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ACTEX Study Manual for

CAS Exam 7

Spring 2018 Edition

Victoria Grossack, FCAS

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Introduction and Notes on Past Exam Questions and Answers and the Material

Greetings! In this actuarial study manual you will find summary outlines and questions and answers for the readings for Part 7. They are divided into three groups: Policy Liabilities (PL), Insurance Company Valuation (ICV), and Enterprise Risk Management (ERM).

Questions and parts of some solutions have been taken from material copyrighted by the Casualty Actuarial Society. They are reproduced in this study manual with the permission of the CAS solely to aid students studying for the actuarial exams. Some editing of questions has been done. Students may also request past exams directly from the society. I am very grateful to the CAS for its cooperation and permission to use this material. It is, of course, in no way responsible for the structure or accuracy of the manual.

Exam questions are identified by numbers in parentheses at the end of each question. CAS questions have four numbers separated by hyphens: the year of the exam, the number of the exam, the number of the question, and the points assigned.

In addition to the old exam questions and the summary outlines, review questions are included for most of the newer material. Some of the review questions are designed to help students process and memorize the material, while others have been designed to be more like potential exam questions.

Page numbers (p.) with solutions refer to the reading to which the question has been assigned unless otherwise noted. Note that parts of some exam questions may make use of material that is no longer included in the syllabus. Although I have made a conscientious effort to eliminate mistakes and incorrect answers, I am certain some remain. I encourage students who find errors to bring them to my attention. Please check our web site for corrections subsequent to publication.

♦ ♦ ♦

I would like to thank Chris Van Kooten for previous contributions to this manual, which include many summary outlines and past examination answers.

To the students who make use of this manual, feedback is welcome. Good luck on May 2, 2018!
VAG

SECTION I

ESTIMATION OF POLICY LIABILITIES

Eric Brosius
Loss Development Using Credibility

Outline

I. Introduction

A. What loss development method do you select when there are large random fluctuations in year to year loss experience?
B. Least squares development is shown to provide the best linear approximation to the Bayesian estimate and is contrasted with other standard development techniques.

II. Notation

A. $L(x)$- estimate of ultimate losses $\hat{y}$, given losses to date of x and historical experience (x_i, y_i)
B. Y – random variable representing claims incurred
C. X – random variable representing number of claims reported at year end
D. $Q(X) = E(Y|X = x)$, expected total number of claims
E. $R(X) = E(Y - X|X = x) = Q(X) - x$, expected number of claims outstanding
F. MSE – mean squared error
G. EVPV – Expected Value of the Process Variance - $E_Y(Var(X|Y))$
H. VHM – Variance of the Hypothetical Means - $Var_Y(E(X|Y))$

III. Least Squares Development

A. $L(x) = \text{a} + \text{b}x$, where
B. $\text{b} = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2}$
C. $\text{a} = \bar{y} - \text{b}\bar{x}$

IV. Special Cases of Least Squares Development

A. When x and y are totally uncorrelated, b = 0
   1. $L(x) = \text{a}$, the "budgeted loss method"
B. When the observed link ratios y/x are all equal, a = 0
   1. $L(x) = \text{b}x$, the "link ratio method"
C. When b=1,
   1. $L(x) = \text{a} + x$, the "Bornhuetter-Ferguson method"

V. Hugh White's Question

A. If actual losses are higher than expected losses what do you do?
   1. Reduce the bulk reserve a corresponding amount (Budgeted Loss Method)
   2. Leave the bulk reserve at the same percentage level of expected losses (Bornhuetter-Ferguson Method)
   3. Increase the bulk reserve in proportion to the increase in actual reported over expected reported (Link Ratio Method)

B. These options are 3 points on the least squares continuum and the actual answer is likely to lie somewhere on that continuum.

VI. Bayesian Development Examples

A. Various examples using Bayesian estimation are used to show that the least squares estimate is superior to the link ratio, budgeted loss and Bornhuetter-Ferguson estimates:

B. Simple Model
   1. included to demonstrate method
   2. $Q(x) = \frac{2}{3}x + \frac{1}{3}$, $R(x) = -\frac{1}{3}x + \frac{1}{3}$, based on parameters in example
   3. The function $Q(x)$ does not align with any of the three special cases, but does lie on the least squares continuum.

C. Poisson-Binomial Example
   1. Poisson process determines ultimate claims (y) and reported claims (x) are determined by a Binomial process with the Poisson outcome y as the first parameter.
   2. $Q(x) = x + 2$, based on the parameters given in the paper
   3. This example is used to show that the link ratio method can't reproduce the Bayesian estimate Q(x), since there is no c, such that $cx \equiv x + 2$.
   4. Alternative Options for c
      a. Unbiased Estimate - $E\big((c-1)X\big) = \mu$
      b. Minimized MSE - minimize $E\left(\big((c-1)X - \mu\big)^2\right)$
      c. $c = E\left(\frac{Y}{X}\middle| X \neq 0\right)$
      d. Salzmann's Iceberg Technique - $d = E\left(\frac{X}{Y}\middle| Y \neq 0\right)$, $c = d^{-1}$

D. General Poisson-Binomial Case
   1. $Q(x) = x + \mu(1-d)$, $R(x) = \mu(1-d)$
   2. This is consistent with the form of the Bornhuetter-Ferguson estimate.

E. Negative Binomial-Binomial Case
   1. $R(x) = \frac{(1-d)(1-p)}{1-(1-d)(1-p)}(x + r)$
   2. By plugging in sample parameter values it can be seen that the special cases of the least squares do not apply, but the result does lie on the least squares continuum.

F. Fixed Prior Case – the ultimate number of claims is known
   1. $Q(x) = k,\ R(x) = k - x$
   2. This is consistent with the budgeted loss method.

G. Fixed Reporting Case – percentage of claims reported at yearend is always d
   1. $Q(x) = d^{-1}x,\ R(x) = (d^{-1} - 1)x$
   2. This is consistent with the link ratio method.

VII. The Linear Approximation – Development Formula 1

A. Pure Bayesian analysis requires significant knowledge about the loss and loss reporting process, which may not be available. A linear approximation can be used instead (Bayesian Credibility).
B. Development Formula 1 gives the best linear approximation to Q:
C. $L(x) = \big(x - E(X)\big)\frac{Cov(X,Y)}{Var(X)} + E(Y)$
D. With historical experience, we can estimate the parts:
E. $Cov(X,Y) = \overline{XY} - \bar{X}\bar{Y},\ Var(X) = \overline{X^2} - \bar{X}^2,\ E(X) = \bar{X},\ E(Y) = \bar{Y}$
F. Which gives the general least squares equation:
G. $L(x) = (x - \bar{X})\frac{\overline{XY} - \bar{X}\bar{Y}}{\overline{X^2} - \bar{X}^2} + \bar{Y}$
H. Potential problems in parameter estimation:
   1. Major changes in loss experience should be adjusted for:
      a. Inflation
      b. Exposure growth
   2. Sampling error
   3. Should substitute link ratio method when $a < 0$
   4. Should substitute budgeted loss method when $b < 0$

VIII. Credibility Form of the Development Formula – Development Formula 2

A. If there is a real number $d \neq 0$, such that $E(X|Y = y) = dy$ for all y, then the best linear approximation to Q is given by development formula 2:
B. $L(x) = Z\frac{x}{d} + (1 - Z)E(Y),\ where\ Z = \frac{VHM}{VHM + EVPV}$
C. This is a credibility weighting of the link ratio method and the budgeted loss method.
D. Special Cases:
   1. Poisson-Binomial and other Bornhuetter-Ferguson Cases
      a. $Z = d$
   2. Negative Binomial-Binomial Case
      a. $Z = \frac{d}{(d + p(1 - d))}$

IX. The Case Load Effect – Development Formula 3

A. If the rate of claim reporting is a decreasing function of the number of claims and there are real numbers $d \neq 0$ such that $E(X|Y = y) = dy + x_0$, then define development formula 3:

1. $L(x) = Z\frac{x-x_0}{d} + (1-Z)E(Y)$

X. Mechanics of the Least Squares Approach

A. Adjust data for exposure growth and inflation
B. Develop most mature years to ultimate based on assumed tail factor
C. Develop next oldest year to ultimate using least squares on the complete years
D. Repeat one year at a time until all years have been developed

Past CAS Examination Questions

1. According to Brosius, in "Loss Development Using Credibility," when using historical data to estimate ultimate losses as of a certain development point, if incurred losses are uncorrelated from one age of development to the next, then the least-squares estimate will equal the budgeted loss estimate. (00–6–2–.5)

2. You are given the information below. The tail factor from 48 months to ultimate is 1.0375.

Incurred Losses ($000)

Accident Year	Age of Development (Months) 12	24	36	48	60
1995	100	120	130	140	145
1996	110	130	150	160	
1997	120	140	150		
1998	130	150			
1999	140				

a. Based on the methodology described in Brosius's "Loss Development Using Credibility," estimate the ultimate losses for accident year 1997 using the methods below. Show all work.

i) Least-squares approach ii) Link ratio approach iii) Budgeted loss approach

b. Using the results from a., calculate the credibility value (Z) and use it to prove that the credibility weighted average of your results from the link ratio and budgeted loss ratio approaches equals the least-squares approach. Show all work. (00–6–41–4)

3. According to Brosius, in "Loss Development Using Credibility," the relationship between covariance(X, Y), where X is the reported loss and Y is the ultimate loss, and variance(X) determines which of three reserving methodologies is optimal. Assuming that reported losses at the valuation date are higher than expected, match each of the three loss reserving methods on the left with the covariance/variance relationship on the right under which the method is optimal.

1. Budgeted loss method	a. Cov(X, Y) = Var(X)
2. Bornhuetter-Ferguson method	b. Cov(X, Y) < Var(X)
3. Link ratio method	c. Cov(X, Y) > Var(X)

A. 1a, 2c, 3b B. 1b, 2a,3c C. 1b, 2c, 3a D. 1c, 2a, 3b E. 1c, 2b, 3a (01–6–22–.5)

4. You are given the following information:

   i) A $250,000 cap on noneconomic damages in medical malpractice suits was eliminated effective with January 1, 2000 and subsequent occurrences.
   ii) Expected accident year 2000 losses if cap was still in effect: $25 million.
   iii) Expected increase in accident year 2000 losses from cap elimination is 40%.
   iv) Expected percentage of accident year losses reported at 12 months before cap elimination is 40%.
   v) Expected percentage of accident year losses reported at 12 months after cap elimination is 30%.
   vi) Estimated standard deviation of ultimate losses is $10 million after the elimination of the cap.
   vii) Estimated standard deviation of the ratio of reported loss to ultimate loss at 12 months of development is .20 after the elimination of the cap.
   viii) Reported accident year 2000 losses at 12 months of development are $15 million.
   ix) There is no loss development beyond 48 months.

   Calculate the ultimate loss estimate for accident year 2000 using the Bayesian credibility method as discussed in Brosius's "Loss Development Using Credibility." Show all work. (01–6–30–3)

5. You are given the following information:

	Incurred Losses ($000)	
Accident Year	48 Months	Ultimate Loss
1993	65	90
1994	50	80
1995	70	85
1996	75	95
1997	60	

   Assume level premium writings throughout the 1993–1997 time period. According to Brosius, answer the following.

   a. Calculate a link ratio estimate and a budgeted loss estimate of the ultimate incurred loss for accident year 1997 using an all-year weighted average. Show all work.
   b. Calculate the least-square estimate of ultimate incurred loss for accident year 1997. Show all work.
   c. Display the least-square estimate in the form of a credibility-weighted average of the link ratio estimate and budgeted loss estimate calculated in a. Show all work. (02–6–21–1/1.5/.5)

6. Let $L(x) = a + bx$ be the result of a line fit to accident year pairs (x, y) of reported claims from successive development periods. Let L(x) be our estimate of y, given that we have already observed x. According to Brosius, which one of the following statements is true?

   A. If $a > 0$ and $b = 1$, then L(x) is identical to a Bornhuetter-Ferguson estimate.
   B. If $a > 0$ and $b < 0$, then L(x) is identical to a budgeted loss estimate.
   C. If $a = 1$ and $b > 0$, then L(x) is identical to a link ratio estimate.
   D. If $a = 0$ and $b > 0$, then L(x) is identical to a budgeted loss estimate.
   E. If $a < 0$ and $b > 0$, then L(x) is identical to a link ratio estimate. (03–6–3–1)

7. You are given the following information:

Accident Year	Earned Exposures (000)	Incurred Losses ($000) 27 Months	39 Months
1997	100	35	55
1998	200	65	80
1999	200	75	85
2000	250	85	95
2001	300	97	

Incurred losses will increase by an additional 20% from 39 months to ultimate. Based on Brosius, calculate the accident year 2001 ultimate loss estimate using each of the following methods. Show all work.

a. All-year weighted average link ratio method
b. Budgeted loss method. (03–6–22–1ea.)

8. You are given the following information:

Accident Year	Cumulative Losses Reported (Age of Development in Months) 12	24	36
2001	$1,200	$ 1,800	$2,000
2002	1,100	1,650	1,900
2003	1,300	1,860	
2004	1,400		

Using the least-squares method presented by Brosius, calculate the calendar year 2005 loss emergence for accident year 2004. (05–6–12–2)

9. X and Y are the two random variables describing reported losses and ultimate losses, respectively. Which of the following statements are true regarding the best linear approximation to the Bayesian estimate of Y?

1. If Cov(X, Y) < Var(X), a greater-than-expected reported amount should lead to an increase in the IBNR reserve.
2. If Cov(X, Y) = Var(X), a change in the reported amount should not affect the IBNR reserve.
3. If Cov(X, Y) > Var(X), a greater-than-expected reported amount should lead to an increase in the IBNR reserve.

A. 1 B. 2 C. 3 D. 1, 2 E. 2,3 (06–6–4–1)

10. Given the following information:

	Incurred Losses	
	Age of Development in Months	
Accident Year	12	24
2001	10,000	25,000
2002	11,000	28,000
2003	12,000	27,000
2004	11,500	28,000
2005	12,500	

According to the least-squares method, what is the expected incurred loss for accident year 2005 at 24 months?

A. < \$27,500 B. ≥ \$27,500, but < \$28,500 C. ≥ \$28,500, but < \$29,500
D. ≥ \$29,500, but < \$30,500 E. ≥ \$30,500 (06–6–5–1)

11. As the result of recent tort reform, general liability expected ultimate losses decreased from \$60 million to \$50 million for accident year 2005. Without the reform, 55% of ultimate accident year 2005 losses would have been reported within twelve months. With the reform, this percentage is expected to rise to 63%. At December 31, 2005, \$35 million of losses have been reported for accident year 2005.

a. What is the link ratio estimate of the ultimate loss for accident year 2005?
b. What is the Bornhuetter-Ferguson estimate of the ultimate loss for accident year 2005?
c. Given that Y is expected ultimate losses and X is reported losses at 12 months, and using the estimates below, what is the ultimate loss for accident year 2005, using Brosius's Bayesian credibility method?

$Var_Y[E(X|Y)] = 14.3$ $E_Y[Var(X|Y)] = 57$

d. Why is it inappropriate to use the least-squares method in the situation described in this case? (06–6–15–.5/.5/1/.5)

12. An insurer has been experiencing a deteriorating loss ratio for the last five years on its personal auto business, due to the weakening of underwriting standards. Explain why the least-squares development method may not be appropriate. (07–6–42b–.5)

13. Given the following:

	Cumulative Reported Losses ($000) Age of Development in Months			
Acc Year	12	24	36	48
2004	8,847	12,204	14,332	17,021
2005	10,280	14,650	16,807	
2006	11,747	14,826		
2007	12,077			

a. Estimate the cumulative reported loss as of 24 months for accident year 2007 using the link ratio method.
b. Estimate the cumulative reported loss as of 24 months for accident year 2007 using the budgeted loss method.
c. Estimate the cumulative reported loss as of 24 months for accident year 2007 using the least-squares method. (08–6–9–.5/.5/1)

14. Given the following reported loss information:

Accident Year	As of 60 Months	As of 72 Months
2000	$40,000	$45,000
2001	30,000	60,000
2002	40,000	42,000
2003	30,000	32,000
2004	50,000	

a. Use Brosius' least-squares method to calculate the expected losses for accident year 2004 at 72 months.

b. Briefly explain whether least squares is an appropriate method to use in this situation. (09–6–3–2/.5)

15. Given the following information ($000):

Accident Year	Incurred Loss at 12 Months	Incurred Loss at 24 Months
2006	10,000	12,000
2007	16,000	20,000
2008	10,000	16,000
2009	15,000	

Use the method of least squares development to calculate the estimated incurred loss at 24 months for the accident year 2009. (10-6-11-2)

16. Given the following information ($000) for a line of business:

Accident Year	Written Premium	Earned Premium	Cumulative Reported Losses 12 Months	24 Months	36 Months
2007	5,756	4,779	413	2,310	5,845
2008	6,907	5,735	0	541	1,309
2009	8,289	6,882	936	2,311	
2010	9,946	8,258	50		

- The tail factor from 36 months to ultimate is 1.050.

a. Use the least squares method to estimate ultimate losses for the 2009 accident year.
b. Discuss the reasonability of the estimate derive in part a. above, relative to the estimate that would be produced by the link ratio method.
c. Illustrate graphically the relationships between the link ratio method, budgeted loss method and least squares method in modeling the loss development process. (11-7-1-1/0.5/1.5)

17. Given the following information:

	Incurred Loss Ratio	
Accident Year	As of 36 Months	As of 48 Months
2006	0.222	0.375
2007	0.451	0.675
2008	0.446	0.605
2009	0.228	

a. Estimate the loss ratio for accident year 2009 as of 48 months using the least squares method.
b. An alternate approach to estimating the accident year 2009 loss ratio as of 48 months is to use the arithmetic average of the link ratio method and the budgeted loss ratio method. Using the answer from part a. above, demonstrate whether this averaging approach is optimal. (12-7-4-1.5/1.5)

18. Given the following information:

Cumulative Losses ($000,000)		
Accident Year	Reported at 24 Months	Ultimate Loss
2008	12	18
2009	10	16
2010	14	20
2011	12	18
2012	21	

An insurer writes annual policies that incept on January 1. Exposure and coverage levels were constant for 2008 through 2011. On January 1, 2012, policy coverage was expanded and pricing actuaries estimated the following:

Loss amounts will increase by 25% due to the expanded coverage.
75% of ultimate losses are expected to be reported by 24 months, with a standard deviation of 8% of estimated ultimate loss.
Standard deviation of accident year 2012 ultimate loss will be $3 million.

a. Calculate the projected accident year 2012 ultimate loss using Bayesian credibility methods.

b. Explain why the least squares method is not appropriate for calculating the accident year 2012 loss.

(14-7-1-2:1.5/.5)

19. Given the following information ($000,000):

Accident Year	Cumulative Reported Loss @ 24 Months	Ultimate Loss
2011	36	75
2012	40	71
2013	35	64
2014	25	

a. Using the least-squares method, estimate ultimate loss for Accident Year 2014.

b. For each of the following scenarios, briefly describe a potential problem with the least-squares method:
   i. The slope parameter is negative
   ii. The intercept parameter is negative

c. Due to a regulatory change, the following is anticipated:
   - No change in the reporting pattern
   - Standard deviation of reported loss as of 24 months will be 10% of estimated ultimate loss
   - Expected ultimate loss for 2014 will decrease 20%
   - Standard deviation of accident year 2014 ultimate loss is expected for be $6,000,000

Using the Bayesian credibility method, estimate the revised ultimate loss for accident year 2014.
(16-7-2-3.25:1.25/0.5/1.5)

20. Given the following loss ratio triangle:

	Cumulative Reported Loss Ratios				
Accident Year	12 months	24 months	36 months	48 months	60 months
2010	3.0%	10.0%	15.7%	37.0%	37.0%
2011	5.1%	5.1%	25.0%	44.2%	48.0%
2012	2.5%	3.0%	40.0%	57.0%	59.2%
2013	1.6%	15.7%	22.2%	21.0%	
2014	0.0%	7.8%	16.7%		
2015	6.3%	12.4%			
2016	4.7%				

Assume a tail factor of 1.15 from 60 months to ultimate

Calculate the accident year 2014 ultimate loss ratio using the least squares method.
(17-7-2-1.75)

Solutions to Past CAS Examination Questions

1. T, p. 3.

2. a. i) Ultimate Incurred Losses_{97} = (160)(1.0375) = 166

$\bar{x}$ = (130 + 150)/2 = 140 $\bar{y}$ = (145 + 166)/2 = 155.5

$\overline{xy}$ = [(130)(145) + (150)(166)]/2 = 21,875

$\overline{x^2}$ = $[(130)^2 + (150)^2]/2$ = 19,700

b. $$\frac{\overline{xy} - (\bar{x})(\bar{y})}{\overline{x^2} - (\bar{x})^2} = \frac{21{,}875 - (140)(155.5)}{19{,}700 - 140^2} = 1.05$$

a = $\bar{y}$, b = $\bar{x}$ = 155.50 - (1.05)(140) = 8.5

L(x) = a + bx = 8.5 + (1.05)(150) = 166

ii) c= $\frac{\bar{y}}{\bar{x}}$ = 155.5/140 = 1.1107 L(x) = cx = (1.1107)(150) = 166.6

iii) L(x) = $\bar{y}$ = 155.5, pp. 2–3. $\bar{y}$

b. Z = b/c = 1.05/1.107 = .9485

L(x) = (Z)(cx) + (1 - Z)$\bar{y}$ = (.9485)(166.6) + (1 - .9485)(155.5) = 166, pp. 16–17.

3. 1b, 2a, 3c, pp. 4, 11.

Answer: B

4. 1) Calculate link ratio and budget ratio estimates:

x/d = $15\bar{M}/.3$ = $50\bar{M}$ E[Y] = $(25\bar{M})(1.4)$ = $35\bar{M}$

2) Calculate VHM:

VHM = Var(X) = Var(.3Y) = $(.3)^2(10\bar{M})^2$ = $9(\bar{M})^2$

3) Calculate EVPV:

EVPV = $E[X^2]$ = $E[(.2Y)^2]$ = $\{.2\}^2\{\text{Var}(Y) + (E[Y])^2\}$

EVPV = $[.2]^2[(10\bar{M})^2 + (35\bar{M})^2]$ = $53(\bar{M})^2$

4) Calculate Z:

Z = VHM/(VHM + EVPV) = $9(\bar{M})^2/[9(\bar{M})^2 + 53(\bar{M})^2]$ = .145

5) Calculate the ultimate loss estimate:

L(x) = Zx/d + (1 - Z)E[Y] = $(.145)(50\bar{M})$ + $(1 - .145)(35\bar{M})$ = $37.175\bar{M}$,

5. a. $\bar{x}$ = (65 + 50 + 70 + 75)/4 = 65 $\bar{y}$ = (90 + 80 + 85 + 95)/4 = 87.5

$c = \bar{y}/\bar{x}$ = 87.5/65 = 1.346

For a link ratio estimate, we get: L(x) = cx = (1.346)(60) = 80.76

For a budgeted loss estimate, we get: L(x) = $\bar{y}$ = 87.5

b. $\overline{xy}$ = [(65)(90) + (50)(80) + (70)(85) + (75)(95)]/4 = 5,731.25

$\overline{x^2}$ = $[(65)^2 + (50)^2 + (70)^2 + (75)^2]/4$ = 4,312.5

$$b = \frac{\overline{xy} - (\bar{x})(\bar{y})}{\overline{x^2} - (\bar{x})^2} = \frac{5{,}731.25 - (65)(87.5)}{4{,}312.5 - 65^2} = .5$$

$a = \bar{y} - b\bar{x}$ = 87.5 - (.5)(65) = 55 L(x) = a + bx = 55 + (.5)(60) = 85

c. Z = b/c = .5/1.346 = .3715

L(x) = (Z)(cx) + (1 - Z) $\bar{y}$ = (.3715)(80.76) + (1 - 0.3715)(87.5) = 85, pp. 2–3, 16–17.

6. A. T, pp. 3–4

B. F, p. 3 – Substitute "b = 0" for "b < 0."

C. F, p. 3 – Substitute "a = 0" for "a = 1."

D. F, p. 3 – Substitute "a > 0" for "a = 0" and "b = 0" for "b > 0."

E. F, p. 3 – Substitute "a = 0" for "a < 0."

Answer: A

7. a. Since the exposure level changes, use loss ratios rather than losses:

1997	.350	.550
1998	.325	.400
1999	.375	.425
2000	.340	.380
2001	.323	

$\bar{x}$ = (.350 + .325 + .375 + .340)/4 = .348

$\bar{y}$ = (1.2)(.550 + .400 + .425 + .380)/4 = .527

$c = \bar{y}/\bar{x}$ = .527/.348 = 1.514 L(x) = cx = (1.514)(97,000) = 146,858

b. $L(x) = 300\,\overline{y} = (300{,}000)(.527) = 158{,}100$, pp. 2, 16–17.

8. $\overline{x} = (1{,}200 + 1{,}100 + 1{,}300)/3 = 1{,}200$ $\overline{y} = (1{,}800 + 1{,}650 + 1{,}860)/3 = 1{,}770$

$\overline{xy}\ \overline{M} = [(1{,}200)(1{,}800) + (1{,}100)(1{,}650) + (1{,}300)(1{,}860)]/3 = 2{,}131{,}000$

$\overline{x}^2 = [(1{,}200)^2 + (1{,}100)^2 + (1{,}300)^2]/3 = 1{,}446{,}667$

$$b = \frac{\overline{xy} - (\overline{x})(\overline{y})}{\overline{x^2} - (\overline{x})^2} = \frac{2{,}131{,}000 - (1200)(1770)}{1{,}446{,}667 - 1200^2} = 1.05$$

$a = \overline{y} - b\overline{x} = 1{,}770 - (1.05)(1{,}200) = 510$

$L(x) = a + bx = 510 + (1.05)(1{,}400) = 1{,}980$, pp. 2–3.

9. 1. F, p. 11 – Substitute "decrease" for "increase."

2. T, p. 11

3. T, p. 11

Answer: E

10. $\overline{x} = (10 + 11 + 12 + 11.5)/4 = 11.125$ $\overline{y} = (25 + 28 + 27 + 28)/4 = 27$

$\overline{xy} = [(10)(25) + (11)(28) + (12)(27) + (11.5)(28)]/4 = 301$

$\overline{x}^2 = [(10)^2 + (11)^2 + (12)^2 + (11.5)^2]/4 = 124.3125$

$$b = \frac{\overline{xy} - (\overline{x})(\overline{y})}{\overline{x^2} - (\overline{x})^2} = \frac{301 - (11.125)(27)}{124.3125 - 11.125^2} = 1.14286$$

$a = \overline{y} - b\overline{x} = 27 - (1.14286)(11.125) = 14.28568$

$L(x) = a + bx = 14.28568 + (1.14286)(12.5) = 28.57143$, pp. 2–3.

Answer: C

11. a. $x/d = 35\overline{M}/.63 = 55{,}555{,}556$, p. 2.

b. $L = 35\overline{M} + dE[Y] = 35\overline{M} + (.37)(50\overline{M}) = 53.5\overline{M}$, p. 3.

c. $Z = VHM/(VHM + EVPV) = 14.3/(14.3 + 57) = .201$

$L(x) = Zx/d + (1 - Z)E[Y] = (.201)(55{,}555{,}556) + (1 - .201)(50\overline{M}) = 51{,}116{,}667$, pp. 13–15.

d. It is inappropriate because there are significant changes in the loss history, p. 19.

12. It is not appropriate when "year to year changes are due largely to systematic shifts in the book of business," pp. 12, 19.

13. a. $\overline{x} = (8{,}847 + 10{,}280 + 11{,}747)/3 = 10{,}291$

$\overline{y} = (12{,}204 + 14{,}650 + 14{,}826)/3 = 13{,}893$

$c = \overline{y}/\overline{x} = 13{,}893/10{,}291 = 1.35$ $L(x) = cx = (1.35)(12{,}077) = 16{,}304$

b. $L(x) = \overline{y} = 13{,}893$

c. $\overline{xy} = [(8{,}847)(12{,}204) + (10{,}280)(14{,}650) + (11{,}747)(14{,}826)]/3 = 144{,}243{,}937$

$\overline{x^2} = [(8{,}847)^2 + (10{,}280)^2 + (11{,}747)^2]/3 = 107{,}313{,}273$

$$b = \frac{\overline{xy} - (\overline{x})(\overline{y})}{\overline{x^2} - (\overline{x})^2} = \frac{144{,}243{,}937 - (10{,}291)(13{,}893)}{107{,}313{,}273 - 10{,}291^2} = .902$$

$a = \overline{y} - b\,\overline{x} = 13{,}893 - (.902)(10{,}291) = 4{,}611$

$L(x) = a + bx = 4{,}611 + (.902)(12{,}077) = 15{,}504$, pp. 2–3.

14. a. $\overline{x} = (40{,}000 + 30{,}000 + 40{,}000 + 30{,}000)/4 = 35{,}000$

$\overline{y} = (45{,}000 + 60{,}000 + 42{,}000 + 32{,}000)/4 = 44{,}750$

$\overline{xy} = [(40{,}000)(45{,}000) + (30{,}000)(60{,}000) + (40{,}000)(42{,}000) + (30{,}000)(32{,}000)]/4$

$\overline{xy} = 1{,}560\overline{M}$

$\overline{x^2}$ = [(40,000)^2 + (30,000)^2 + (40,000)^2 + (30,000)^2]/4 = 1,250M

$$b = \frac{\overline{xy} - (\bar{x})(\bar{y})}{\overline{x^2} - (\bar{x})^2} = \frac{1{,}560M - (35{,}000)(44{,}750)}{1{,}250M - 35{,}000^2} = -0.25$$

a = $\bar{y}$ - b $\bar{x}$ = 44,750 - (–.25)(35,000) = 53,500

L(x) = a + bx = 53,500 + (–.25)(50,000) = 41,000

b. Since b < 0, the least-squares estimate is not appropriate. Because of this the estimate produced by the budgeted loss method ($\bar{y}$ = 44,750) may be substituted, pp. 2–4.

15. $\bar{x} = (10{,}000 + 16{,}000 + 10{,}000)/3 = 12{,}000$
$\bar{y} = (12{,}000 + 20{,}000 + 16{,}000)/3 = 16{,}000$
$\overline{xy} = [(10{,}000)(12{,}000) + (16{,}000)(20{,}000) + (10{,}000)(16{,}000)]/3 = 200{,}000{,}000$
$\overline{x^2} = [(10{,}000)^2 + (16{,}000)^2 + (10{,}000)^2]/3 = 152{,}000{,}000$

$$b = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2} = \frac{200M - (12{,}000)(16{,}000)}{152M - (12{,}000)^2} = 1$$

$a = \bar{y} - b\bar{x} = 16{,}000 - 12{,}000 = 4{,}000$

$L(x) = a + bx = 4{,}000 + 15{,}000 = 19{,}000$

16. a. Ultimate losses for AY 2007 and 2008:
2007: $Ult = 5{,}845(1.05) = 6{,}137.25$
2008: $Ult = 1{,}309(1.05) = 1{,}374.45$

Loss ratios for AY 2007 and 2008 (Divide by earned premium):

Year	24 Months	36 Months	Ultimate
2007	48.3%	122.3%	128.4%
2008	9.4%	22.8%	24.0%
2009	33.6%		

$\bar{x} = (0.483 + 0.094)/2 = 0.289$

$\bar{y} = (1.284 + 0.240)/2 = 0.762$

$\overline{xy} = [(0.483)(1.284) + (0.094)(0.240)]/2 = 0.321$

$\overline{x^2} = [(0.483)^2 + (0.094)^2]/2 = 0.121$

$$b = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2} = \frac{0.321 - (0.289)(0.762)}{0.121 - (0.289)(0.289)} = 2.689$$

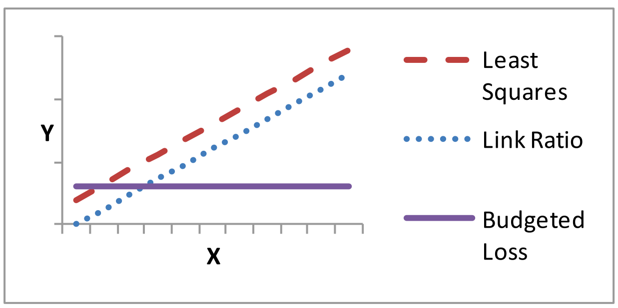
$$a = \bar{y} - b\bar{x} = 0.762 - 2.689(0.289) = -0.015$$

$$L(x) = a + bx = -0.015 + 2.689(0.336) = 0.889$$

$$Ult\ Loss\ 2009 = 6{,}882(0.889) = 6{,}118.10$$

b. Since the estimate of a is less than 0 the least squares method will produce estimates of y that are less than 0 when x is small. Brosius suggests substituting the link-ratio method when a < 0. The link-ratio method will produce positive estimates of y even for small values of x.

c.



17. a. $L(x) = a + bx$

$$\bar{x} = (0.222 + 0.451 + 0.446)/3 = 0.373$$

$$\bar{y} = (0.375 + 0.675 + 0.605)/3 = 0.552$$

$$\overline{xy} = [(0.222)(0.375) + (0.451)(0.675) + (0.446)(0.605)]/3 = 0.219$$

$$\overline{x^2} = [(0.222)^2 + (0.451)^2 + (0.446)^2]/3 = 0.151$$

$$b = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2} = \frac{0.219 - (0.373)(0.552)}{0.151 - (0.373)(0.373)} = 1.104$$

$$a = \bar{y} - b\bar{x} = 0.552 - 1.104(0.373) = 0.140$$

$$L(x) = a + bx = 0.140 + 1.104(0.228) = 0.392$$

$$Ult\ Loss\ Ratio\ 2009 = 39.2\%$$

b. In a credibility weighting $Z = b/c$, where $c = \bar{y}/\bar{x}$
$Z = 1.104/(0.552/0.373) = 0.746$
Since $Z = 0.746 \neq 0.5$ the arithmetic average does not produce an optimal solution.

18. a. X= loss reported at 24 months

Y= Ultimate losses

L(x)=Z(x/d)+(1 - Z)E[Y]
Z=VHM/(VHM+EVPV)
VHM =$(E[D] \times \sigma(y))^2$ = $((.75)(3))^2$ =5.0625
EVPV=Var(D)[Var(y) +E[y] ^{2}]=$(0.08)^2$ [3^2+[(1.25)({18+16+20+18}/4)]^2]=3.2976

Z=5.0625/(5.0625+3.2976)=.606

L(x)=(.606)(21/.75)+(1-.606)(22.5)=25.833 million

b. The least squares method is appropriate when the distribution of loss is not changing year over year. Given the coverage expansion and change in 2012 loss distribution, we cannot use the least squares method.

19. a.
$$\overline{X} = \frac{36+40+35}{3} = 37$$
$$\overline{Y} = \frac{75+71+64}{3} = 70$$
$$\overline{XY} = \frac{36\times 75+40\times 71+35\times 64}{3} = 2593.33$$
$$\overline{X^2} = \frac{36^2+40^2+35^2}{3} = 1373.67$$
$$b = \frac{\overline{XY} - \overline{X}\,\overline{Y}}{\overline{X^2} - \overline{X}^2} = 0.713$$
$$a = \overline{Y} - b\times\overline{X} = 43.62$$
2014 Ultimate Loss = a + b × 25 = 61.45

b. i. If b < 0, then y decreases as x increases.
ii. If a < 0, then y is negative for small values of x.

c. $\sigma_d = 0.1$

$Y = 0.8 \times 70 = 56$

$\sigma_Y = 6$

$d = \frac{37}{70} = 0.5286$

$VHM = \sigma_Y^2 d^2 = 6^2 (0.5286)^2 = 10.058$

$EVPV = \sigma_d^2 \left[\sigma_Y^2 + Y^2 \right] = (0.1)^2 \left(6^2 + 56^2 \right) = 31.72$

$Z = \frac{VHM}{VHM + EVPV} = \frac{10.058}{10.058 + 31.72} = 0.2407$

$L = 0.2407 \left(\frac{0.25}{0.5286} \right) + \left(1 - 0.2407\right)\left(56\right) = 53.904$

20. Need 2013 ultimate first:

$\overline{X}$ = 1/3 × (0.37+0.442+0.57) = 0.4607

$\overline{Y}$ = 1/3 ×1.15 × (0.37+0.48+0.592) = 0.5528

$\overline{XY}$ = 1/3 × (0.37 × 1.15 × 0.37+...) = 0.2632

$\overline{X}^2$ = 1/3 × (0.37^2+...) = 0.2191

b = ($\overline{XY}$ - $\overline{X}$ × $\overline{Y}$) / (X^2 bar - ($\overline{X}$ bar) ^2)) = 1.2435

a = $\overline{Y}$ - b × $\overline{X}$ = -0.0201

Since a < 0, using link ratio method instead

2013 ultimate = 0.21 × 1.15 × (0.37+0.48+0.592)/(0.37+0.442+0.57) = 0.2520

Calculate 2014 ultimate

$\overline{X}$ = ¼ × (0.157+0.25+0.4+0.222) = 0.2573

$\overline{Y}$ = ¼ × (0.37 × 1.15+0.48 ×1.15+0.592 × 1.15+0.2520) = 0.4776

$\overline{XY}$ = ¼ × (0.157 × (0.37 × 1.15)+...) = 0.1333

$\overline{X}^2$ = ¼ × (0.157^2+...) = 0.0741

b = ($\overline{XY}$ - $\overline{X}$ × $\overline{Y}$) / ($\overline{X^2}$ - ($\overline{X}$) ^2)) = 1.3187

a = $\overline{Y}$ - b × $\overline{X}$ = 0.1383

2014 ultimate = a + b × 0.167 = 0.359