year 1 to 2 with the ratios from development year 2 to 3 is 0.391 based on four observations. Indicate whether this calculation supports the Mack assumptions. Justify your answer.

## Problem 2-21. SOA Spring 2015 Exam GIADV, Questions 4(f) and 4(g).

**Relevant conditions from earlier in the problem:** Let  $C_{i,k}$  be the cumulative paid claims for accident year *i* and development year *k*. The chain-ladder method estimates  $C_{i,k+1}$  as  $f_k * C_{i,k}$ . Mack notes that this can be viewed as a regression model where the intercept term is forced to be zero. Mack further notes that weighted least squares could be used to derive an estimate of  $f_k$ . The weight  $1/C_{i,k}$  leads to the standard chain-ladder estimate. The following table displays estimates of  $f_1$  using three different weights.

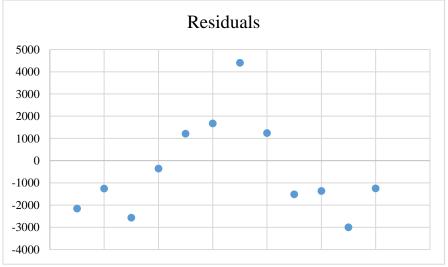
Weight	1	1/C <sub>i,1</sub>	$1/(C_{i,1})^2$
Estimate of f <sub>1</sub>	1.627	1.361	1.151

(f) Venter suggests that various models can be compared by adjusting the sum of squared errors by the number of estimated parameters and/or the sample size. Describe one such adjustment.

(g) Explain why this method is inappropriate for comparing the three weighted estimates.

**Problem 2-22.** Which method generally has a larger estimation error: the chain-ladder method or a parametrized Bornhuetter-Ferguson method? Justify your answer. (Venter, p. 828)

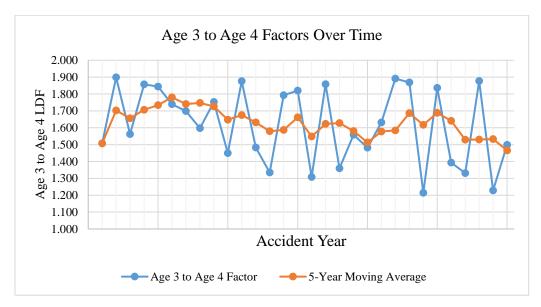
**Problem 2-23.** A chain-ladder model was fitted to a triangle of cumulative losses, producing the graph of residuals below.



Does this graph support the linearity assumption the chain-ladder method? Why or why not? (Venter, pp. 829-830)

**Problem 2-24.** For a given book of business, loss-development factors (LDFs) from age 3 to age 4 are provided for 30 accident years. The following graph shows the individual LDFs as well as a moving average of the latest five years' LDFs as of each accident year.

Consider the graph and arrive at a recommendation as to whether or not all of the data from the 30-year period should be used to calculate development factors for use in analyzing the most recent loss experience for this book of business. If you conclude that all the data should be used, justify your answer. If you conclude that all the data should not be used, suggest an alternative approach.



**Problem 2-25.** (a) Suppose that a loss-development triangle has m pairs of columns that can be compared with one another. Venter (p. 834) discusses that, at a 10% significance level, the number of columns that show a significant correlation can be considered a binomial random variable with parameters m and 0.1. What is the mean of the resulting binomial distribution?

(b) What is the standard deviation of the resulting binomial distribution?

(c) If a result that exceeds the mean by  $3\frac{1}{3}$  standard deviations is considered to show a strong correlation, what is the formula for how many columns would need to be shown to be correlated before the independence assumption of the chain-ladder method would need to be rejected?

(d) Using a 10% significance level and the criterion that a result would need to exceed the mean by  $3\frac{1}{3}$  standard deviations to show a strong correlation, if there are 24 possible pairwise column comparisons in a loss-development triangle, how many would need to be shown to be correlated for the independence assumption of the chain-ladder method to be rejected?

(e) What is a possible situation that Venter discusses as being an exception to this formula approach, suggesting that a correlation problem exists even if the requisite number of columns is not found to be correlated? (Venter, p. 834)

#### Problem 2-26. Based on CAS Spring 2011 Exam 7, Question 4(a).

For a particular loss-development triangle, a linear regression of incremental losses against the previous cumulative losses was performed, yielding the following table:

	<u>0-1 Years</u>	<b>1-2 Years</b>	2-3 Years	3-4 Years	4-5 Years	5-6 Years
Constant a	79.52	691.4	16.09	260.18	102.29	-24
Standard Deviation of a	405.66	3529.08	84.13	1362.41	537.63	124.14
Factor b	1.58	1.39	0.63	0.44	0.3	0.17
Standard Deviation of b	0.474	0.556	0.126	0.132	0.03	0.0085

Use the data above to demonstrate whether the chain-ladder method or the linear-plus-constant method better models the emergence of losses.

#### Problem 2-27. CAS Spring 2011 Exam 7, Question 4. Given the following loss-development triangle:

r										
	Age in Years									
Year	0	1	2	3	4	5	6	7	8	9
0	5,400	10,800	17,280	24,192	29,030	33,385	36,056	37,498	37,873	37,873
1	5,800	12,039	19,898	28,773	37,369	46,089	54,105	58,895	64,579	
2	3,888	7,582	12,199	16,834	20,933	24,367	27,335	29,207		
3	4,900	9,930	16,062	22,669	27,533	32,092	35,391			
4	5,655	10,631	17,123	23,802	29,320	33,980				
5	4,950	10,131	16,568	23,712	29,215					
6	6,650	13,475	22,073	30,902						
7	4,890	10,269	16,512							
8	5,780	11,502								
9	4,964									

#### **Cumulative Reported Losses (\$000)**

A linear regression of incremental losses against the previous cumulative losses was performed, yielding the following table:

	<u>0-1</u>	<u>1-2</u>	<u>2-3</u>	<u>3-4</u>	<u>4-5</u>	<u>5-6</u>
	<b>Years</b>	<b>Years</b>	<u>Years</u>	<b>Years</b>	<b>Years</b>	<b>Years</b>
Constant a	-66	-733	-740	-2,638	-4,183	-4,247
Standard Deviation of a	920	442	960	2,494	2,305	3,195
Factor b	1.024	0.692	0.453	0.352	0.323	0.250
Standard Deviation of b	0.171	0.041	0.055	0.106	0.079	0.092

(a) Use the data above to demonstrate whether the chain-ladder method or the linear-plus-constant method better models the emergence of losses.

(**b**) Support graphically the answer from part (a) above.

Class	Frequency	Severity (\$)
1	0.20	15,000
2	0.05	25,000
3	0.40	5,000
4	0.25	3,000
5	0.10	12,000

Calculate the Pearson product-moment correlation coefficient between the frequency and severity.

**Note:** This problem is included here as practice for candidates to calculate the Pearson coefficient, which is used in Venter's test for correlations among age-to-age factors. The formula for the Pearson coefficient r between two random variables X and Y is  $r = [E(XY) - E(X)*E(Y)]/(\sigma_X*\sigma_Y).$ 

Problem 2-29. CAS Spring 2013 Exam 7, Question 17(a). Given the following data:

Line of Business	<u>2008</u>	<u>2009</u>	<u>2010</u>	<u>2011</u>	<u>2012</u>
Personal Auto	900	1,150	6,000	850	1,100
Homeowners'	340	200	1,060	150	250

#### Policy-Year Ultimate Loss (\$000)

Calculate the Pearson product-moment correlation coefficient between the personal auto loss data and the homeowners' loss data.

**Problem 2-30. CAS Spring 2012 Exam 7, Question 15(a).** Given the following auto liability claims information:

Claim #	Case Incurred Indemnity (\$)	Case Incurred ALAE (\$)
1	800	100
2	5,000	250
3	2,100	0
4	12,000	1,800
5	77,000	4,000

Calculate the Pearson product-moment correlation coefficient.

Problem 2-31. CAS Spring 2014 Exam 7, Question 4. Given the following loss	ss information:
--	-----------------

Year	12 M	onths	24 Months	5 36 M	onths	48	Months	60 Months	72 Months	s
2008		200	1,000	)	1,500		1,800	2,100	2,10	0
2009		400	800	)	1,000		1,500	1,800		
2010		800	1,500	)	2,000		2,200			
2011		1,200	2,000	)	2,200					
2012		400	1,800	)						
2013		200								
t-statistic for 0.90 at Various Degrees of Freedom (DF)										
DF		1	2	3	4		5	6	7	
t-statis	t-statistic		1.89	1.64	1.53	3	1.48	1.44	1.42	]

Assume  $T = r[(n-2)/(1-r^2)]^{1/2}$  is t-distributed with (n-2) degrees of freedom, where r is the sample correlation coefficient.

(a) Evaluate whether the triangle above meets the assumption that the age-to-age factors are independent from 12 months to 36 months, using a 10% t-statistic significance standard.

(b) State the other basic chain-ladder assumptions needed for least-squares optimality.

**Problem 2-32. SOA Fall 2016 Exam GIADV** – **Question 4(f).** Venter proposes several approaches for comparing or evaluating different models. One proposal is to calculate the sum of squared errors for each model and then adjust them to reflect the number of observations and the number of estimated parameters.

Describe two other approaches that Venter proposes for comparing or evaluating different models.

**Problem 2-33. SOA Spring 2016 Exam GIADV, Questions 4(d) through (f).** You are interested in determining the variability of unpaid claim estimates. The triangle of paid claims data you are working with, by accident year (AY) and development year, is presented below. The shaded cells have been completed using the standard chain-ladder method. It is assumed that all claims are fully developed after seven years.

Mack's method of estimating reserve variability has been applied to this triangle. The key results are provided in the table.

Development Year									
Accident Year	1	2	3	4	5	6	7	Standard Error	
1	9,791	12,431	13,033	14,212	14,486	14,867	15,155	0	
2	11,314	19,266	23,518	27,910	28,117	28,697	29,253	15	
3	12,654	14,924	18,489	22,433	24,281	24,829	25,310	111	
4	13,305	14,234	15,293	15,900	16,474	16,845	17,172	903	
5	14,693	26,298	37,108	42,448	43,980	44,972	45,843	3,208	
6	16,037	18,544	22,861	26,151	27,094	27,705	28,242	4,399	
7	17,360	23,587	29,077	33,262	34,462	35,239	35,922	9,393	
f <sub>k</sub>	1.35868	1.23279	1.14392	1.03608	1.02256	1.01937			
$(\alpha_k)^2$	1,264.53	404.682	111.855	37.514	0.308	0.00252			

Venter proposes comparing various models by calculating the sum of squared errors (SSE). For the standard chain-ladder method applied to these observations, SSE = 126,347,521. Venter then proposes three methods by which the SSE can be adjusted to account for the number of observations and the number of estimated parameters.

(d) State the number of observations and the number of estimated parameters for this situation.

(e) Calculate the adjusted SSE using one of the three methods suggested by Venter.

(f) Venter proposes investigating models other than the standard chain ladder (where each value is multiplied by a factor that depends only on the development year). Describe one such alternative model, using words, not formulas.



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