TABLE OF CONTENTS

VOLUME I

Portfolio Theory And Equilibrium In Capital Markets

1.	Bodie 6	"Risk Aversion and Capital Allocation to Risky Assets"	1
2.	Bodie 7	"Optimal Risky Portfolios"	23
3.	Bodie 8	"Index Models"	73
4.	Bodie 9	"The Capital Asset Pricing Model"	91
5.	Bodie 10	"Arbitrage Pricing Theory and Multifactor Models of Risk and Return"	129
6.	Bodie 11	"The Efficient Market Hypothesis"	149
7.	Bodie 12	"Behavioral Finance and Technical Analysis"	171

Asset-Liability Management

8.	Bodie 15	"The Term Structure of Interest Rates"	181
9.	Bodie 16	"Managing Bond Portfolios"	217
10.	Panning	"Managing Interest Rate Risk: ALM, Franchise Value, and Strategy"	277

VOLUME II

Financial Risk Management

11.	Bodie 14	"Bond Prices and Yields"	291
12.	Coval	"The Economics of Structured Finance"	303
13.	Cummins CAT	"CAT Bonds and Other Risk-Linked Securities"	319
14.	Bodoff	"Capital Allocation by Percentile Layer"	333
15.	Butsic	"Solvency Measurement for P-L Risk-Based Capital Applications"	341
16.	Cummins Capital	"Allocation of Capital in the Insurance Industry"	367
17.	Goldfarb	"Risk-Adjusted Performance Measurement for P&C Insurers"	389

Rate Of Return, Risk Loads, And Contingency Provision

18.	Feldblum Financial	"Pricing Insurance Policies: The Internal Rate of Return Model"	423
19.	Ferrari	"Relationship of Underwriting, Investment, Leverage, and Exposure"	471
20.	McClenahan	"Insurance Profitability"	503
21.	Robbin IRR	"IRR, ROE, and PVI/PVE"	515
22.	Robbin UW	"The Underwriting Profit Provision"	527
23.	Kreps Ratios	"Riskiness Leverage Models"	565
24.	Mango	"An Application of Game Theory"	573

NOTES

I have updated the 2016 manual to reflect the 2017 syllabus material and the relevant material from the 2016 CAS Exam 8. I moved problems from the material no longer on the syllabus to other readings when appropriate. This is my third manual update from that originally written by Peter J. Murzda, Jr., FCAS, ASA and updated previously by Chris Van. Kooten, FCAS, FCIA.

Questions and parts of some solutions have been taken from material copyrighted by the Casualty Actuarial Society. They are reproduced in this study manual with the permission of the CAS solely to aid students studying for CAS exams. Students may also request past exams directly from the CAS or find them on the CAS website. I am very grateful to the CAS for its cooperation and permission to use this material. The CAS is not responsible for the structure or accuracy of this manual. In some cases, questions and answers have been edited or altered to be more accurate, reflect syllabus changes, or provide a better organized manual. Students should keep in mind that there may be more than one correct way to answer a question even if only one is shown.

Exam questions are identified by numbers in parentheses at the end of each question. Questions have four numbers separated by hyphens: the year of the exam, the number of the exam, the number of the question, and the points assigned. MC indicates that a multiple choice question has been converted into a true/false question.

Page numbers (p.) with solutions refer to the reading to which the question has been assigned unless otherwise noted.

I have made a conscientious effort to eliminate mistakes and incorrect answers, but a few may remain. I am grateful to students who previously pointed out errors and encourage those who find others to bring them to my attention. Please check the ACTEX website for corrections subsequent to publication.

Margaret Tiller Sherwood
FCAS, FSA, MAAA, FCA, CPCU, ARM, ERMP, CERA
January 2017

Bodie Kane Marcus

***Investments*, Tenth Edition: Chapter 6**
Risk Aversion and Capital Allocation of Risky Assets

OUTLINE

I. Notation

A.	$R(r)$	Expected return
B.	σ^2	Variance of returns
C.	U	Utility value
D.	A	Index of investor's risk aversion
E.	P	Portfolio of risky assets
F.	r_P	Risky rate of return of P
G.	$E(r_P)$	Expected rate of return of P
H.	σ_P	Standard deviation of P
I.	F	Risk-free asset
J.	r_f	Risk-free return
K.	r_f^B	Rate charged to borrow
L.	E	Proportion of risky portfolio in equities
M.	B	Proportion of risky portfolio in bonds
N.	C	Complete portfolio
O.	r_C	Rate of return of C
P.	y	Proportion of C that is made up of the risky portfolio
Q.	y^*	Optimal allocation to the risky portfolio
R.	CAL	Capital allocation line
S.	S	Reward to volatility ratio = slope of the CAL

II. Risk and Risk Aversion

A. Risk, Speculation and Gambling

1. Speculation – assumption of considerable investment risk to obtain commensurate gain
   a. Considerable risk – risk sufficient to affect decision
   b. Commensurate gain – positive risk premium
2. Gamble – bet or wager on an uncertain outcome (no positive risk premium)
3. Fair game – risky investment with 0 risk premium

B. Risk Aversion and Utility Values

1. History shows that investors are risk averse and require a risk premium to compensate for risk taken
2. Risk averse investors penalize expected returns for the risk involved
3. Choose among risky portfolios by assigning a utility score $U = E(r) - \frac{1}{2}A\sigma^2$, ($E(r)$ written as a decimal)
   a. Risk averse investors – $A > 0$
   b. Risk neutral investors – $A = 0$ (no penalty for risk)
   c. Risk lover – $A < 0$
4. Utility score can be interpreted as a certainty equivalent rate of return
   a. Investment is desirable only if certainty equivalent return exceeds the risk-free rate

5. Mean-Variance Criterion:
   a. Portfolio A dominates B if $E(r_A) \geq E(r_B)$ and $\sigma_A \leq \sigma_B$ with at least one inequality strict

6. Indifference curve – connects points in the mean-standard deviation plane with the same utility values
   a. Investor has no preference amongst portfolios on the same indifference curve

C. Estimating Risk Aversion

1. Questionnaires
2. History of portfolio composition changes over time
3. Tracking of groups of individuals to determine averages

III. Capital Allocation Across Risky and Risk-Free Portfolios

A. Broad asset allocation between investment classes is most important part of portfolio construction

1. Treasury bills – low risk
2. Long-term bonds – moderate risk
3. Stocks – higher risk

B. First task is to determine allocation between a generic risky portfolio and the risk-free asset

1. The composition of the risky portfolio doesn't change with the amount invested in the risky portfolio, y

   a. Treat the risky portfolio as a single fund holding both bonds and equities in fixed proportion

IV. The Risk-Free Asset

A. Treasury bills are viewed as the most risk-free asset

1. Not truly risk-free in real terms, but government issued debt has default free guarantees that makes it the closest thing
2. The short-term nature makes it insensitive to interest rate fluctuations (can simply hold to maturity to lock in return)

B. Money market funds are also close to risk-free due to very low default rates and short durations

1. Often comprised of the following:

   a. Treasury bills

   b. Bank certificates of deposit (CDs)

   c. Commercial paper (CP)

V. Portfolios of One Risky Asset and a Risk-Free Asset

A. Expectations can be used to determine the rate of return of the complete portfolio, C

$$E(r_C) = r_f + y\left[E(r_P) - r_f\right] \qquad \sigma_C = y\sigma_P$$

B. The equations can be rearranged to define the return standard deviation trade-off:

$$E(r_C) = r_f + \sigma_C\left(\frac{\left[E(r_P) - r_f\right]}{\sigma_P}\right) = r_f + \sigma_C S$$

1. This is the equation of the line (**Capital Allocation Line**) passing through the risk-free portfolio and the risky portfolio in the return-variability plane
2. The slope of the line (**reward-to-volatility ratio a.k.a. Sharpe Ratio**) is given by the portion in brackets and provides the increased return for each additional unit of risk

C. If the investor can borrow at the risk-free rate it is possible to have values beyond the risky portfolio P along the CAL (same slope)

1. In reality only governments can borrow at the risk-free rate and other investors must borrow at a higher rate

a. In this case the CAL kinks at P, where the slope changes to: $\dfrac{\left[E(r_P) - r_f^B\right]}{\sigma_P}$

b. Borrowing is typically done on margin with a broker

i. Margin purchases cannot exceed 50% of the purchase value

ii. Purchased securities must be maintained in a margin account and if the value declines below a maintenance margin a deposit is required through a margin call

VI. Risk Tolerance and Asset Allocation

A. The formulas for utility (U) and expected return $\left[E(r_C)\right]$ and variance (σ_C) of the complete portfolio can be combined to determine the optimal allocation y^*

$$U = E(r) - \frac{1}{2} \text{ x } A \text{ x } \sigma^2$$

1. Utility is maximized by setting the first derivative with respect to y equal to 0 and solving:

$$y^* = \frac{\left[E(r_P) - r_f\right]}{A\sigma_P^2}$$

2. Optimal solution is directly proportion to the risky asset risk premium and inversely proportional to risk aversion

B. Utility curve analysis can be used to validate the optimal portfolio

1. A utility curve is a curve in the return-variability plane that describes all possible return-variability combinations having the same utility value

2. The indifference curve shows the expected return ($E(r)$) compared to the risk (σ^2)

3. Investors will always a prefer a higher indifference curve since for any given variability a higher curve corresponds to increased utility

4. The highest indifference curve that is tangent to the CAL will be tangent at y^*

C. Analysis thus far has considered returns to be normally distributed, which is frequently not the case

VII. Passive Strategies: The Capital Market Line

A. Portfolio decisions that avoid any security analysis are known as passive strategies
   1. Investing in value-weighted indices is the most common passive strategy e.g. S&P 500
   2. This may be the reasonable strategy for many investors
      a. Cost to pursue an active strategy is not zero
      b. There is a free-rider benefit from following active investors who keep assets fairly priced by their buying and selling activities
   3. Passive index funds have outperformed most actively managed funds historically
   4. Passive investors allocate their investments according to their degree of risk aversion

B. The CAL that uses treasury bills as the risk-free asset and a broad market index (or proxy) for the risky asset is known as the **capital market line**

VIII. Appendix A: Risk Aversion, Expected Utility, and the St. Petersburg Paradox

A. The appendix tries to show that investors are risk averse

   1. The St. Petersburg Paradox considers a coin toss game that has infinite expected returns, but most would only pay a small entry fee to partake

   2. What needs to be considered is the utility gained or lost from partaking in such a game
      a. The utility of an extra dollar varies inversely with wealth, i.e., the more wealth, the less utility of another dollar.
      b. Rather than looking at the expected return in a game that might be 'fair,' consider the expected utility
      c. If the expected utility of the outcome is less than the utility of the wager, a risk averse investor will reject it
      d. The certainty equivalent value of the wager can be computed by determining the wealth that corresponds to the expected utility of the wager

   3. Can use a log utility function to describe an investor's preferences.

IX. Appendix B: Utility Functions and Equilibrium Prices of Insurance Contracts

A. A dollar in bad times (when wealth is low) is more valuable than a solar in good times (when wealth is high)
B. The equilibrium value of a dollar in the low economy would be higher than the value of a dollar when the economy performs better
C. Riskier bonds are sold at lower prices than safer ones
D. Riskier stocks historically have provided higher rates of return on long periods of time

X. Appendix C: The Kelly Criterion

A. Consider a sequence of identical one-period investments, each with two positive payoffs:
- A positive excess return b with probability p
- A negative excess return $-a$ greater than zero with a probability q=1-p

B. Kelly thought this was a basic form of a capital allocation problem and used a log utility function to evaluate it

C. The expected utility of the prospect per dollar of initial wealth is
$$E\left[U\left(y\right)\right] = p\,\ln\left(1+r+yb\right)+q\,\ln\left(1=r-ay\right)$$

D. The Kelly Criterion maximizes the expected utility
$$y=\left(1-r\right)\left(\frac{p}{a}-\frac{q}{b}\right)$$
1. The fraction of total wealth invested in the risky investment is independent of wealth
2. Invest more when p and b are large and less when q and a are large
3. When gains are losses are equal ($a = b$), the larger the win/loss spread, the smaller the fraction invested because $y=(1+r)(p-q)/a$

E. Properties of the Kelly Criterion

1. It never risks ruin
2. The probability that it will outperform any other strategy goes to 1 as the investment horizon goes to infinity
3. The optimal strategy is the same regardless of the investment horizon
4. The strategy has the shortest expected time to a specific wealth goal

PAST CAS EXAMINATION QUESTIONS

A. Portfolios of One Risky Asset and a Risk–Free Asset

A1. Assume the expected market return is 15%, the Treasury bill rate is 6%, and that you can borrow at the risk-free rate. Further assume that you invest all your own money in the market portfolio, as well as half that amount of borrowed money. The expected return on your investment is:

A. ≤ 14% B. ≥ 14.1% but ≤ 16.0% C. ≥ 16.1% but ≤ 18.0% D. ≥ 18.1% but ≤ 20.0% E. ≥ 20.0% (90–5B–65–1)

A2. Assume the expected market return is 15% and the Treasury bill rate is 6%. Further assume that you invest 75% of your own money in the market portfolio, and lend the remaining 25% at a risk-free rate of interest. The expected risk premium on your investment is:

A. < 7.0% B. ≥ 7.0% but < 8.0% C. ≥ 8.0% but < 9.0% D. ≥ 9.0% but < 10.0% E. ≥ 10.0% (91–5B–54–1)

A3. You have $100 to invest and can borrow or lend money at the risk-free rate of 4%. The market portfolio offers an expected return of 12% with a standard deviation of 20%.

a. Combining borrowing, lending, and/or investing in the market portfolio, how can you construct a portfolio to achieve an expected return on your investments of 18%? Show all work.
b. What is the standard deviation of your portfolio? Show all work. (96F–5B–30–1/.5)

A4. Given the following regarding a risky portfolio (P) and the risk-free asset, calculate the slope of the capital allocation line for $\sigma < \sigma_P$.

$E(r_P) = 12\%$ $\sigma_P = 10\%$ $r_f = 5\%$

A. .05 B.07 C.10 D.50 E.70 (97–220–38)

A5. Company X invests all of its money in common stock portfolio A with an expected return and expected variance equal to that of the general market. Portfolio A is perfectly positively correlated with the market. The risk-free interest rate is 5% and the expected market risk premium is 8%. The standard deviation of returns of portfolio A is 20%. Calculate the expected return and standard deviation of company X's investments if the following investment strategies are implemented. Show all work.

a. Invest half of the money in portfolio A and half of the money in risk-free securities.
b. Borrow an amount equal to half of the company's current wealth at the risk-free rate and invest everything into portfolio A. (98F–5B–27–1/1)

A6. According to Bodie et al., each investor is content to put money into two benchmark investments.

a. Describe these two investments. Use a diagram to depict the investment strategy.
b. Explain how an investor can vary his/her expected rate of return and commensurate risk. (99S–5B–21–1/1)

A7. Given the following with respect to an optimal risky portfolio, calculate the slope of the capital allocation line (CAL) for this portfolio.

Expected return	7%	Risk premium	3%
Variance	100		

A. .03 B.04 C.30 D.40 E. 3.00 (03–6–1)

A1. r_C = (Percentage Invested in Market)$E(r_m)$ − (Percentage Borrowed)r_f
r_C = (1.5)(15%) − (.5)(6%) = 19.5%, pp. 172–73.

Answer: D

A2. Expected Investment Risk Premium = (Percentage Invested in Market)$(r_M - r_f)$
EIRP = (.75)(15% – 6%) = 6.75%, p. 171.

Answer: A

A3. a. r_C = (Percentage Invested in Market)$E(r_M)$ − (Percentage Borrowed)r_f

.18 = (1 + x)(.12) − x(.04) x = .75

Amount Borrowed = (.75)(Original Investment) = (.75)(100) = 75, pp. 172–73.

b. σ = (Percentage Invested in Market)(σ_M) = (1.75)(20%) = 35%, p. 171.

A4. Slope = $\frac{E(r_P) - r_f}{\sigma_P} = \frac{.12 - .05}{.10}$ = .7, p. 172.

Answer: E

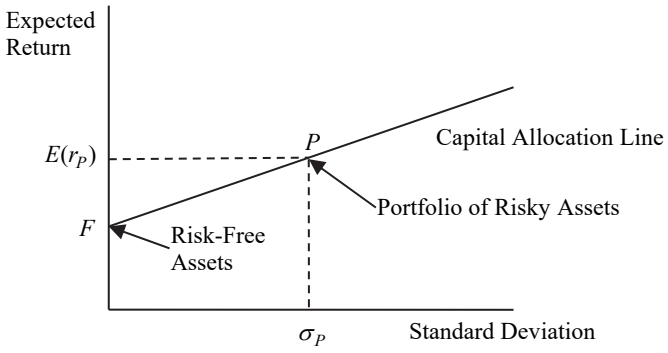
A5. a. $r_A = E(r_M) = r_f$ + Risk Premium = .05 + .08 = .13

$E(r_C) = [E(r_A) + r_f)]/2 = (.13 + .05)/2 = .09$ $\sigma_P = (.5)(\sigma_A) = (.5)(.20) = .10$, p. 171.

b. $E(r_C)$ = (Percentage Invested in A)$E(r_A)$ − (Percentage Borrowed)(r_f)
$E(r_C)$ = (1.5)(13%) − (.5)(5%) = 17%

σ_C = (Percentage Invested in A)(σ_A) = (1.5)(20%) = 30%, pp. 171–73.

A6. a. 1) Risk-free assets, i.e., Treasury bills
2) Portfolio of risky assets, i.e., the market portfolio



b. By changing the proportions of risk-free assets and the portfolio of risky assets in his portfolio, an investor can vary his expected return and associated risk. For example, if the proportion invested in risk-free assets is increased, his complete portfolio moves down the capital allocation line, reducing his expected return and lowering risk, p. 172.

A7. Slope = $\frac{E(r_P) - r_f}{\sigma_P} = \frac{3}{\sqrt{100}}$ = .3, p. 172.

Answer: C

Bodie Kane Marcus

Investments, Tenth Edition: Chapter 9
The Capital Asset Pricing Model

OUTLINE

I. Introduction

A. The Capital Asset Pricing Model (CAPM) is a well-known benchmark for evaluating potential investments

1. Provides insight into the 'fair' rate of return of a security given it's risk
2. Can also be used to evaluate returns on potential projects

B. Variations of the CAPM are also examined

II. Notation

A.	M	Market portfolio
B.	CML	Capital market line
C.	CAL	Capital allocation line
D.	y	Proportion an individual investor allocates to the optimum portfolio M
E.	σ_M^2	Variance of the market portfolio
F.	$\overline{A}$	Average degree of risk aversion across all investors
G.	SML	Security market line
H.	α	Difference between fair and expected rate of return on a stock
I.	R_M	Return on the market index
J.	$NEER$	New expected estimator of expected return
K.	P_H	Value of aggregate human capital
L.	P_M	Market value of traded assets (market portfolio)
M.	R_H	Excess rate of return on aggregate human capital
N.	C	Consumption-tracking (consumption-mimicking) portfolio
O.	RP_C	Risk premium associated with consumption uncertainty

III. The Capital Asset Pricing Model

A. Represents a step forward from the Markowitz model

1. Assumes all investors have an identical investable universe and used the same input list to draw their efficient frontiers
2. All investors hold portfolio of risky assets that duplicates assets in the market portfolio M
3. Market portfolio is in the efficient frontier and represents the tangency point of the optimal capital allocation line (CML is optimal CAL)

B. Why do all investors hold the market portfolio?

1. This is really a function of all the assumptions that go into the CAPM – all investors act in the same way so arrive at the same portfolio
2. The portfolio they arrive at has to be the market portfolio because they all hold identical securities in identical proportions
3. All assets are included in the market portfolio in proportion to their value relative to the entire portfolio
4. All assets will be included albeit at different prices – if a stock becomes undesirable the price will decline until investors demand it once again

C. The passive strategy is efficient

1. Since there is a common input list in determining the market portfolio all relevant information is included and the market portfolio is efficient (**mutual fund theorem**)
2. This indicates that for many investors there is no need to do any security analysis (not true if nobody did any market analysis)
3. In reality different investors will come up with different input lists and thus develop risky portfolios that differ from the market portfolio
4. If the passive strategy is efficient, attempts to beat it generate trading and research costs and end up with inferior results

D. The risk premium of the market portfolio

1. Each investor chooses a proportion y to allocation to the optimum portfolio M

$$y = \frac{E(r_M) - r_f}{A\sigma_M^2}, \text{ from chapter 6, and } E(r_M) - r_f = E(R_M)$$

2. Since all borrowing on risk-free assets must be offset by lending, the average position in the risky portfolio will be $\bar{y} = 1$
3. Substitute $\bar{y}$ for y and the $\bar{A}$ for A provides the desired result

$$E(R_M) = \bar{A}\sigma_M^2$$

E. Expected returns on individual securities

1. The expected return on a portfolio is commensurate to the risk that it adds to the portfolio
2. The risk added is given by the terms of the covariance matrix multiplication that pertain to that particular stock
3. It can be shown that the sum of all the covariance terms can be simplified to the covariance of the stock with the market portfolio
4. The particular stock's contribution to variance is given by the weight of the stock in the portfolio times the covariance with the market portfolio:

$$i\text{'s contribution to variance} = w_i Cov(r_i, r_M)$$

5. The reward to risk ratio for security i in the portfolio is then given by:

$$\frac{w_i\left[E(r_i) - r_f\right]}{w_i Cov(r_i, r_M)} = \frac{E(R_i)}{Cov(r_i R_M)}$$

6. The reward to risk ratio for the investment in the market portfolio is often called the **market price of risk**

$$\frac{E(r_M)-r_f}{\sigma_M^2}=\frac{E(R_M)}{\sigma_M^2}$$

7. Since all investments must offer the same reward to risk ratio the two equations can be equated and rearranged to get the expected return of portfolio i

$$E(r_i)=r_f+\beta_i\left[E(r_M)-r_f\right]$$

This is the **expected return-beta** (or **mean-beta**) **relationship**

8. Beta measures how the returns of a stock move with the market:

$$\beta_i=\frac{Cov(r_i,r_M)}{\sigma_M^2}$$

9. $\beta_M=1$

F. The security market line

1. The security market line graphs the return-beta relationship
   a. y-intercept is at the risk-free rate r_f
   b. The slope of the line is the risk-premium of the market portfolio
2. Comparison to CML
   a. CML graphs efficient portfolios as a function of portfolio standard deviation
   b. SML graphs individual asset premiums as a function of asset risk (as measured by beta – contribution of asset to portfolio variance)
3. SML is valid for efficient portfolios and individual assets
   a. Provides benchmark rate of return for a given beta
4. In equilibrium, all assets plot on the SML
5. SML can be applied to real-world situations as a benchmark
   a. In reality individual investors will have differing views on the expected return of an asset – if this expected return places the security above the SML it is underpriced and a good purchase
      i. The difference between the investors expected return and the return indicated by the SML is the securities **alpha**, α
   b. Can provide the required rate of return for a project for it to be accepted by investors.

G. What if the markets are wrong?

1. CAPM's ideas
   a. Investors can eliminate some risks by diversifying across many regions and sectors
   b. Some risks cannot be eliminated by diversification
   c. People must be rewarded for investing in riskier investments by earning more than they can in safer assets
   d. The rewards on a specific investment depend only on the extent it affects the risks on all an investor's investments
   e. That contribution is "beta"

2. In practice beta is not much use for explaining rates of returns on firms' shares
3. Ratio of book value (the value of assets at the time they enter the balance sheet) to market value workers better
4. If incorporate the book-to-market effect into the **hurdle rate** (minimum accepted rate of return to invest in a security), the alternative hurdle rates is called the **new estimator of expected return**, or NEER
5. If investors are rational, usually use NEER and if investors are irrational, usually use beta

H. The CAPM and the Index Model

a. Index model can be used to take realized returns to expected returns (demonstrated in chapter 8):

$$R_i = \alpha_i + \beta_i R_M + e_i$$

$$E(R_i) = \alpha_i + \beta_i E(R_M)$$

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_i^2$$

b. To increase the risk premium of an individual investor's portfolio

1. Diversify nonsystematic risk
2. Choose stocks with a positive α or take a short position

IV. Assumptions and Extensions of the CAPM

A. Types of Model Testing

1. Normative tests examine the assumptions of the model – if valid and development is error-free the predictions will be accurate

   a. Very few models can pass this test and CAPM isn't one of them
   b. Assumptions are knowingly untrue, which is required to make the model solvable
   c. Need assumptions that if violated do not drastically alter the outcomes (robust)

2. Positive tests examine the predictions and are really tests of the robustness of the model to its assumptions

B. Assumptions of CAPM

1. Individual behavior
   a. All investors are rational, mean-variance optimizers (follow Markowitz portfolio selection)
   b. All investors plan for one identical holding period
   c. All investors analyze securities in the same way (homogenous expectations)
2. Market structure
   a. All assets are publicly held and trade on public exchanges, short positions are allowed, and investors can borrow or lend at a common risk-free rate
   b. All information is publicly available
   c. No taxes
   d. No transactions costs

C. Challenges and Extensions to the CAPM

1. Short positions are not as easy to take as long ones
   a. A large short position requires large collateral
   b. There is a limited supply of shares to be borrowed by short sellers
   c. Many investment companies are prohibited from short sales
   d. Many countries further restrict short sales by regulation
2. A **bubble** occurs when
   a. Investors exhibit irrational exuberance about an asset and prices of it rise
   b. Rational investors take short positions, which holds down the price
   c. With effective restrictions, short sales fail to prevent prices rising to unsustainable levels
   d. There is market correction or market crash
3. Unrealistic assumptions in CAPM
   a. All assets trade
   b. No transaction costs
   c. There is a single period horizon

D. The Zero-Beta Model

1. Efficient frontier portfolios have the following characteristics
   a. Any portfolio that is a combination of two frontier portfolios is also efficient
   b. Every portfolio on the efficient frontier has a companion portfolio on the inefficient frontier which is uncorrelated
      i. The companion portfolio is referred to as the **zero-beta portfolio** of the efficient portfolio
      ii. Choosing the market portfolio M and its zero-beta portfolio for P and Q gives:

$$E(r_i) - E(r_Z) = \left[E(r_M) - E(r_Z)\right]\frac{Cov(r_i, r_M)}{\sigma_M^2} = \beta_i\left[E(r_M) - E(r_Z)\right]$$

      iii. Resembles the SML with the risk-free rate replaced by the zero-beta portfolio
2. This is the CAPM equation that results when there are restrictions on borrowing

E. Labor Income and Non-Traded Assets

1. CAPM assumes that all risky assets are traded, but not all assets are traded
   a. Human capital (labor) – Value exceeds that of traded assets
   b. Privately held businesses – Value similar to that of traded assets
2. Private business is lesser problem for CAPM than human capital
   a. Can be borrowed against and sold
   b. Small business owners will endeavor to find portfolios that hedge their investment in their own business (will seek less weight in their portfolio for assets correlated to small business)
3. Labor income poses more problems because of its size
   a. Can be borrowed against via a home mortgage and hedged via life insurance but less portable across time

b. Labor intensive firms with high wage expenses may be good hedges for uncertain labor income, and their stocks may require lower expected return than predicted by CAPM

c. SML exhibiting expected return-beta relationship for economy with labor income varying relative to non-labor capital

$$E(R_i) = E(R_M)\frac{Cov(R_i,R_M)+\frac{P_H}{P_M}Cov(R_i,R_H)}{\sigma_M^2+\frac{P_H}{P_M}Cov(R_M,R_H)}$$

i. Basically represents an adjustment to beta

ii. $\frac{P_H}{P_M}$ may be greater than 1 and expect $Cov(R_i,R_H)$ to be positive

iii. Adjusted beta greater when CAPM beta is less than 1 and vice versa

iv. SML is less steep than standard CAPM

F. A Multiperiod Model and Hedge Portfolios

1. Merton relaxes single-period myopic assumption about investors to produce intertemporal CAPM (ICAPM)
2. If uncertainty in returns is only source of risk and investment opportunities remain unchanged through time, ICAPM achieve same return-beta relationship as CAPM
3. Introduce two kinds of risk

d. Changes in parameters describing investment opportunities

i. Such risks might include

a) Risk-free rate

b) Expected returns

c) Risk of market portfolio

ii. If investors future spending will be impacted negatively, they will demand hedging opportunities

e. Changes in prices of consumption goods (inflation)

i. Investors will demand hedging opportunities that respond to changes in inflation

4. Anytime investors seek hedging opportunities expected returns and risk will change
5. The ICAPM introduces K sources of extra-market risk and finds K associated hedge portfolios to produce the SML

$$E(R_i) = \beta_{iM}E(R_M)+\sum_{k=1}^{K}\beta_{ik}E(R_k)$$

β_M is security index on market-index portfolio

β_{iK} is beta on the kth hedge fund

G. A Consumption-Based CAPM (CCAPM)

1. Centers CAPM directly on consumption
2. Optimize utility of consumption now versus in the future
3. Consumption has greater utility during difficult economic times when consumption is lower (value income more)

10. A well-capitalized reinsurance company allocates its risk capital into the following three risk categories:

- Premium risk
- Reserve risk
- Interest rate risk

The company writes only the following two lines of business, which have similar expected losses:

- Line A: Short-tailed property catastrophe reinsurance
- Line B: Long-tailed casualty reinsurance

The company wants to further allocate the capital for each of those three risks sources to Line A and Line B.

a. Explain which line of business (A or B) should receive a higher allocation of capital for each of premium, reserve, and interest rate risk.

b. Discuss how the choice of risk measure between value at risk (VaR) and conditional tail expectation (CTE) is expected to impact the allocated capital to line A.

(15-9-17-1.5/0.75)

11. An insurance company writes three lines: Auto Liability (AL), Auto Physical Damage (PD) and Workers' Compensation (WC).

- The total required capital is $5,000,000.
- Capital is allocated based on a Co-GTE risk measure.
- The expense ratio is 27% for each line of business.
- The interest rate for discounting is 2.5%.
- The cost of capital is 15%.
- Expenses are paid at the beginning of the year and losses are paid at the end of each year.

Line	Premium	Undiscounted Loss Ratio	Loss Reserve Duration	99.5% Co-GTE
AL	5,000,000	70.0%	2.5	2,275,000
PD	2,000,000	62.5%	0.8	1,625,000
WC	8,000,000	77.5%	3.5	2,600,000
Total	15,000,000	73.0%	2.8	6,500,000

Determine whether the Workers' Compensation line of business adds value to the company on a risk-adjusted basis.

(15-9-18-2.25)

10. a. Premium Risk
Line A b/c greater chance of mispricing. Line A has catastrophe risk which is very volatile and difficult to predict. Reinsurance can be priced for the "average" year of experience based on models. However, in most years there will be very little losses but infrequently there will be years with very large losses (when a hurricane hits, for example). Also, there is model risk involved b/c catastrophe models are likely used due to lack of historical experience, so that introduces additional pricing risk.

Reserve Risk
B receives more because it will have more reserves due to its longer payout pattern.

Interest Rate
Interest Rate Risk → Line B should receive more capital because it is longer tailed so the investments supporting the reserves need to have a longer duration, and are therefore more dependent on changes in interest rate.

b. VaR allocates capital to achieve a given percentile threshold (exceedence probability). The severity of losses above the threshold has no impact on the allocation. However CTE looks at the average severity above the threshold. A would get more allocated capital using CTE due to the low frequency, high-severity losses of the CAT exposed line, which would impact CTE but would not impact VaR as much.

11. Discounted loss ratio as Loss Ratio / (1+Interest)^(duration)
WC: 0.775/(1.025)^3.5 = 0.7108

Net profit as (1 – Discounted Loss Ratio – Expense Ratio) * Premium
WC: (1-0.7108-0.27) * 8,000,000 = 153,238

Capital allocated as (Co-CTE / Total Co-CTE) * Capital Required
WC: (2,600/6,500) * 5,000,000 = 2,000,000

RAROC as (Net Profit/ Capital Allocated)
WC: 153,238/2,000,000 = 7.7%

Conclusion: 7.7% < 15% so WC does not add value because RAROC is lower than cost of capital.

12. Management is choosing between two new lines of business with the following characteristics:

	Line A	Line B
Expected Premium (million)	6	4
Required Capital (million)	4	2
Expense Ratio	20%	30%
Discounted Loss Ratio	75%	65%

For both lines:

- Investment return is 5%.
- Expenses are paid immediately.
- Losses are paid at the end of the year.
- Capital is released at the end of the year.

Using a risk-adjusted evaluation, determine which line should be pursued by management.

(16-9-23-1.25)

12. Net Income
Line A: 6,000,000 (1-.20) (1.05) – 6,000,000(.75) = 540,000
Line B: 4,000,000 (1-.30) (1.05) – 4,000,000(.65) = 340,000

Line A: RAROC: 540,000/4,000,000 = 13.75%
Line B: RAROC: 340,000/2,000,000 = 17%

17%> 13.5%

Therefore B is preferable and should be pursued if the cost of capital is less than 17%.