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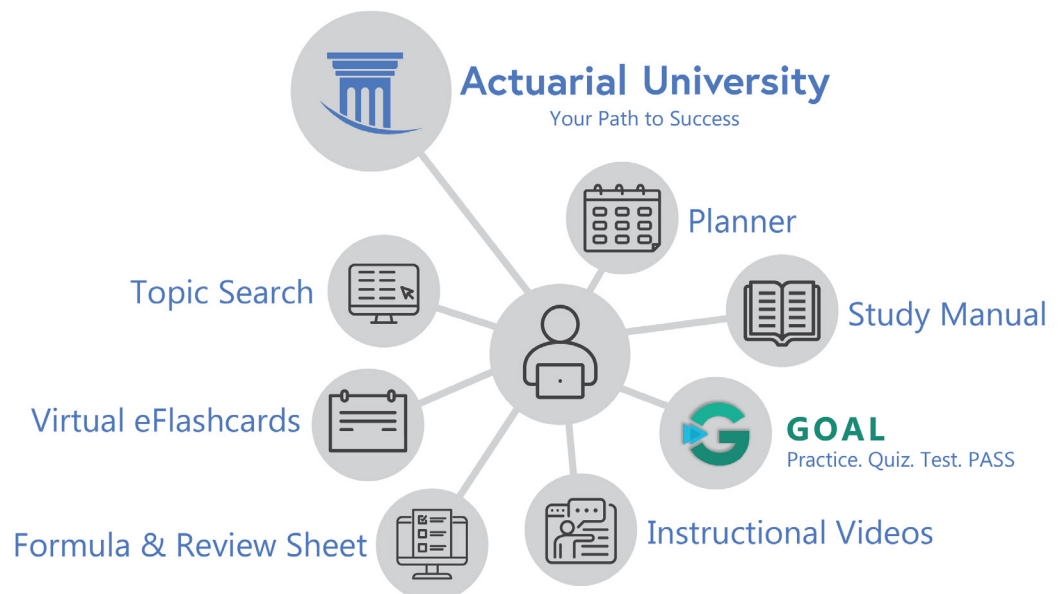
EXAM
IFM

Study Manual for SOA Exam IFM

by
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GOAL | Problem # | Go! | Problem 1 of 5 | Prev Next Leave

Question — Difficulty: Mastery ⓘ

A loan is being repaid with 2 payments: A first payment of $1,500$ at the end 2 years and a second payment of $1,600$ at the end of 7 years.

Determine the loan amount given an annual effective rate of discount of 2%.

A $2,834.65$

B — You used the formula: $1,500(0.98)^{-2} + 1,600(0.98)^{-7}$ that gives AV's instead of PV's using a 2% discount rate.

C $2,829.60$

D $2,759.65$

Help Me Start

Sketch a time diagram showing the two payments.

Solution

1500 — 1600

0   1   2   ***   ***   7

The loan is the PV of the two payments using the annual effective discount rate of 2%:

$$L = 1,500(1-.02)^2 + 1,600(1-.02)^7 = 1,500(0.98)^2 + 1,600(0.98)^7$$
$$= 2,829.60.$$

Common Questions & Errors

Common Error 1: Finding the amount $2000v^{15-4+1}$ which is the principal repaid in the 4th payment. Subtract the answer from the annual payment amount to get the interest paid.

- Flag for review, record notes & email the professor
- Monitor difficulty level
- Got it wrong? Often there is a simple reason why
- Helpful strategies to get you started
- Graphs and other solution techniques demonstrated when applicable
- Commonly encountered errors

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Preface

Thank you for choosing ACTEX Learning.

The Investment and Financial Markets (IFM) Exam is a new exam that is first launched by the Society of Actuaries (SoA) in July 2018. Although the IFM Exam draws heavily from the MFE Exam (which is no longer offered after July 1, 2018), it covers a lot of topics (including corporate finance and the interface between derivatives and insurance) which the MFE Exam does not cover. This study manual is created to help you best prepare for the IFM Exam.

Given that the IFM Exam covers a very wide range of topics, it is crucial to learn them in a logical sequence and to see through the connections among them. We have meticulously categorized the exam topics into two broad themes: Quantitative and Qualitative.

The first part of this manual focuses on the Quantitative theme, which encompasses all of the topics covered in *Derivatives Markets* (the required text authored by R.L. McDonald) and the technical topics from *Corporate Finance* (the required text authored by J. Berk and others). To help you develop a strong foundation, we begin with the easiest calculations that are just straightforward extensions of what you have learnt in Exam FM. These are then followed by progressively harder calculations, ranging from the binomial model to various versions of the Black-Scholes formula.

The following features concerning the Quantitative part of the manual are noteworthy:

1. The connections between the option pricing models (from *Derivatives Markets*) and real options (from *Corporate Finance*) are clearly explained.

2. We do not want to overwhelm readers with verbose explanations. Whenever possible, concepts and techniques are demonstrated with examples and/or integrated into the practice problems.

3. We provide sufficient practice problems (which are similar to the real exam problems in terms of format and level of difficulty), so that you do not have to go through the textbooks' end-of-chapter problems. We find that the end-of-chapter problems in *Derivatives Markets* are either too trivial (simple substitutions) or too computationally intensive (Excel may be required), compared to the real exam questions.

4. We do not follow the order in Derivatives Markets, because the focus of this textbook is somewhat different from what the SoA expects from the candidates. According to the SoA, the purpose of the exam is "to develop candidates' knowledge of the theoretical basis," but the book emphasizes more on applications.

We believe that the materials in the Quantitative theme should be studied first, as a lot of time has to be spent on the practice problems in order to develop a solid mastery of these materials.

The second part of the manual is devoted to the Qualitative theme, which encompasses a lot of definitions and hard facts that you have to memorize (unfortunately). There are some calculations in the Qualitative theme, but they are typically trivial. To help you breeze through this theme, the materials in this theme are presented in an easy-to-read point form, with the

most important points being clearly highlighted. Of course, we have practice problems to test how well you can remember the materials.

We recommend that you go through the Qualitative theme after the Quantitative theme, simply because everyone's short-term memory is limited.

The manual is concluded with several mock exams, which are written in a similar format to the released exam and sample questions provided by the SoA. This will enable you to, for example, retrieve information more quickly in the real exam. Further, we have integrated all of the relevant released exam and sample questions into the examples, practice problems, mock exams in the manual. These exam/sample questions include:

– The released MFE sample and released exam questions that are still relevant to the IFM exam syllabus.

– The released FM sample and released exam questions that are relevant to the IFM exam syllabus.

– The sample questions on Finance and Investment (Corporate Finance, IFM-21-18, IFM-22-18).

This integration seems to be a better way to learn how to solve those questions, and of course, you will need no extra time to review those questions.

We recommend you to use of this study manual is as follows:

1. Read the lessons in order.

2. Immediately after reading a lesson, complete the practice problems for that lesson.

3. After studying all lessons, work on the mock exams.

If you find a possible error in this manual, please let us know at the "Customer Feedback" link on the ACTEX homepage (www.actexmadriver.com). Any confirmed errata will be posted on the ACTEX website under the "Errata & Updates" link.

A Note on Rounding and the Normal Distribution Calculator

To achieve the desired accuracy, we recommend that you store values in intermediate steps in your calculator. If you prefer not to, please keep at least six decimal places.

In this study guide, normal probability values and z-values are based on a normal distribution calculator instead of a normal table in other exams. In the actual examination you will be able to use the same normal distribution calculator.

The calculator is very easy to use. Simply go to

https://www.prometric.com/en-us/clients/SOA/Pages/calculator.aspx

Recall that $N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$ is the cumulative distribution function of a standard normal random variable. To find $N(x)$, you may use the first panel of the calculator. Type in the value of x and press "Normal CDF". Then you would get $N(x)$. For example, when $x = -1.282$, the calculator would report 0.09992.

To find the $100p$th percentile of the standard normal random variable (i.e. to find the value of x such that $N(x) = p$), enter p into the cell adjacent to $N(x)$, and press "Inverse CDF". Then you would get x. For example, when $N(x) = 0.25$, the calculator would report -0.67449.

If you do not want to go online every time when you follow the examples and work on the practice problems, you can set up your own normal distribution calculator using Excel. Open a blank workbook, and set up the following:

Cell A1:   x

Cell A2:   N(x)

Cell B1: -1.282

Cell B2:   = round(norm.s.dist(B1, 1), 5)

Cell A5:   N(x)

Cell A6:   x

Cell B5:   0.25

Cell B6:   = round(norm.s.inv(B5), 5)

Cell B2 would report 0.09992 and Cell B6 would report -0.67449 if you are using Excel 2010 or more recent versions of Excel. You can alter the values in B1 and B5 to calculate any probability and percentile. Save your workbook for later use.

Syllabus Reference

In what follows,

– BM stands for the textbook *Corporate Finance* (4th Ed) coauthored by Berk and De-Marzo,

– McD stands for the textbook *Derivatives Markets* (3rd Ed) authored by McDonald,

– SN1 stands for the SoA published Study Note IFM-21-18 *Measures of Investment Risk, Monte Carlo Simulation and Empirical Evidence on the Efficient Market Hypothesis*, and

– SN2 stands for the SoA published Study Note IFM-22-18 *Actuarial Applications of Options and Other Financial Derivatives.*

Module 1: Topics in Investments

Lesson 1: Risk and Return	
1.1.1	BM 10.3, 10.4
1.1.2	BM 10.2, 11.2 (starting from the middle of p.360)
1.1.3	BM 11.1, 11.2, 11.3 (up to p.365, and start again at p.369)
1.1.4	BM 11.3, 10.5, 10.6
Lesson 2: Portfolio Theory	
1.2.1	BM 11.5 (up to the middle of p.379)
1.2.2	BM 11.4
1.2.3	BM 11.5 (starting from the middle of p.379)
1.2.4	BM 12.2 (up to the middle of p.411)
Lesson 3: The Capital Asset Pricing Model	
1.3.1	BM 11.7, 10.7 (up to p.343), 10.8
1.3.2	BM 11.8, 12.3, 10.7 (starting from p.344)
1.3.3	BM 12.2 (starting from the middle of p.411), 12.3, 11.8, 13.1

Module 2: Project Analysis and Investment Risk

Lesson 1: Cost of Capital	
2.1.1	BM 12.1
2.1.2	BM 12.4
2.1.3	BM 12.5 – 12.7
Lesson 2: Risk Analysis	
2.2.1	BM 8.5
2.2.2	BM 8.5
2.2.3	SN1 (IFM-21-18) Section 3
2.2.4	BM 22.2
Lesson 3: Investment Risk Measures	
2.3.1	SN1 (IFM-21-18) Section 2.1 – 2.2
2.3.2	SN1 (IFM-21-18) Section 2.3 – 2.4
2.3.3	SN1 (IFM-21-18) Section 2.5

Module 3: Introductory Derivatives

Lesson 1: Stock as an Underlying Asset	
3.1.1	McD 1.2 (up to the middle of p.4)
3.1.2	McD 1.5
3.1.3	McD 1.1, 2.2, 2.3
Lesson 2: Forward and Prepaid Forward	
3.2.1	
3.2.2	McD 5.2
3.2.3	McD 5.3 (up to the middle of p.136)
3.2.4	McD 5.1
Lesson 3: Options and Related Strategies	
3.3.1	McD 3.2, 9.1 (through the top of p.269)
3.3.2	McD 3.1, 2.4
3.3.3	McD 3.3, 3.4
3.3.4	McD 3.4
Lesson 4: Futures and Foreign Currencies	
3.4.1	McD 5.4 (through the top of p.143), first eqt on p.287
3.4.2	McD 9.1 (middle of p.269)
3.4.3	McD 1.4

Module 4: Risk-neutral Valuation in Discrete-time

Lesson 1: Introduction to Binomial Trees	
4.1.1	McD 10.1 (up to the middle of p.297)
4.1.2	McD 10.1 (from the middle of p.297 to the middle of p.298)
4.1.3	McD 10.1 (from p.299 to the middle of p.300)
Lesson 2: Multiperiod Binomial Trees	
4.2.1	McD 10.3, 10.4
4.2.2	McD 10.5
4.2.3	McD 10.2 (p.303)
4.2.4	McD 10.2 (p.303 – 304)
Lesson 3: Options on Other Assets	
4.3.1	McD 10.5 (p.312)
4.3.2	McD 10.5 (p.312 and 313), McD 9.1 (formula 9.4 only)
4.3.3	McD 10.5 (from the middle of p.314 to the middle of p.315)

Module 5: Risk-neutral Valuation in Continuous-time

Lesson 1: The Black-Scholes Model	
5.1.1	McD 18.2
5.1.2	McD 18.3, 18.4 (up to eqt (18.30) on p.561)
5.1.3	McD 18.4
Lesson 2: The Black-Scholes Formula	
5.2.1	McD 18.4
5.2.2	McD 12.1 (up to p.352)
5.2.3	McD 12.2
Lesson 3: Greek Letters and Elasticity	
5.3.1	McD 12.3 (p.356 – 360 before "Rho", p.361 "Greek measures for portfolio"), 13.4 (up to p.393)
5.3.2	McD 12.3 (p.359, middle of p.360)
5.3.3	McD 12.3 (p.362, to the end of the section 12.3)
Lesson 4: Risk Management Techniques	
5.4.1	McD 13.2, 13.3 (up to the middle of p.387)
5.4.2	McD 13.4 (p.394 to p.395)
5.4.3	McD 13.3 (middle of p.387 to the end)
5.4.4	McD 13.5 (beginning at the bottom of p.413)

Module 6: Further Topics on Option Pricing

Lesson 1: Exotic Options I	
6.1.1	McD 14.2
6.1.2	McD Exercise 14.20
6.1.3	McD 14.3, top half of p.714
6.1.4	McD 14.4 (except "Options on dividend-paying stock" and Example 14.2)
Lesson 2: Exotic Options II	
6.2.1	McD 14.6
6.2.2	McD Exercise 14.21
6.2.3	McD 14.5
Lesson 3: General Properties of Options	
6.3.1	McD 9.3 (p.281 – 285 "Different strike prices")
6.3.2	McD 9.3 (p.276 – 277 "European versus American options" and "maximum and minimum option prices")
6.3.3	McD 9.3 (p.280 - 281 "Time to expiration")
6.3.4	McD 9.3 (from the middle of p.277 to the middle of p.278), 11.1
6.3.5	McD 9.3 (p.278 "Early exercise for puts")
Lesson 4: Real Options	
6.4.1	BM 22.1, 22.3
6.4.2	BM 22.4

Module 7: Capital Structure

Lesson 1: The Modigliani and Miller Propositions	
7.1.1	BM scattered in 14
7.1.2	BM 14.2
7.1.3	BM 14.3
7.1.4	BM 15.1 – 15.2
Lesson 2: Costs of Financial Distress	
7.2.1	BM 16.1
7.2.2	BM 16.2
7.2.3	BM 16.3
7.2.4	BM 16.4
Lesson 3: Agency Costs and Benefits	
7.3.1	BM 16.5
7.3.2	BM 16.6
7.3.3	BM 16.7
7.3.4	BM 16.8 – 16.9

Module 8: Market Efficiency

Lesson 1: The Capital Asset Pricing Model, Revisited	
8.1.1	BM 13.1
8.1.2	BM 13.1
8.1.3	BM 13.2
Lesson 2: Investor Behavior	
8.2.1	BM 13.3
8.2.2	BM 13.4
8.2.3	BM 13.5
Lesson 3: Multifactor Models of Risk	
8.3.1	BM 13.7
8.3.2	BM 13.7
8.3.3	BM 13.8
Lesson 4: Efficient Market Hypothesis	
8.4.1	BM 13.6
8.4.2	BM 13.6
8.4.3	SN1 (IFM-21-18) Section 4.1
8.4.4	SN1 (IFM-21-18) Section 4.2
8.4.5	SN1 (IFM-21-18) Section 4.2

Module 9: Long-term Financing

Lesson 1: Equity Financing for Private Companies	
9.1.1	BM 23.1
9.1.2	BM 23.1
Lesson 2: Equity Financing for Public Companies	
9.2.1	BM 23.2
9.2.2	BM 23.3
Lesson 3: Debt Financing	
9.3.1	BM 24.1
9.3.2	BM 24.2

Module 10: Actuarial Applications

Lesson 1: Variable Annuity Guarantees	
10.1.1	SN2 (IFM-22-18) Sections 1 and 2.1
10.1.2	SN2 (IFM-22-18) Section 2.1
10.1.3	SN2 (IFM-22-18) Section 2.1
10.1.4	SN2 (IFM-22-18) Section 2.1
10.1.5	SN2 (IFM-22-18) Section 2.1
10.1.6	SN2 (IFM-22-18) Section 2.1
Lesson 2: Other Types of Insurance Guarantees	
10.2.1	SN2 (IFM-22-18) Section 2.2
10.2.2	SN2 (IFM-22-18) Section 2.3
10.2.3	SN2 (IFM-22-18) Section 2.3
Lesson 3: Risk Management and Hedging	
10.3.1	SN2 (IFM-22-18) Section 3.1
10.3.2	SN2 (IFM-22-18) Section 3.1
10.3.3	SN2 (IFM-22-18) Section 3.1
10.3.4	SN2 (IFM-22-18) Section 3.2

Cost of Capital

1. To determine the equity cost of capital
2. To determine the debt cost of capital
3. To determine a project's cost of capital

Armed with the materials in Module 1, we are now in a position to link the materials in Exam FM to the quantitative part of Exam IFM.

When a firm is deciding whether to accept a project with **fixed cash flows**, they either

(1) compute the **NPV** of the project using a **risk-free rate of return**, and accept the project if the NPV is positive; or

(2) compute the **internal rate of return** of the project (assume that there is one and only one IRR), and accept the project if the IRR is higher than some appropriate discount rate.

Since the IRR approach suffers from many problems (e.g. multiple IRR, selection of mutually exclusive projects), the NPV method is more widely used in capital budgeting.

However, for most projects, cash flows are **random**. Suppose that the risk-free rate is 10%. Consider two projects, both of which need an initial investment of 160.

Project 1: You would receive a certain cash flow of $200 after 1 year. The NPV is

$$200/1.1 - 160 = 181.82 - 160 = 21.82.$$

Project 2: You would receive $100, $200 or $300. Each possible outcome has a probability of 1/3. The expected cash flow after 1 year is also $200.

Which project would you prefer? Since rational investors are risk-adverse, they would prefer the first project. This means that the NPV of the second project should be less than 21.82. Equivalently speaking, it is wrong to use a risk-free rate to discount risky cash flows.

So, how should we value random cash flows? The two methods commonly used in practice are:

(1) To reduce the expected cash flows appropriately (through adjusting the probabilities of possible outcomes) and use the risk-free rate to discount them. This is the **derivative approach**, which is detailed in Modules 4 and 5.

(2) To adjust the interest rate appropriately using the CAPM so that the expected value is discounted more heavily. The adjusted interest rate is called the **risk-adjusted interest / discount rate** or the **cost of capital**.

This lesson explores (2).

2.1.1   The Equity Cost of Capital

A firm needs capital to operate. There are mainly two ways to raise capital.

(1) A firm can issue only **stocks** (equity). Such a firm may pay dividends to its stockholders, but the payment of dividends is not an obligation.

(2) A firm can issue **debts** to raise capital. In this case, the firm is obligated to pay interest (unless the debt is zero-coupon) and repay the principal upon maturity.

(Both ways to raise capital will be discussed in Module 9.)

To attract investors, the stocks and debts have to offer a reasonable expected rate of return. Such returns are called the **equity cost of capital** and the **debt cost of capital**. Since debt is relatively safer than equity, for a given firm, the debt cost of capital is less than the equity cost of capital.

To determine the appropriate cost of capital for a project of a firm, we often need to first determine the costs of capital of the firm's (and in many cases many similar firm's) equity and debt. In Module 1 Lesson 3 we estimated the **beta** of a firm by performing a linear regression of the excess return on the firm's stock against the excess return on the market portfolio. The beta obtained is also called the **equity beta** of the firm. The expected return obtained from the SML is the **equity cost of capital** of the firm.

If a firm issues only stocks, then the firm's asset and equity must be equal. This means that the expected return on the firm's asset must be the same as the equity cost of capital. If the firm has a new project that has very similar risk characteristic as the firm's current projects (represented by the firm's asset), then we can use the equity cost of capital as the project's cost of capital. Let us take Walmart as an example. Suppose that an analyst is estimating the value of new Walmart outlet. The Walmart outlet is largely a clone of the existing outlets and its risk should closely match the overall risk of the entire Walmart Corporation, which is basically a collection of the cloned chain stores. (We describe such a project as **scale-enhancing**, meaning that the project carries the same amount and type of risk as other projects that the firm is currently running.) Hence, the cost of capital for Walmart as a whole seems to be an appropriate discount rate for evaluating the profitability of the new outlet.

Example (2.1.1)

An all-equity firm has a beta of 1.2. To value projects of the firm, you are given:

(i) The market risk premium is 9.5%.

(ii) The risk-free interest rate is 5%.

(iii) The firm is considering the following three mutually exclusive projects:

Project	Initial cost	Expected cash flow(s)
A	100	130 at time 1
B	94	120 at time 1
C	55	15 at times 1, 2, …, 9

(a) Calculate the IRR for all three projects. Which project has the largest IRR?

(b) Assuming that each project has a similar risk as the firm, determine the project that the firm should pick.

Solution

(a) For project A: $100 = 130/(1 + \text{IRR}_A) \Rightarrow \text{IRR}_A = 30\%$

For project B: $94 = 120/(1 + \text{IRR}_B) \Rightarrow \text{IRR}_B = 27.66\%$

For project C: $55 = 15 \times a_{\overline{9}|}$. Using a financial calculator, we find that $\text{IRR}_C = 23.06\%$. So Project A has the largest IRR.

(b) The equity cost of capital is $0.05 + 1.2 \times 0.095 = 16.4\%$. This is also the rate we use for discounting.

$\text{NPV(A)} = 130/1.164 - 100 = 11.68$.

$\text{NPV(B)} = 120/1.164 - 94 = 9.09$.

$\text{NPV(C)} = 15 \times \frac{1 - \frac{1}{1.164^9}}{0.164} - 55 = 15 \times 4.5431 - 55 = 13.15$.

So the firm should pick Project C.

[END]

A firm can certainly consider a project that is not the same as the existing projects. This is rather like when the McDonald's first opened a new line called McCafé in 2001 in Chicago. It can also happen that a firm has multiple lines of businesses, each with different risk characteristics. In such situations, the calculation in Example (2.1.1) would not work: we cannot use the firm's cost of capital as the project's cost of capital.

2.1.2   The Debt Cost of Capital

If a firm uses both debt and equity for financing, then the firm's asset and equity would have different risks. The use of debt increases the chance of bankruptcy and makes the equity risker. To calculate the beta of the firm's asset, we would need to use the debt cost of capital of the firm. In this section we illustrate how the debt cost of capital can be estimated and how we can relate the equity beta and debt beta to the firm's asset beta.

Industry Beta of Debt

The risk of default is higher for debts issued by firms with poor credit ratings and for debts with longer times to maturity. Some studies publish the average **debt betas** by rating and maturity. These published values can be used with the CAPM to estimate the **debt cost of capital**.

Tuning down the Yield to Maturity (YTM)

The **YTM** of a bond is the **IRR** of it, assuming that all the promised payments would be made in full. Taking the possibility of default into consideration, the expected return on the bond is in general smaller than the bond's YTM. Consider a one-year bond. Let L be the expected loss rate (measured as the fraction of the face value) given default, and p be the default probability. Then

$$\text{expected return of the bond} = (1 - p)\text{YTM} + p(\text{YTM} - L) = \text{YTM} - pL.$$

(This formula also holds for a multi-year bond if p, L and YTM do not vary with time.)

Example (2.1.2)

An A- rated 5-year bond has a yield to maturity of 2.5%. You estimate that there is a 1% chance that the bond would default per year, and the expected loss rate given default is 70%.

(a) Estimate the expected return on the bond.

(b) If the risk-free interest is 1% and the market risk premium is 8%, estimate the bond's beta.

Solution

(a) The expected return is $2.5\% - 0.01 \times 70\% = 1.8\%$.

(b) Solving the equation $1.8\% = 1\% + \beta_D \times 8\%$, we get $\beta_D = 0.1$.

[END]

The beta of the debt is typically small. Even for a CCC bond (a bond that has an annual default rate bigger than 75% in bad economy), the beta is no more than 0.5.

Unlevered Cost of Capital and Beta

A firm's **asset** is made up of its equity and debt. The asset is like a portfolio of equity and debt. If we let r_U be the firm's expected return on the asset or the asset cost of capital (yet another name is unlevered cost of capital of the firm), then

$$r_U = \frac{E}{E + D}r_E + \frac{D}{E + D}r_D,$$

where r_E (r_D) is the equity (debt) cost of capital. Also, the firm's asset is sensitive to the market portfolio because its equity and debt are sensitive to the market portfolio. If we let β_U be the asset beta of a firm (or the unlevered beta), then

$$\beta_U = \frac{E}{E + D}\beta_E + \frac{D}{E + D}\beta_D,$$

where β_E (β_D) is the equity (debt) beta of the firm. The above two formulas work as long as $\frac{D}{E+D}$ is constant through time.

Sometimes a firm maintains a large cash balance. The cash can be treated as a risk-free asset that counter-weights the debt and lowers the firm's risk. We can calculate the firm's net debt by

$$\text{Net debt} = \text{Debt} - \text{Excess cash and short-term investments} = D - C.$$

In extreme cases, especially for high-tech firms that have very little debt and lots of cash for R&D, the net debt can be negative. The firm's **enterprise value** is the firm's equity plus debt, minus the excess cash, and its beta can be computed from

$$\beta_U = \frac{E}{E + D - C}\beta_E + \frac{D}{E + D - C}\beta_D - \frac{C}{E + D - C}\beta_C,$$

where β_C is the beta of the cash balance. In general, the cash balance is nearly risk-free and β_C is close to 0. The textbook mentions that if we assume that the cash balance and the firm's debt have similar market risk, or if the debt beta reflects the combined risk of the firm's debt and cash balance, then $\beta_D \approx \beta_C$, and we have the following shortcut:

$$\beta_U = \frac{E}{E + D - C}\beta_E + \frac{D - C}{E + D - C}\beta_D.$$

Example (2.1.3)

A high-tech firm has a current market capitalization of 450 billion, 40 billion in debt and 70 billion in cash. Assuming that the firm has a debt beta of 0.01 and an equity beta of 2.3, and that the cash is risk-free, estimate the beta of the firm's enterprise value.

Solution

We have $E = 450$, $D = 40$ and $C = 70$. The beta of the firm's enterprise value is

$$\beta_U = \frac{450}{450 + 40 - 70} \times 2.3 + \frac{40}{450 + 40 - 70} \times 0.01 = 2.465.$$

[END]

2.1.3 A Project's Cost of Capital

Weighted Average Cost of Capital (WACC)

If a project is of similar risk to a firm's core business activities, then it is reasonable to use the firm's equity cost of capital to value the project if the firm has no debt. But what if the firm uses debt financing? We can argue that the firm's asset and the project are of similar risk, and hence it seems reasonable to use r_U as the cost of capital. This is nearly correct, except one thing: in many countries, including the US, interest expense of a firm is tax-deductible. Let τ_c be the corporate tax rate. For $1 interest payment made to the debtholders, τ_c is saved. As such, $r_D (1 - \tau_c)$ is the after-tax debt cost of capital because the tax saving partly defrays the borrowing cost. While the asset cost of capital is $r_U = \frac{E}{E+D}r_E + \frac{D}{E+D}r_D$, the firm's effective after-tax cost of capital is

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Weighted Cost of Capital

$$r_{wacc} = \frac{E}{E+D}r_E + \frac{D}{E+D}r_D(1-\tau_c) = r_U - \frac{D}{E+D}\tau_c r_D$$

In this regard, r_U is also called the pretax WACC.

Example (2.1.4)

You are given the following information for a firm:

(i) Debt: 5000 6% coupon bonds outstanding, 1000 par value, 25 years to maturity, selling for 105% of par; the bonds make annual payments.

(ii) Common stock: 75000 shares outstanding, selling for 60 per share; the beta is 1.10.

Also, the risk-free interest rate is 5% and the market risk premium is 7%. Compute the weighted average cost of capital of the firm, assuming a corporate tax rate of 35%.

Solution

We first find the pretax cost of debt. Solving $1050 = 60\frac{1-(1+i)^{-25}}{i} + 1000(1+i)^{-25}$ with the help of a financial calculator, we get $i = 5.6228\%$.

The cost of equity, according to CAPM, is $r_E = 5\% + 1.1 \times 7\% = 12.7\%$.

The market value of debt is $D = 5,000 \times 1050 = 5,250,000$.

The market value of equity is $E = 75,000 \times 60 = 4,500,000$. So, $D + E = 9,750,000$.

The WACC is $r_{wacc} = \frac{450}{975} \times 12.7\% + \frac{525}{975} \times 5.6228\% \times (1 - 0.35) = 7.8295\%$.

[END]

Industry Beta

If a new project is not similar to the firm's existing business, then one way to find the project's cost of capital is to find a firm whose main business is similar to the new project and estimate that firm's asset beta. Such a firm is called a comparison firm. If there are multiple comparison firms, we may even use the average of the comparison firms' asset costs of capital or asset betas. Many data vendors publish equity betas for main industries.

Example (2.1.5)

Marriott wants to compute the cost of capital for its new restaurant division. It has identified three comparison firms for its restaurant division. The table below provides the equity beta estimates, book values of debt, and market values of equity for the three comparison firms:

	β_E	Debt(billions)	Equity(billions)
Firm 1	0.75	0.004	0.096
Firm 2	1.00	2.300	7.700
Firm 3	1.08	0.210	0.790

The risk-free interest rate is 4%, the risk premium on the market portfolio is 8.4%, the CAPM holds, debts are risk-free, and corporate tax rate is zero. Estimate the cost of capital for Marriott's restaurant division, assuming that all three firms provide equally good comparisons for Marriott's restaurant division, and that the restaurant division is financed by equity only.

Solution

We have $\beta_D = 0$. So we can use $\beta_U = \frac{E}{E+D}\beta_E$ to compute the unlevered beta of each firm:

Firm 1: $\beta_U = \frac{0.096}{0.096+0.004} \times 0.75 = 0.72$,

Firm 2: $\beta_U = \frac{7.7}{7.7+2.3} \times 1.00 = 0.77$,

Firm 3: $\beta_U = \frac{0.79}{0.79+0.21} \times 1.08 = 0.8532$

We assume that the asset beta of Marriott's restaurant division can be approximated by
$$\frac{0.72 + 0.77 + 0.8532}{3} = 0.7811.$$

Since Marriott's restaurant division has no debt, its equity beta and asset beta are the same. By the CAPM, the cost of capital is 0.04+0.7811 × 0.084=10.56%.

[END]

Example (2.1.6)

☘️ Assume that Time-Warner is interested in acquiring the ABC television network from Disney. It has estimated the expected cash flows from acquiring ABC and wants to compute the cost of capital to value those cash flows. However, the major networks that are most comparable, NBC and CBS are owned by GE and Westinghouse, respectively, which have substantial cash flows from other courses. For these comparison firms, the table below presents hypothetical equity betas, debt to asset ratios, and the ratios of the market values of the network assets to all assets:

	Equity Beta	$D/(D+E)$	Network Assets / All Assets
GE	1.1	0.1	0.25
Westinghouse	1.3	0.4	0.50

Estimate the appropriate beta for the ABC acquisition. Assume that the debt of each of these comparison firms is risk-free, that the non-network assets of them are substantially similar.

Solution

First, we compute the asset beta of the two comparison firms:

$$\text{GE: } 0.9 \times 1.1 = 0.99, \text{ Westinghouse: } 0.6 \times 1.3 = 0.78$$

Viewing the asset side of the comparison firms as portfolios of network and non-network assets, we have

$$\begin{cases} 0.25\beta_N + 0.75\beta_{NA} = 0.99 \\ 0.50\beta_N + 0.50\beta_{NA} = 0.78 \end{cases}$$

and thus $\beta_N = 0.36$. This is the beta for the network asset.

[END]

Operating Leverage

Operating leverage is the relative proportion of fixed versus variable costs. A project with a relatively high proportion of fixed cost versus variable costs is said to have high operating leverage. Because the expenses of businesses with higher operating leverage do not proportionately increase as sales increase, the beta of a project with high operating leverage should be larger.

Execution Risk

Execution risk is the risk that a new project fails because of missteps in the firm's execution. Marketing mistakes and manufacturing delays are examples of execution risk. Some firms adjust for this risk by assigning a higher cost of capital to new projects. This is incorrect because execution risk is diversifiable and such risk should be taken into account by adjusting the expected cash flows of new projects.

Uses of the CAPM in Practice

When we calculate the NPV of a project, we need to estimate the project's future cash flows and also the cost of capital. You may wonder if the **CAPM** is a good model for projecting the cost of capital and the error involved (such as the estimation error of beta, the choice of the risk-free rate etc) is significant. It turns out that in general the errors in estimating the project's future cash flows have a far greater impact than the small discrepancies in the cost of capital. While not perfect, the CAPM is simple and robust. It does not allow a lot of leeway for managers to manipulate the cost of capital. Thus it is the predominant model used in practice.

Exercise 2.1

For Questions 1 to 3, consider the following information for bonds:

Rating	A or above	BBB	BB	B	CCC
Average default rate	0.2%	0.5%	2.3%	6.0%	12.2%
Average beta	0.05	0.1	0.18	0.25	0.30

1. ✦ A BB bond has a yield to maturity of 8% and an expected loss rate in the event of default of 65%. What is the expected return on the bond?

2. ✦ If the market risk premium is 5.5% and the risk-free interest rate is 3%, what is the expected return on a B bond?

3. ✦ There is a new bond issue that is BBB. You are given that

   (i) A B bond has an expected return of 5%.

   (ii) The market risk premium is 4.6%.

   (iii) The new BBB bond has an expected loss rate given default of 55

   Find the yield to maturity of the BBB bond.

For Questions 4 to 6, refer also to the following apart from the table in Questions 1 to 3:

Firm	Debt rating	Equity beta	Market Capitalization	Total Enterprise Value
1	BBB	1.3	2000	8000
2	A	0.8	2400	4000
3	B	1.75	3000	4500

4. ✦ Estimate the asset beta of Firm 1. Repeat for Firm 2 and Firm 3.

5. ✦ Firm 1 and Firm 3 decide to merge into one firm. What is the asset beta for the conglomerate?

6. ✦ Firm 2 wins a law suit and receives 1000 cash. How would its asset beta change? Assume that the beta of cash is 0.02.

7. ✦ A company has a debt beta of 0.1 and an equity beta of 2.1. It has 20 million in debt outstanding, and the total enterprise value is 140 million.

   Calculate the pretax WACC of the company if the risk-free interest rate is 2% and the expected return on the market portfolio is 10%.

8. ✦ The standard deviation of the returns on the shares of DSD is 0.534% and the correlation coefficient between the returns on its shares and the return on the market is 0.789. The average return on the market is 14%, with a variance of $0.145\%^2$, and the risk-free rate is 6%.

   What is the cost of equity of the company?

9. A company's market value balance sheet is as follows:

Asset value	500	Debt	200
		Equity	300
Firm value	500	Firm value	500

The risk-free interest rate is 3.5%, the equity beta is 1.2, the debt beta is 0.2, and the return on market portfolio is 14.4%.

Calculate the company's cost of capital.

10. A company has an annual dividend yield of 7.5% and a constant dividend growth rate of 3% per year. It also has 5-year bonds outstanding that have an annual coupon rate of 8%, a redemption value of 1000 and are selling at a premium of 1025.

The company has a 35% corporate tax rate and a debt-to-asset ratio of 30%.

Calculate the firm's after-tax weighted average cost of capital.

11. A company is considering a project that will result in initial after-tax cash savings of $4 million at the end of the first year, and these savings will grow at a rate of 2% per year indefinitely. The firm has a target debt-value ratio of 50%, a cost of equity of 13%, and an after-tax cost of debt of 4.5%. The cost-saving proposal is somewhat riskier than the usual projects the firm undertakes; management uses the subjective approach and applies an adjustment factor of +2% to the cost of capital for such risky projects.

Find the cost of capital for the project.

12. You are given the following information about a company's expected returns and debt:

   (i) Return on assets: 11%

  (ii) Return on debt: 7%

 (iii) The beta of the company's equity is 0.9.

Also, the market risk premium is 12.3%, and the risk-free interest rate is 4.5%.

If the company's debt amounts to 800, find the value of the company.

13. A firm has 30 million shares outstanding trading for 17 per share. It also has 90 million in outstanding debt. If the firm's equity cost of capital is 18.5%, debt cost of capital is 5.7%, and the corporate tax rate is 34%, find the firm's weighted average cost of capital.

14. Consider the following data from an industry:

Firm	Debt beta	Equity beta	Market Capitalization	Total Enterprise Value
1	0.05	2.3	1600	3500
2	0.30	1.8	2600	5200
3	0.06	2.0	3000	4500

Find the average asset beta for the industry based on the three firms.

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