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Practice Problems for Advanced Topics in General Insurance

Spring 2019 Edition

Gennady Stolyarov II

FSA, ACAS, MAAA, CPCU, ARe, ARC, API, AIS, AIE, AIAF

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New Hartford, Connecticut

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ISBN: 978-1-63588-615-3

Printed in the United States of America.

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Fourth Edition
Spring 2019

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Introduction to Practice Problems in Advanced Topics in General Insurance

Gennady Stolyarov II, FSA, ACAS, MAAA, CPCU, ARe, ARC, API, AIS, AIE, AIAF

The purpose of this book is to help you pass Exam GIADV: Advanced Topics in General Insurance, the final exam within the Society of Actuaries' new General Insurance Track. Thus far, as of February 2019, ten sittings of this relatively new exam have been administered biannually since 2014. Cumulatively throughout these exam sittings, 39 candidates have passed the exam to date. I was among them, passing the Spring 2015 exam on my first attempt. I am hopeful that the existence of this book will greatly increase the number of candidates who register for the exam and pass it – including you.

While preparing for the exam, I noticed a distinct absence of available systematic study resources apart from the syllabus readings themselves. I therefore embarked on a project to craft my own study materials, in addition to assembling any useful practice problems I could find from previous exams. I found that a thorough, piece-by-piece consideration of key concepts within the syllabus readings could give rise to a breadth of exercises – both basic and challenging. Furthermore, in addition to past questions from sittings of Exam GIADV, I discovered questions relevant to each of the syllabus readings, scattered throughout past sittings of Casualty Actuarial Society (CAS) Exams 7, 8, and 9. Studying the relevant past CAS exam questions adds to the range of possible problems with which candidates can become familiar in order to facilitate greater mastery of the Exam GIADV syllabus.

This book of practice problems is the most comprehensive culmination of my efforts to date, and I am pleased to have the opportunity to work with ACTEX Publications to bring all of these resources to you in one convenient compilation so that you will spend less time gathering problems from many separate sources. The Spring 2019 edition of this book includes relevant problems and solutions from each of the past Exam GIADV sittings, relevant recent CAS exam sittings, and original problems that I developed. This book is structured to align precisely with the five syllabus topics and eight syllabus papers – each of which has a section of problems devoted to it. The following is a summary breakdown of what you will find:

Section (and Syllabus Paper)	Original Problems	SOA Problems	CAS Problems	Total Problems
1 (Mack)	21	11	5	37
2 (Venter)	22	10	5	37
3 (Clark LDF)	60	10	6	76
4 (Marshall et al.)	103	10	4	117
5 (Lee)	44	6	12	62
6 (Clark Reinsurance)	139	20	9	168
7 (D'Arcy / Dyer)	99	10	6	115
8 (Mango)	43	10	2	55
TOTAL	**531**	**87**	**49**	**667**

Each section presents all of the problems in succession, followed by the solutions at the end. You are encouraged to attempt each problem on your own and write down or type your solution, and then look at the answer key for step-by-step explanation and/or calculations. As this book is a learning tool, I have provided relevant citations from the syllabus readings for many of the practice problems. Also, I am not an advocate of leaving any problems as unexplained "exercises to the reader." While *each* of these problems is intended to be an exercise for you, this book's purpose is to show you how they can be solved as well – so give each of them your best attempt, but know that detailed answers are available for you to check your work and fill in any gaps that may have prevented you from solving a problem yourself.

It is important to emphasize that the exam is always based on the syllabus readings and not primarily on any external study materials. As such, you are strongly encouraged to read and re-read the syllabus papers and internalize their contents. This book should be viewed as a companion and supplement to, *not* a substitute for, the syllabus readings. Here is a suggested approach for how to use this book in conjunction with the syllabus papers.

Step 1. Read a particular syllabus paper from start to finish, as you would an article or book. This helps you gain a familiarity with the contents and the structure of the paper, as well as where to find particular concepts and methods.
Step 2. Perform a second, closer reading of the syllabus paper, this time in conjunction with this book. The original exercises in this book were structured to align with the sequence of each syllabus paper's content. Look at the citations within each exercise to see where you will find the corresponding discussion within the syllabus paper. Once you have visited the relevant portions of the syllabus paper, attempt the exercise, and check your answer. This process will facilitate active reading of each paper. At this stage, you should be engaged with the material in detail and check your understanding at every step of the way.
Step 3. Create flashcards from the conceptual questions in this book and review them daily so as to internalize key ideas, methods, formulas, and even calculation shortcuts that may help if deployed properly during the exam. Making your own flashcards helps you actively engage with the material further. You have many options regarding how to create them – from the traditional pen-and-notecard approach, to often-free online and mobile applications such as Anki, StudyDroid, or StudyBlue. Even you as move on to subsequent syllabus topics, you should be regularly reviewing flashcards from previous papers and topics to keep these materials fresh in your mind.
Step 4. Once you have completed all of the exercises in this book, re-read each of the syllabus papers once more and focus on any areas that may still require additional work for you to understand and recall. Think about how else any particular idea might be tested. I encourage you to extend your practice by developing your own original problems as well. Nothing helps you learn the material as much as trying to teach it in a stepwise manner, even to yourself.

Other Study Recommendations

The key for success on any actuarial exam is to set ambitious but flexible study goals that require a regular exertion of effort but can also adapt to changing circumstances without sacrificing other priorities in life. My greatest successes on exams came during sittings for which I studied using a self-developed point system, assigning a certain number of points for every page I read, every practice problem I solved or created, and every flashcard I reviewed. The point assignment could vary based on the type of activity and its difficulty level. For each day, I would set a point goal and try to exceed it, ideally raising my all-day average of points every day. Of course, my point system is not scientific and does not precisely match the difficulty level of each studying activity, but the existence of a point goal is a subjective motivator for continual effort while also giving one an eventual sense of satisfaction with what one has done on any given day. If one does need to attend to other priorities during the day, one can tailor one's activities to match (for instance, reviewing electronic flashcards during a trip, or reading a syllabus paper on a tablet during an elliptical-trainer run) while still meeting the point goal. It is also important to deploy one's available energy and resources wisely, always being heedful of the

Aristotelian "golden mean" – a useful principle to follow with regard to any physical or mental exertion. Avoiding excessive stress and burnout is vital for any candidate who seeks to make steady exam progress. Try to keep your mind fresh and find ways to build buffers of time into your schedule to enable you to swiftly react to the inevitable changes of circumstance. Remember that this endeavor is an ultramarathon, not a sprint.

Use a variety of study techniques to keep the information fresh in your mind. Simple memorization creates anchors in your mind that can render the application of a skill more instantaneous. You should also be solving practice problems on a daily basis, if possible. The more different problem types and approaches for solving them that you are able to internalize, the more capable you will be when facing an unfamiliar problem. With enough practice you might, indeed, be able to recognize some seemingly completely new problems as variations on familiar themes.

Exams are time-limited, and it is important to pace yourself appropriately. During the 15-minute reading period, make a mental note of the problems that you know how to approach right away, and do those first. At the end, you should strive to give yourself a sufficient time buffer to think through the problems you find more challenging and unusual. Try, as much as possible, to always keep moving forward somewhere. If you hit a block on one problem, shift to another and work through it; perhaps an insight on the first problem will arrive later.

If you are preparing to take Exam GIADV, you have already come far. Hopefully, this book will assist you in mastering the exam syllabus and achieving another milestone on your journey to Fellowship along the SOA's General Insurance Track.

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February 4, 2019

Section 1: Variability of Chain-Ladder Reserve Estimates
Topic 1: Basic Stochastic Reserving

Syllabus Learning Objective Addressed: The candidate will understand how to use basic loss-development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes Addressed: The candidate will be able to
(a) Identify the assumptions underlying the chain-ladder estimation method.
(b) Test for the validity of these assumptions.
(c) Identify alternative models that should be considered depending on the results of the tests.
(d) Estimate the standard deviation of a chain-ladder estimator of unpaid claims.

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Practice Problems

Problem 1-1. What is the objective of Mack's paper, in terms of a response to the fact that the estimated ultimate claim amount can never be known with certainty? (Mack, p. 103)

Problem 1-2. Let $C_{i,k}$ and $C_{i,k+1}$ be the claim amounts for accident year i and development years k and (k+1), respectively. Let f_k be the age-to-age factor for this time period, derived using the chain-ladder method.

(a) Fill in the blanks (Mack, p. 106):
Each increase from $C_{i,k}$ to $C_{i,k+1}$ is considered a ____________ of an expected increase from $C_{i,k}$ to ______, where f_k is an unknown "true" factor of increase which is the same for all accident years and which estimated from the available data.

(b) Using the $C_{i,\_}$ notation, formulate the first assumption of the chain-ladder method, as described by Mack (p. 108). Let I be the year in which all claims have developed to ultimate.

Problem 1-3. (a) Is it reasonable to assume, for the chain-ladder method that the variables $\{C_{i,1}, ..., C_{i,I}\}$ and $\{C_{j,1}, ..., C_{j,I}\}$ for different accident years i and j, are independent?

(b) What, if any, exceptions exist to the assumption in (a)? (Mack, pp. 110-111)

Problem 1-4. (a) Which of the following is an unbiased estimator of the development factor? (i) The weighted-average chain-ladder factor; (ii) The simple-average chain ladder factor. (Mack, p. 112)

(b) Give a mathematical reason to prefer one of the factors in (a) over the other.

(c) State the *proportionality condition* of a chain-ladder estimate. Let α_k be a proportionality constant. (Mack, p. 113)

Problem 1-5. (a) Give the formula for mean square error $MSE(c_{i,I})$ where D is the set of observed data: $D = \{c_{i,k} \mid i + k \leq I + 1\}$. Express the MSE in terms of the random variable $C_{i,I}$, the specific estimated value $c_{i,I}$, and D.

(b) The formula in (a) involves conditionality. Why is the conditionality important here?
(Mack, p. 114)

(c) Reformulate the equation in (a) such that a variance expression is one of the terms.

(d) What does this MSE *not* take into account?

(e) What is the square root of MSE called? (Mack, p. 115)

Problem 1-6.

Let $R_i = C_{i,I} - C_{i,I+1-i}$ be the outstanding claim reserve for accident year i. Let $r_i = c_{i,I} - C_{i,I+1-i}$ be the estimate of the outstanding claim reserve.

(a) What is the MSE of r_i? Give the formula in terms of r_i, R_i, and D.

(b) To what other MSE is the MSE of r_i equal? What is the verbal meaning of this? (Mack, p. 116)

Problem 1-7.

(a) Given the ultimate claim estimate $c_{i,I}$, known claim data points $C_{j,k}$, estimated development factors f_k, and estimators $\hat{\alpha}_k^2$ of the proportionality constants α_k^2, what is the formula for estimating MSE($c_{i,I}$) solely from known data?

(b) In the formula in (a), how is $\acute{\alpha}_k^2$ determined (also solely from known data)? This is itself a rather involved equation.

(c) Give the special formula for the latest of the $\hat{\alpha}_k^2$ estimators: $\hat{\alpha}_{I-1}^2$. Conceptually, why is a special formula needed? (Mack, pp. 116-117)

Problem 1-8.
(a) Give the expression for the symmetric 95%-confidence interval for the reserve R_i.
(b) What distributional assumption may lead the expression in (a) to not reflect reality?
(c) What solution does Mack recommend for the problem in (b)? What useful property does this approach have?
(d) What mathematical formulas are used to obtain the estimates in the solution in part (c)? (Mack, pp. 118-119)

Problem 1-9.

(a) If R_i is the reserve for the accident year i, provide the formula for the overall reserve R of the accident years 1 through I represented in a loss-development triangle.

(b) In order to obtain the variance of R, why is it not possible to simply add the squares of the standard errors of each R_i?

(c) Give the formula for $(se(R))^2$, the square of the standard error of R. (Mack, p. 120)

Problem 1-10.
Mack (pp. 122-124) describes three additional estimators for development factors: $f_{k,0}$ (the $c_{i,k}^2$-weighted average), $f_{k,1}$ (the $c_{i,k}$-weighted average), and $f_{k,2}$ (the ordinary unweighted average). Give formulas for each estimator.

Problem 1-11.
(a) Mack (p. 124) recommends analyzing what plot to check for a linear relationship?
(b) Mack (p. 125) recommends analyzing what three plots to check for random behavior (and to test whether the variance assumption is met)?

Problem 1-12. What is the formula for the weighted residual using Mack's methodology, where $C_{i,k}$ and $C_{i,(k+1)}$ are the cumulative losses for accident year i and maturities k and (k+1), respectively, and f_k is the chain-ladder weighted-average loss-development factor? (The quantity i can range between 1 and I-k, where I is the latest accident year in the loss-development triangle.) (Mack, p. 124)

Problem 1-13. In Mack's test for diagonal effects, what is the formula for n for a given diagonal j? What does each term other than n stand for? (Mack, p. 167)

Problem 1-14. In Mack's test for diagonal effects, what is the formula for m for a given diagonal j, in terms of the quantity n? (Mack, p. 167)

Problem 1-15. In Mack's test for diagonal effects, what is the formula for Z_j for a given diagonal j? What does each term other than Z_j stand for? (Mack, p. 167)

Problem 1-16. (Generalization of SOA Fall 2014 Exam GIADV Question 4(e)) Explain why the variance of the combined chain-ladder reserve estimate for multiple accident years is greater than the sum of the individual variances by accident year.

Problem 1-17. Review. What are the three basic implicit chain-ladder assumptions, as described by Mack (p. 121). (Note that this would be a reasonable exam question.)

Problem 1-18. SOA Spring 2014 Exam GIADV, Questions 3(a), (b), (c), and (d).

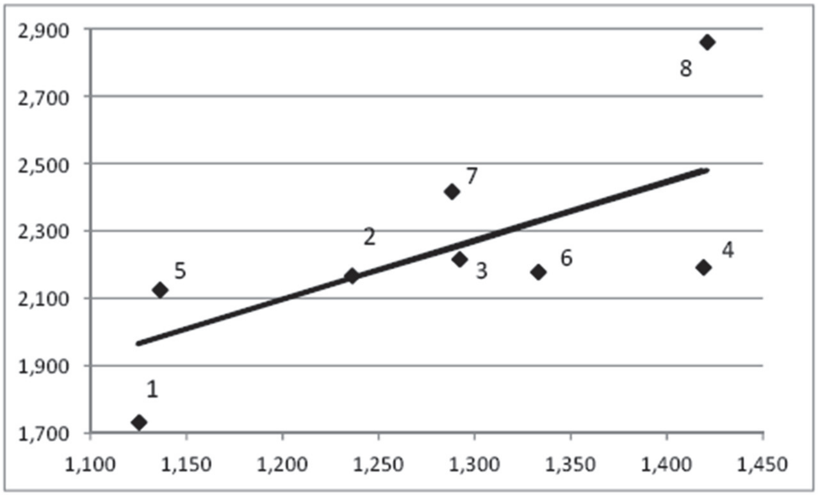
In his paper "Measuring the Variability of Chain Ladder Reserve Estimates," Mack states that there are three statistical assumptions that are implicit in the chain-ladder method. One of the assumptions is that $E(C_{i,k+1} \mid C_{i,1}, \ldots, C_{i,k}) = C_{i,k} * f_k$ for all i and k.

(a) Describe this assumption in words.

(b) Describe a reserving situation in which this assumption may not hold.

(c) Mack suggests a regression test to evaluate this assumption. A plot is constructed for each lag. For a known loss-development triangle, the following plot was made to assess the development from lag 2 to lag 3. The chain-ladder estimate of f_2 is 1.75.

The plot is of $C_{i,3}$ (vertical axis) against $C_{i,2}$ (horizontal axis) with the line y = 1.75x added. The numbers indicate the accident year for that plotted line.



Determine if this plot provides evidence that the assumption $E(C_{i,k+1} \mid C_{i,1}, \ldots, C_{i,k}) = C_{i,k}*f_k$ holds. Support your answer.

(d) Describe, using words and/or formulas as appropriate, the other two statistical assumptions identified by Mack.

Problem 1-19. Based on Problem 3(g) from the Spring 2014 SOA Exam GIADV.

You are evaluating a loss-development triangle, which uses 7 accident years. The following table provides development factors (f_k in Mack's paper) and the variance estimates (α_k^2 in Mack's paper).

k	f_k	α_k^2
1	5.800816178	350.4
2	2.704101342	240.2
3	1.972175009	50.6
4	1.772175009	12.4
5	1.137961477	6.614
6	1.00938884	2.364

You are also given part of the loss-development triangle for the last three maturities. Claims are assumed to be at ultimate levels at Maturity 7. The shaded values are populated using the standard chain-ladder method.

Accident Year	Maturity 5	Maturity 6	Maturity 7
2090	4992	5645	5698
2091	7468	8534	8614.124358
2092	4903	5579.42512	5631.809448

Calculate the variance of the chain-ladder estimate of the reserve for claims from Accident Year 2092.

Problem 1-20. SOA Spring 2014 Exam GIADV, Question 3(g).
You are interested in determining the variability of reserve estimates. The triangle of data you are working with is presented below (AY = accident year). The shaded cells have been completed using the standard chain-ladder method. It is assumed that all claims are fully developed after ten years.

	Development Year									
AY	1	2	3	4	5	6	7	8	9	10
1	358	1,125	1,735	2,218	2,746	3,320	3,466	3,606	3,834	3,901
2	352	1,236	2,170	3,353	3,799	4,120	4,648	4,914	5,339	5,432
3	291	1,292	2,219	3,235	3,986	4,133	4,629	4,909	5,285	5,378
4	311	1,419	2,195	3,757	4,030	4,382	4,588	4,835	5,206	5,297
5	443	1,136	2,128	2,898	3,403	3,873	4,207	4,433	4,773	4,857
6	396	1,333	2,181	2,986	3,692	4,075	4,427	4,665	5,022	5,110
7	441	1,288	2,420	3,483	4,089	4,513	4,902	5,166	5,562	5,659
8	359	1,421	2,864	4,174	4,900	5,408	5,875	6,191	6,665	6,782
9	377	1,363	2,382	3,471	4,075	4,498	4,886	5,149	5,543	5,640
10	344	1,200	2,098	3,057	3,589	3,961	4,303	4,534	4,882	4,967

The following table provides development factors (f_k in Mack's paper) and the variance estimates (α_k^2 in Mack's paper).

k	1	2	3	4	5	6	7	8	9
f_k	3.489	1.748	1.457	1.174	1.104	1.086	1.054	1.077	1.017
α_k^2	159.63	37.79	41.90	15.18	13.69	8.21	0.44	1.13	0.44

Calculate the variance of the chain-ladder estimate of the reserve for claims from accident year 3.

Problem 1-21. Based on Questions 4(a), (b), and (c) from the Fall 2014 SOA Exam GIADV.
You are working with the following development triangle, in which the shaded cells have been completed using the standard chain-ladder method. It is assumed that all claims are fully developed after three years. Mack's method of estimating reserve variability was applied to this triangle, and the results are provided below.

Accident Year	Maturity 1	Maturity 2	Maturity 3	Standard Error
2095	4992	5691	5739	0
2096	8113	9265	9343.144439	17.09134231
2097	5009	5716.490195	5764.705188	14.89769161
f_k	1.1412438	1.00843437		
$(\alpha_k)^2$	0.011997204	0.011997204		

(a) Demonstrate that the value of $(\alpha_1)^2$ was correctly calculated.

(b) Demonstrate that the standard error for Accident Year 2097 was correctly calculated.

(c) Calculate the upper limit of an 85% confidence interval for outstanding claims for Accident Year 2097 using a Normal distribution. The 92.5th percentile of the standard normal distribution is at 1.439531471.

Problem 1-22. SOA Fall 2014 Exam GIADV, Questions 4(a) through (e). You are interested in determining the variability of unpaid claim estimates. The triangle of data you are working with is presented below. The shaded cells have been completed using the standard chain-ladder method. It is assumed that all claims are fully developed after six years.

Mack's method of estimating reserve variability has been applied to this triangle. The key results are provided in the table.

Accident Year	Development Year 1	2	3	4	5	6	Standard Error
1	8,600	12,221	13,221	14,317	14,784	14,815	0
2	8,306	13,049	13,455	13,768	14,034	14,063	50
3	7,709	13,847	15,300	15,619	16,027	16,060	183
4	8,623	14,159	15,096	15,717	16,128	16,161	632
5	8,791	16,224	17,380	18,095	18,568	18,607	888
6	9,021	14,917	15,980	16,638	17,072	17,108	2,080
f_k	1.65362	1.07125	1.04117	1.0261	1.0021		
$(\alpha_k)^2$	250.8709	12.8207	16.8267	1.2412	0.0916		

(a) Demonstrate that the value of $(\alpha_4)^2$ was correctly calculated. (Your calculation need not match all four decimal places.)

(b) Demonstrate that the standard error for accident year 3 was correctly calculated.

(c) Calculate the upper limit of a 95% confidence interval for outstanding claims for accident year 3 using a Normal distribution. The 97.5th percentile of the standard normal distribution is at 1.96.

(d) Propose a method for constructing an improved confidence interval. Justify your proposal.

(e) The total developed claims over the six accident years is 96,815. Explain why the variance of this estimate is greater than the sum of the six variances by accident year.

Problem 1-23. Based on CAS Spring 2011 Exam 7 - Question 3(a). One implicit assumption underlying the chain-ladder loss-development method is independence of accident years. Given the following link ratios, use a 95% confidence interval to test the null hypothesis that the corresponding loss-development triangle does *not* have significant calendar-year effects.

Note: The z-score for a 95% confidence interval is 1.959963985.

Accident Year	12-24 Months	24-36 Months	36-48 Months	48-60 Months
2016	1.705547512	1.26749638	1.204774529	1.0117781
2017	1.846657014	1.549300265	1.307608148	
2018	1.494452488	1.286715547		
2019	0.975201698			

Problem 1-24. SOA Spring 2015 Exam GIADV – Questions 4(a) through (e).

You are interested in determining the variability of unpaid claim estimates. The triangle of data you are working with is presented below. The shaded cells have been completed using the standard chain-ladder method. The missing columns are not needed to respond to the items. It is assumed that all claims are fully developed after 12 years. Mack's method of estimating reserve variability has been applied to this triangle. The key results are provided in the table.

	Development Year						
Accident Year	1	2	3	10	11	12	**Standard Error**
1	17,652	41,350	50,387	82,540	82,090	82,826	0
2	11,532	15,432	17,590	26,642	26,715	26,955	143
3	9,074	20,036	25,951	41,123	40,981	41,348	337
4	8,655	4,996	5,904	9,273	9,241	9,324	227
5	5,451	6,987	9,388	14,812	14,760	14,893	374
6	4,778	4,413	5,446	9,285	9,253	9,336	389
7	6,758	6,281	7,646	12,088	12,046	12,154	783
8	5,041	2,001	2,937	4,948	4,931	4,975	509
9	5,536	3,196	4,540	7,362	7,337	7,403	659
10	5,937	6,109	7,621	12,218	12,176	12,285	1,187
11	4,403	4,635	5,748	9,215	9,183	9,266	1,341
12	8,928	12,151	15,069	24,158	24,075	24,291	12,790
f_k	1.361	1.24015	1.26622	0.99655	1.00897		
$(\alpha_k)^2$	4,020.395	59.847	51.295	1.352	0.575		

(a) Demonstrate that the value of $(\alpha_{10})^2$ was correctly calculated. (Your calculation need not match all four decimal places.)

(b) Demonstrate that the standard error for accident year 3 was correctly calculated.

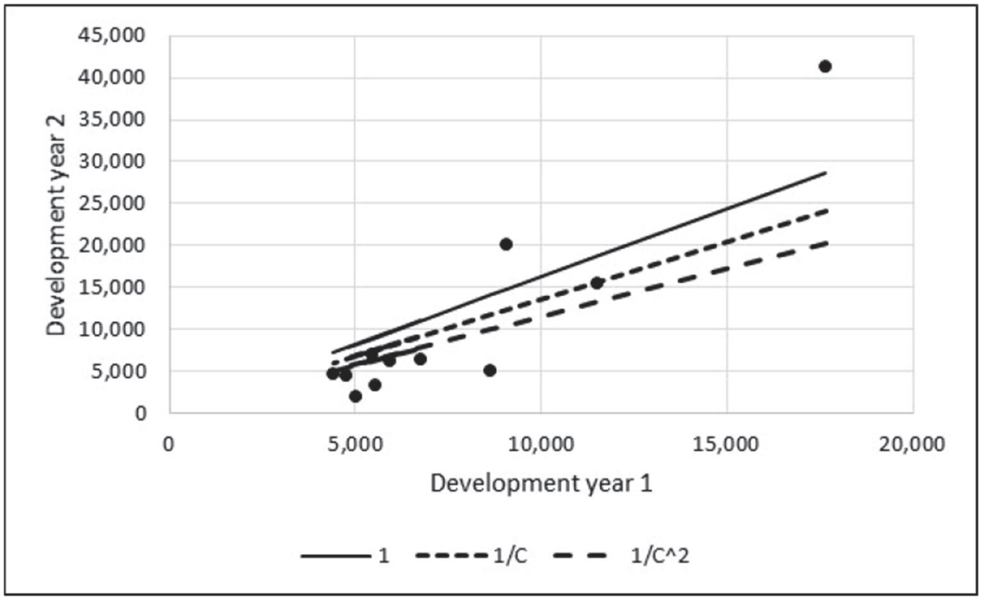
(c) Let $C_{i,k}$ be the cumulative paid claims for accident year *i* and development year *k*. The chain-ladder method estimates $C_{i,k+1}$ as $f_k * C_{i,k}$. Mack notes that this can be viewed as a regression model where the intercept term is forced to be zero. Mack further notes that weighted least squares could be used to derive an estimate of f_k. The weight $1/C_{i,k}$ leads to the standard chain-ladder estimate. The following table displays estimates of f_1 using three different weights.

Weight	1	$1/C_{i,1}$	$1/(C_{i,1})^2$
Estimate of f_1	1.627	1.361	1.151

Explain why the weight $1/C_{i,k}$ is consistent with the variance assumption Mack uses to obtain his standard-error estimate.

(d) State the formula for the age-to-age factor f_1 that results from one of the other two weights. Verify that the calculated number (1.627 or 1.151) is correct using that formula.

(e) Mack further suggests that standard regression plots can be used to determine which, if any, of the three weights produces a reasonable model. In the following plot the points are the values of $(C_{i,1}, C_{i,2})$, and the lines are of the form $y = f_1 * x$ where the value of f_1 is determined using each of the three weights.



Determine, from this graph, which, if any, of the three models is reasonable. Support your answer.

Problem 1-25. SOA Fall 2015 Exam GIADV – Questions 4(a) through (e).

You are interested in determining the variability of unpaid claim estimates.
The triangle of paid claims data you are working with, by accident year (AY) and development year, is presented below. The shaded cells have been completed using the standard chain-ladder method. It is assumed that all claims are fully developed after seven years.

Development Year

Accident Year	1	2	3	4	5	6	7	**Standard Error**
1	12,652	20,548	26,243	30,915	31,365	32,082	32,784	0
2	9,532	12,208	16,229	16,824	16,909	17,223	17,600	6
3	15,074	18,423	25,004	28,617	30,524	31,176	31,858	103
4	27,655	43,895	54,236	58,131	59,990	61,271	62,612	1,775
5	25,451	33,237	35,821	39,581	40,847	41,719	42,632	2,736
6	19,778	22,434	27,543	30,434	31,407	32,078	32,780	4,117
7	16,758	22,936	28,159	31,115	32,110	32,796	33,513	6,939
f_k	1.36864	1.22774	1.10496	1.03198	1.02136	1.02188		
$(\alpha_k)^2$	746.086	308.903	104.826	27.980	0.202	0.00146		

(a) Demonstrate that the value of $(\alpha_4)^2$ was correctly calculated. (Your calculation need not match to all three decimal places.)

(b) Demonstrate that the standard error for accident year 3 was correctly calculated.

(c) One of Mack's assumptions is $E(C_{i,k+1} \mid C_{i,1}, \ldots, C_{i,k}) = C_{i,k} * f_k$. Explain why f_k has only the subscript k and not both i and k.

(d) Mack shows that under his assumptions, $C_{i,k} / C_{i,k-1}$ and $C_{i,k+1} / C_{i,k}$ are uncorrelated. Describe a situation where these ratios may be correlated.

(e) Explain why the formula used to estimate $(\alpha_1)^2$ through $(\alpha_5)^2$ cannot be used to estimate $(\alpha_6)^2$.

Problem 1-26. CAS Spring 2011 Exam 7 – Question 3. One implicit assumption underlying the chain-ladder loss-development method is independence of accident years. Given the following loss-development triangle and link ratios:

Reported Losses ($000)

Accident Year	12 Months	24 Months	36 Months	48 Months	60 Months
2006	3,100	4,185	4,813	5,053	5,104
2007	3,050	3,660	4,282	4,368	
2008	2,800	3,640	4,077		
2009	2,500	3,125			
2010	2,357				

Age-to-Age Loss-Development Factors

Accident Year	12-24 Months	24-36 Months	36-48 Months	48-60 Months
2006	1.35	1.15	1.05	1.01
2007	1.20	1.17	1.02	
2008	1.30	1.12		
2009	1.25			

(a) Use a 95% confidence interval to test the null hypothesis that the corresponding loss-development triangle does *not* have significant calendar-year effects.

Note: The z-score for a 95% confidence interval is approximately 1.96.

(b) Describe two other implicit assumptions underlying the chain-ladder loss-development method.

Problem 1-27. CAS Spring 2012 Exam 7 – Question 3. Given the following information as of December 31, 2011:

Cumulative Incurred Losses ($000)		
Accident Year	**As of 24 Months**	**As of 36 Months**
2004	2,000	2,600
2005	4,000	6,000
2006	6,500	11,700
2007	6,000	10,000
2008	3,600	6,600
2009	7,600	11,000
2010	5,000	

(a) Using a volume-weighted average to calculate the overall age-to-age factor, create a plot of weighted residuals following Mack's methodology.

(b) Based on the residual plot, assess whether the variance assumption has been met.

Problem 1-28. CAS Spring 2012 Exam 7 – Question 5. A loss-reserve actuary has reviewed three cumulative paid loss triangles to test whether the assumptions underlying the chain-ladder method are met.

(a) State the three chain-ladder assumptions as described by Mack.

(b) For each of the following situations, discuss whether any of Mack's assumptions are violated.

i. The first triangle shows a faster claims-settlement pattern in the most recent calendar year, resulting from the use of new claims-management software.

ii. For the second triangle, the actuary found that the most appropriate selection method for loss-development factors was to use an all-year volume-weighted average approach.

iii. In the third triangle, the 36-48-month loss development factors are inversely proportional to the 24-36-month development factors. These relationships do not appear to be random.

Problem 1-29. CAS Spring 2013 Exam 7 – Question 1. Given the following information:

	Age-to-Age Loss-Development Factors				
Accident Year	**12-24 Months**	**24-36 Months**	**36-48 Months**	**48-60 Months**	**60-72 Months**
2007	2.45	1.73	1.19	1.05	1.05
2008	5.42	1.26	1.23	1.09	
2009	2.64	1.35	1.16		
2010	2.04	1.55			
2011	6.23				

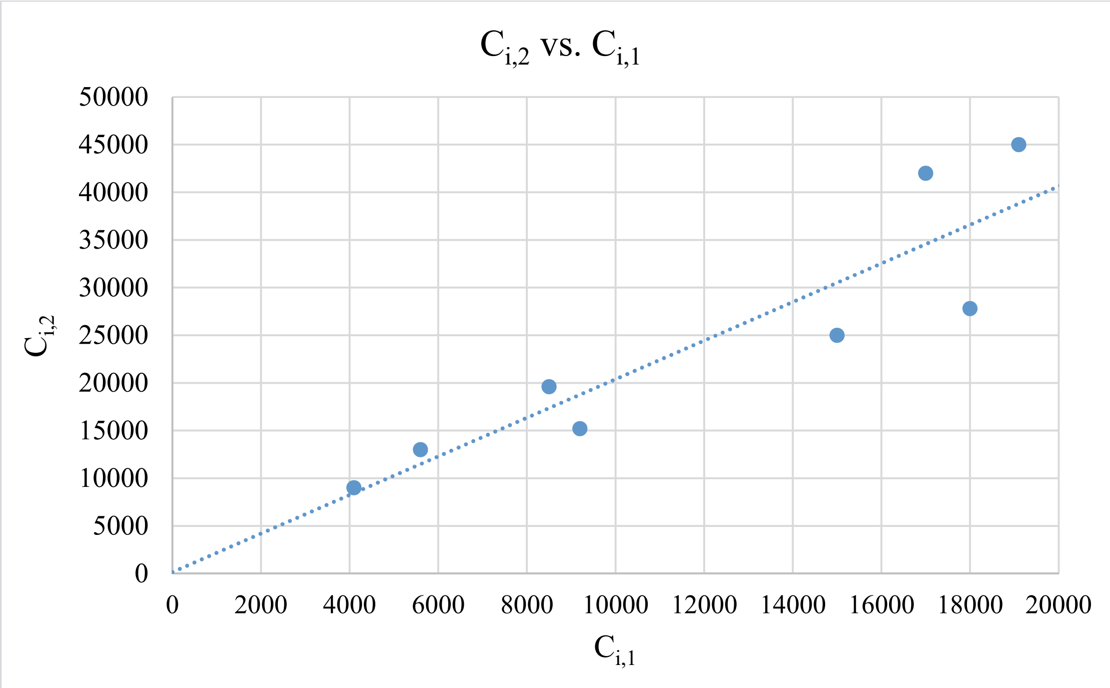
z-value for 90^{th} percentile of the Normal distribution: 1.645

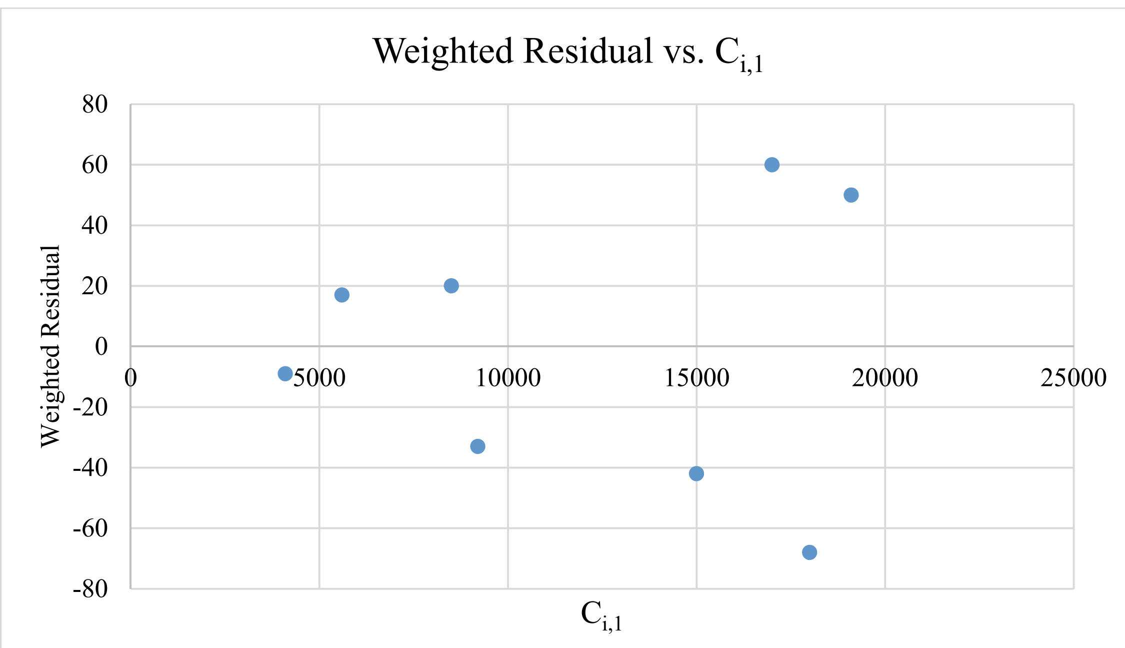
(a) The null hypothesis is that the triangle does not display calendar-year effects. Conduct a test to determine whether the null hypothesis should be accepted or rejected at the 90% confidence level.

(b) Briefly describe two potential causes of calendar-year effects in loss-development data.

Problem 1-30. In Mack's test for calendar-year effects, particular values of n always correspond to particular outputs of E(Z) and Var(Z). Compute E(Z) and Var(Z) for each of n = 1, 2, 3, and 4.

Problem 1-31. CAS Spring 2014 Exam 7, Problem 2. Given the following output from a company's reserving software:





• $C_{i,1}$: Loss evaluated at 12 months for accident year i ($000)

• $C_{i,2}$: Loss evaluated at 24 months for accident year i ($000)

Based on the two charts above, explain whether the chain-ladder method is appropriate for estimating ultimate loss.

Problem 1-32. SOA Spring 2016 Exam GIADV – Questions 4(a) through (c). You are interested in determining the variability of unpaid claim estimates. The triangle of paid claims data you are working with, by accident year (AY) and development year, is presented below. The shaded cells have been completed using the standard chain-ladder method. It is assumed that all claims are fully developed after seven years.

Mack's method of estimating reserve variability has been applied to this triangle. The key results are provided in the table.

	Development Year							
Accident Year	1	2	3	4	5	6	7	**Standard Error**
1	9,791	12,431	13,033	14,212	14,486	14,867	15,155	0
2	11,314	19,266	23,518	27,910	28,117	28,697	29,253	15
3	12,654	14,924	18,489	22,433	24,281	24,829	25,310	111
4	13,305	14,234	15,293	15,900	16,474	16,845	17,172	903
5	14,693	26,298	37,108	42,448	43,980	44,972	45,843	3,208
6	16,037	18,544	22,861	26,151	27,094	27,705	28,242	4,399
7	17,360	23,587	29,077	33,262	34,462	35,239	35,922	9,393
$\mathbf{f_k}$	1.35868	1.23279	1.14392	1.03608	1.02256	1.01937		
$\mathbf{(\alpha_k)^2}$	1,264.53	404.682	111.855	37.514	0.308	0.00252		

(a) Demonstrate that the value of $(\alpha_4)^2$ was correctly calculated. (Your calculation need not match to all three decimal places.)

(b) Demonstrate that the standard error for accident year 3 was correctly calculated.

(c) For a given accident year, it is possible that the value for a given development year will be less than the value for the previous development year.

For each of Mack's three assumptions:

(i) State the assumption; and

(ii) Explain why that assumption does or does not prevent the value from decreasing from one development year to the next.

Problem 1-33. SOA Fall 2016 Exam GIADV – Questions 4(a) through (e). You are interested in determining the variability of unpaid claim estimates. The triangle of paid claims data you are working with, by accident year (AY) and development year, is presented below. The shaded cells have been completed using the standard chain-ladder method. It is assumed that all claims are fully developed after seven years.

Mack's method of estimating reserve variability has been applied to this triangle. The key results are provided in the table.

Development Year

Accident Year	1	2	3	4	5	6	7	**Standard Error**
1	9,659	15,468	17,887	18,236	18,910	19,262	19,644	0
2	10,731	17,668	22,333	24,701	24,827	25,331	25,833	1
3	11,715	11,037	14,503	14,707	16,414	16,735	17,067	27
4	12,450	15,686	19,069	23,888	24,927	25,415	25,919	1,448
5	13,574	15,924	17,706	19,563	20,414	20,814	21,226	2,715
6	14,717	20,165	24,347	26,900	28,070	28,620	29,187	3,782
7	16,100	21,206	25,603	28,289	29,519	30,097	30,694	6,915
$\mathbf{f_k}$	1.31713	1.20737	1.10489	1.04349	1.01957	1.01983		
$\mathbf{(\alpha_k)^2}$	791.093	92.178	222.248	57.324	0.0305	0.000016		

(a) Demonstrate that the value of $(\alpha_5)^2$ was correctly calculated. (Your calculation need not match to all three decimal places.)

(b) Demonstrate that the standard error for accident year 4 was correctly calculated.

(c) Using the Normal approximation, a 95% confidence interval for the accident year 6 ultimate claims is 29,187 ± 1.96*3,782 or (21,774, 36,600). Explain, referring to this example, why using the Normal approximation may not be reasonable.

(d) Recommend an approach that may be superior to using the Normal approximation. Justify your recommendation.

(e) One of Mack's assumptions is $E(C_{i,k+1} \mid C_{i,1}, \ldots, C_{i,k}) = C_{i,k} * f_k$. Mack observes that this is consistent with a regression model with a slope of f_k and an intercept of 0. Mack states that a weighted regression should be used to estimate the slope. Explain why it is necessary to perform a weighted regression.

Problem 1-34. SOA Spring 2017 Exam GIADV – Questions 4(a) through (d)

You are interested in determining the variability of unpaid claim estimates. The triangle of paid claims data you are working with, by accident year (AY) and development year, is presented below. The shaded cells have been completed using the standard chain-ladder method. It is assumed that all claims are fully developed after seven years.

Mack's method of estimating reserve variability has been applied to this triangle. The key results are provided in the table.

Development Year

Accident Year	1	2	3	4	5	6	7	Standard Error
1	20,587	29,243	33,208	35,957	36,328	37,131	37,871	0
2	21,399	23,109	30,971	36,752	38,103	38,877	39,652	2
3	22,259	31,780	42,282	45,157	48,759	49,792	50,784	71
4	23,191	33,060	46,113	48,668	50,866	51,944	52,979	1,936
5	25,065	29,536	38,140	41,630	43,510	44,432	45,317	3,157
6	25,024	40,688	52,885	57,724	60,332	61,610	62,838	6,018
7	25,387	34,597	44,968	49,083	51,300	52,387	53,431	9,745
f_k	1.36278	1.29978	1.09150	1.04517	1.02119	1.01993		
$(\alpha_k)^2$	910.323	289.210	122.133	50.162	0.0596	0.000071		

(a) Demonstrate that the value of $(\alpha_4)^2$ was correctly calculated. (Your calculation need not match to all three decimal places.)

(b) Demonstrate that the standard error for accident year 5 was correctly calculated.

(c) Each of the estimated development factors ($f_1, \ldots, f_6$) is greater than one. Indicate whether or not this observation provides support for the underlying assumptions of Mack's model. Justify your response.

(d) In addition to the estimated development factors being greater than one, the observed paid claims in each row in the table above are increasing. Indicate whether or not this observation provides support for the underlying assumptions of Mack's model. Justify your response.

Problem 1-35. SOA Fall 2017 Exam GIADV, Questions 4(a) through (d)

You are interested in determining the variability of unpaid claim estimates. The triangle of paid claims data you are working with, by accident year (AY) and development year, is presented below. The shaded cells have been completed using the standard chain-ladder method. It is assumed that all claims are fully developed after seven years.

Mack's method of estimating reserve variability has been applied to this triangle. The key results are provided in the table.

Development Year

Accident Year	1	2	3	4	5	6	7	**Standard Error**
1	9,146	12,176	17,670	18,546	18,128	18,517	18,888	0
2	10,834	15,902	20,884	23,304	22,887	23,371	23,839	0.04
3	11,946	15,697	20,478	22,854	20,718	21,159	21,583	5.64
4	12,414	19,333	38,991	42,905	40,935	41,806	42,644	1,761
5	14,284	20,888	25,210	27,675	26,405	26,967	27,507	1,514
6	15,648	17,240	25,293	27,767	26,492	27,056	27,598	7,217
7	17,221	23,473	34,438	37,806	36,070	36,838	37,576	9,765
f_k	1.36304	1.46713	1.09779	0.95408	1.02128	1.02004		
$(\alpha_k)^2$	366.962	2012.50	18.3273	40.0504	0.00098	2.4×10^{-8}		

(a) Demonstrate that the standard error for accident year 4 was correctly calculated.

(b) The formula for the square of the standard error of the overall reserve estimator is a sum taken over accident years 2 through 7. For each accident year, the term is the sum of two components. Calculate the value of the term for accident year 2.

(c) The second component in each term must be positive because the reserve estimators for pairs of accident years are positively correlated. In discussing this formula with your actuarial student, he questions this statement by noting that under Mack's assumptions, future development depends only on current development for that accident year and hence reserve estimators for different accident years are independent. Explain why the estimators are dependent.

(d) Calculate the weighted residual as defined by Mack for the observation at accident year 4 and development year 3.

Problem 1-36. SOA Spring 2018 Exam GIADV, Questions 5(a), 5(b), 5(e), and 5(f)

You are interested in determining the variability of unpaid claim estimates. The triangle of paid claims data you are working with, by accident year (AY) and development year, is presented below. The shaded cells have been completed using the standard chain-ladder method. It is assumed that all claims are fully developed after seven years.

Mack's method of estimating reserve variability has been applied to this triangle. The key results are provided in the table.

Development Year

Accident Year	1	2	3	4	5	6	7	Standard Error
1	9,146	12,176	17,670	18,546	18,128	18,517	18,888	0
2	10,834	15,902	20,884	23,304	22,887	23,371	23,839	0.04
3	11,946	15,697	20,478	22,854	20,718	21,159	21,583	5.64
4	12,414	19,333	38,991	42,905	40,935	41,806	42,644	1,761
5	14,284	20,888	25,210	27,675	26,405	26,967	27,507	1,514
6	15,648	17,240	25,293	27,767	26,492	27,056	27,598	7,217
7	17,221	23,473	34,438	37,806	36,070	36,838	37,576	9,765
f_k	1.36304	1.46713	1.09779	0.95408	1.02128	1.02004		
$(\alpha_k)^2$	366.962	2012.50	18.3273	40.0504	0.00098	2.4×10^{-8}		

Age-to-Age Factors

1	1.3313	1.4512	1.0496	0.9775	1.0215	1.0200
2	1.4678	1.3133	1.1159	0.9821	1.0211	
3	1.3140	1.3046	1.1160	0.9065		
4	1.5574	2.0168	1.1004			
5	1.4623	1.2069				
6	1.1017					

(a) Demonstrate that the value of $(\alpha_4)^2$ was correctly calculated. (Your calculation need not match to all four decimal places.)

(b) Calculate the standard error of the reserve estimator for accident years 4 and 5 combined.

(e) The sample correlation between the first two columns of age-to-age factors is 0.574.

Demonstrate that the test statistic suggested by Mack to test for a calendar-year effect is equal to 1.

(f) Under the null hypothesis that there is no calendar-year effect, the expected value of the test statistic suggested by Mack is 4.875, and the standard deviation of the test statistic is 1.196.

Determine whether there is a significant calendar-year effect and what this indicates about the use of the chain-ladder method in this case.

Problem 1-37. SOA Fall 2018 Exam GIADV, Questions 4(a) through (d).

One of the assumptions of Mack's model of loss development is that $E(C_{i,k+1} \mid C_{i,1}, \ldots, C_{i,k}) = C_{i,k} * f_k$.

(a) Explain whether this implies that observed development factors within a given accident year are uncorrelated, independent, neither, or both.

(b) State Mack's other two assumptions.

(c) Mack proposes a test for correlation between development factors that uses Spearman's rank correlation coefficient.

Demonstrate that the weighted-average test statistic for the triangle below is −0.24.

Development Year

Accident Year	1	2	3	4	5	6	7	Standard Error
1	9,146	12,176	17,670	18,546	18,128	18,517	18,888	0
2	10,834	15,902	20,884	23,304	22,887	23,371	23,839	0.04
3	11,946	15,697	20,478	22,854	20,718	21,159	21,583	5.64
4	12,414	19,333	38,991	42,905	40,935	41,806	42,644	1,761
5	14,284	20,888	25,210	27,675	26,405	26,967	27,507	1,514
6	15,648	17,240	25,293	27,767	26,492	27,056	27,598	7,217
7	17,221	23,473	34,438	37,806	36,070	36,838	37,576	9,765

Age-to-Age Factors

1	1.3313	1.4512	1.0496	0.9775	1.0215	1.0200
2	1.4678	1.3133	1.1159	0.9821	1.0211	
3	1.3140	1.3046	1.1160	0.9065		
4	1.5574	2.0168	1.1004			
5	1.4623	1.2069				
6	1.1017					

(d) State the conclusion that should be drawn from the test. Justify your answer.

Solutions

Solution 1-1. The objective is to enable the construction of *confidence intervals* for the estimated reserves.

Solution 1-2.
(a) Each increase from $C_{i,k}$ to $C_{i,k+1}$ is considered a **random disturbance** of an expected increase from $C_{i,k}$ to $\mathbf{C_{i,k}*f_k}$, where f_k is an unknown "true" factor of increase which is the same for all accident years and which estimated from the available data.

(b) The first assumption of the chain-ladder method is that the information contained in $C_{i,I+1-i}$ in order to project the claims to ultimate cannot be augmented by also using $C_{i,1}$ through $C_{i,I-i}$ or $C_{1,I+1-i}$ through $C_{i-1,I+1-i}$. (That is, past claims for the same accident year or claims of the same maturity for prior accident years are irrelevant. The magnitude of a development factor for a prior maturity should not affect the magnitude of the current estimated development factor.)

Solution 1-3.
(a) Yes. The chain-ladder method explicitly does not take into account any dependency among the accident years.

(b) If there is a change in claim handling, case reserving, or the rate of inflation, this can affect several accident years in the same way and render the assumption of independence dubious.

Solution 1-4. (a) Both (i) and (ii) are unbiased estimators.

(b) The weighted-average factor is preferable because by choosing weights proportional to claim amounts, we minimize the variance of the weighted average.

(c) Proportionality condition: $Var(C_{j,k+1} \mid C_{j,1}, \ldots, C_{j,k}) = \alpha_k^2 * C_{j,k}$.

Solution 1-5. (a) $\mathbf{MSE(c_{i,I}) = E[(C_{i,I} - c_{i,I})^2 \mid D]}$.

(b) The conditionality on D is important because we only want to estimate the error due to *future* random variations. Thus, we assume we already know D, which is the set of past data. Without the conditionality on D, we would be calculating MSE over both the past and the future, which not assist us in predicting the future on the basis of a particular development triangle.

(c) $\mathbf{MSE(c_{i,I}) = Var(C_{i,I} \mid D) + (E(C_{i,I} \mid D) - c_{i,I})^2}$.

(d) The MSE does not take into account future changes in the underlying model, such as the emergence of previously unanticipated claim types.

(e) The square root of MSE is **standard error**.

Solution 1-6.
(a) $MSE(r_i) = E[(r_i - R_i)^2 \mid D]$.

(b) $MSE(r_i) = MSE(c_{i,I})$. This means that the mean square error of the reserve is the same as the mean square error of the ultimate-loss estimate.

Solution 1-7.
(a) $MSE(c_{i,I}) = (c_{i,I})^2 * \sum_{k=I+1-i}^{I-1}[\left(\frac{\hat{\alpha}_k^2}{f_k^2}\right)(\frac{1}{C_{i,k}} + \frac{1}{\sum_{j=1}^{I-k} C_{j,k}})]$.

(b) $\hat{\alpha}_k^2 = (\frac{1}{I-k-1}) * \sum_{j=1}^{I-k}[(C_{j,k})(\frac{C_{j,k+1}}{C_{j,k}} - f_k)^2]$, **for** $1 \le k \le I - 2$.

(c) $\hat{\alpha}_{I-1}^2 = \min[\hat{\alpha}_{I-2}^4/\hat{\alpha}_{I-3}^2, \min(\hat{\alpha}_{I-3}^2, \hat{\alpha}_{I-2}^2)]$. A special formula for this constant is needed, because the estimate is based on only a single observation for development years (I-1) and I: $C_{1,I}/C_{1,I-1}$. This is the LDF calculated using the rightmost top two entries of the loss-development triangle. It is impossible to use the standard formulas to estimate both $\hat{\alpha}_{I-1}$ and f_{I-1} from this observation.

Solution 1-8.

(a) The interval is $(R_i - 2*se(R_i), R_i + 2*se(R_i))$ where $se(R_i) = se(c_{i,I})$ is the standard error of both the reserve and the ultimate-loss estimate.

(b) The expression in (a) relies on a symmetric Normal distribution of the possible reserve amounts. However, the real-world distribution may be skewed and may be highly volatile, with large standard errors. If the standard error exceeds 50% of R_i, then the lower bound of the interval will be negative, which is not always possible in reality.

(c) Mack recommends using a Lognormal distribution. Using a Lognormal distribution prevents negative boundaries for confidence intervals.

(d) The parameters of the Lognormal distribution are μ_i and σ_i^2. Then the estimates are
$R_i = \exp(\mu_i + \sigma_i^2/2)$;
$(se(R_i))^2 = \exp(2\mu_i + \sigma_i^2)*(\exp(\sigma_i^2) - 1)$; and thus
$\sigma_i^2 = \ln(1 + (se(R_i))^2/R_i^2)$; and
$\mu_i = \ln(R_i) - \sigma_i^2/2$.

Solution 1-9. (a) $R = R_2 + \ldots + R_I$. (There is no R_1 term, since, presumably, the latest known value of the claim amount for the earliest displayed accident year is already at ultimate.)

(b) For each i, the estimators R_i are not independent of one another. They are positively correlated, because they are all influenced by the same age-to-age factors f_k.

(c) $(se(R))^2 = \sum_{i=2}^{I}[se(R_i)^2 + c_{i,I}(\sum_{j=i+1}^{I} c_{j,I})(\sum_{k=I+1-i}^{I-1} \frac{[2\alpha_k^2/f_k^2]}{\sum_{n=1}^{I-k} C_{n,k}})]$.

Solution 1-10.

$f_{k,0} = {}_{i=1}^{I-k}\sum(C_{i,k}*C_{i,k+1})/({}_{i=1}^{I-k}\sum[C_{i,k}^2])$.

$\mathbf{f_{k,1} = ({}_{i=1}{}^{I-k}\sum[C_{i,k+1}])/({}_{i=1}{}^{I-k}\sum[C_{i,k}])}$.
(Note: This is the same as the weighted-average chain-ladder factor f_k.)

$\mathbf{f_{k,2} = (1/[I-k])({}_{i=1}{}^{I-k}\sum[C_{i,k+1}/C_{i,k}])}$. (Note: This is the straight-average chain-ladder factor.)

Solution 1-11. (a) To check for a linear relationship, analyze the plot of $\mathbf{C_{i,k+1}}$ **against** $\mathbf{C_{i,k}}$.

(b) To test the variance assumptions, analyze the following plots:
(i) For $f_{k,0}$: $\mathbf{C_{i,k+1} - C_{i,k}*f_{k,0}}$ **against** $\mathbf{C_{i,k}}$.
(ii) For $f_{k,1}$: $\mathbf{(C_{i,k+1} - C_{i,k}*f_{k,1})/\sqrt{(C_{i,k})}}$ **against** $\mathbf{C_{i,k}}$.
(iii) For $f_{k,2}$: $\mathbf{(C_{i,k+1} - C_{i,k}*f_{k,2})/(C_{i,k})}$ **against** $\mathbf{C_{i,k}}$.

Solution 1-12. Weighted residual = $(C_{i,(k+1)} - f_k*C_{i,k})/\sqrt{(C_{i,k})}$.

Solution 1-13. $n = S_j + L_j$.

For each diagonal j, S_j is the number of entries smaller than the median of their respective columns. L_j is the number of entries greater than the median of their respective columns.

Solution 1-14. m = floor[(n-1)/2] (i.e., the closest integer less than or equal to (n-1)/2).

Solution 1-15. $Z_j = \min(S_j, L_j)$.
For each diagonal j, S_j is the number of entries smaller than the median of their respective columns. L_j is the number of entries greater than the median of their respective columns.

Solution 1-16. The individual reserve amounts by accident year are *not* independent. They are positively correlated due to the use of the same development factors f_k and therefore will have positive covariances which lead to the combined reserve estimate's variance exceeding the sum of the individual accident-year reserves' variances.

Solution 1-17.
Assumption 1. $\mathbf{E(C_{i,k+1} \mid C_{i,1}, \ldots, C_{i,k}) = C_{i,k}*f_k}$. (Given observed data, the expected value of the next development period's claims is the current development period's claims, multiplied by the "true" development factor f_k.)

Assumption 2. **Independence of accident years**.

Assumption 3. $\mathbf{Var(C_{i,k+1} \mid C_{i,1}, \ldots, C_{i,k}) = \alpha_k^2*C_{i,k}}$ (Proportionality condition)

Solution 1-18.
(a) Possibility 1: Given observed data, the expected value of the next development period's cumulative claims is the current development period's cumulative claims, multiplied by the "true" development factor f_k which does not vary by accident year.

Possibility 2 (from SOA GIADV Spring 2014 Solutions): The expected cumulative claims at lag (k+1) are proportional to those at lag k. The constant of proportionality depends only on the lag, and not on the accident year. (*Other phrasings would be acceptable, as long as they address the concepts mentioned above.*)

(b) Possibility 1 (from SOA GIADV Spring 2014 Solutions): A change in underwriting or marketing practices in one accident year can change the rate at which claims develop, and so the factor will not be the same over all accident years.

Possibility 2 (from SOA GIADV Spring 2014 Solutions): A change in the claims-handling process can have a calendar-year effect, with the factor for a given lag depending on how the accident year plus lag relates to the year of the change.

(c) The assumption $E(C_{i,k+1} \mid C_{i,1}, \ldots, C_{i,k}) = C_{i,k} * f_k$ appears to hold. Two key observations are:
1. The points seem to be randomly distributed about the line y = 1.75x, suggesting that a linear relationship adequately describes the observations.
2. There is no pattern of deviations by accident year. (For instance, earlier accident years are not consistently above or below the line, nor are later accident years.)

(d) Assumption 2: Independence of accident years.
Assumption 3: $Var(C_{i,k+1} \mid C_{i,1}, \ldots, C_{i,k}) = \alpha_k^2 * C_{j,k}$ (Proportionality condition)
Verbal description (from SOA GIADV Spring 2014 Solutions): The variance of cumulative claims at lag (k+1) is proportional to cumulative claims at lag k, with the constant depending only on the lag.