Part III

Practice Examinations

Prelude

Now that you have learned all the syllabus material in Exam FAM, here are five (5) comprehensive practice exams designed to assess your understanding of the whole exam syllabus and boost your chance of passing the real exam.

What are these practice exams like?

These practice exams have the following characteristics:

- Each exam has exactly 34 multiple-choice questions distributed in line with the weights of the ten topics in Exam FAM according to the following table:

	Topic	Weight range	Approximate no. of questions
	FAM-S Part		
1.	Short-Term Insurance and Reinsurance Coverages	5-10%	2-3
2.	Severity, Frequency, and Aggregate Models	12.5-17.5%	4-6
3.	Parametric Estimation	2.5-7.5%	1-3
4.	Introduction to Credibility	2.5-5%	1-2
5.	Pricing and Reserving for Short-Term Insurance Coverages	10-15%	3-5
6.	Option Pricing Fundamentals	2.5-7.5%	1-3
	Total	48.75%	17
	FAM-L Part		
7.	Long-Term Insurance Coverages and Retirement Financial Security Programs	2.5-5%	1-2
8.	Mortality Models	10-15%	3-5
9.	Present Value Random Variables for Long-Term Insurance Coverages	12.5-20%	4-7
10.	Premium and Policy Value Calculation for Long-Term Insurance Coverages	15-22.5%	5-8
	Total	51.25%	17

- For your convenience, the questions in these practice exams are sorted according to the ten FAM exam topics. That is, Question 1 is set on Topic 1 and Question 34 on Topic 10. This way, you can easily see which topics are your weak spots and identify additional practice questions for those topics. Questions in the real exam will appear in a random order.

- They strike a good balance between standard questions testing topics regularly featured in past exams and harder questions testing more unfamiliar topics that were rarely tested in the past, but may play a more important role in Exam FAM nowadays. The amount of calculations required by most questions should be reasonable—not too tedious, not trivial.
- The five exams have more or less the same level of difficulty, so you need not work them out in order. You can start with Practice Exam 5 if you like.

How to use these practice exams?

To make the most of these practice exams, here are my recommendations:

- Attempt them when and only when you have completed the core of this study manual. Working on the practice exams when you are not fully ready defeats their purpose.
- Set aside exactly 3.5 hours and work on each exam in a simulated exam environment detached from distractions. Put away your notes and phone—no Facebook, Instagram, Twitter, or Snapchat. You can only have the FAM tables, the Prometric standard normal calculator, scratch papers, and your calculator with you, as if this was a real exam.
- Budget your time wisely. Don't spend a disproportionate amount of time (say, more than 10 minutes) on a single question, no matter how difficult it seems, and don't be afraid to skip questions. For a 210-minute exam with 34 questions, you should spend about 6 minutes on each question.
- When you are done, check your answers with the detailed illustrative solutions I provide. If you miss a question, it is important to understand the cause. Is it due to a lack of familiarity with the syllabus material, carelessness, or just bad luck? In quite a number of questions, the wrong answers are distractors that correspond to common mistakes that students make. (The SOA may do the same!) Even if you get a question right, it is beneficial to look at my solutions, which may be shorter or neater than yours, and may include some problem-solving remarks.

A NOTE OF ENCOURAGEMENT

Don't feel too frustrated if you find the practice exams hard. The experiences of students who took FAM recently (and my own experiences) suggest that these practice exams are likely a bit more difficult than the real exam. It is better to see something difficult when you practice than to be under-prepared and caught off guard on the real exam, right? ☺

On the positive side, if you consistently do well (say, you get **at least 25 out of 34 questions** correct in each exam), you should be on your way to passing Exam FAM with ease. Good luck!

Practice Exam 1

1. Losses follow a uniform distribution on the interval $[0, 1000]$.

An insurance pays each loss up to a policy limit.

Calculate the policy limit that results in an expected cost per loss of 455.

(A) 700	(B) 750	(C) 800
(D) 850	(E) 900	

2. You are given:

(i) Losses follow an exponential distribution.

(ii) LER_d is the loss elimination ratio for an ordinary deductible of d.

(iii) LER_{2d} is the loss elimination ratio for an ordinary deductible of $2d$.

(iv) $\dfrac{\mathrm{LER}_{2d}}{\mathrm{LER}_d} = 1.8$

Calculate the loss elimination ratio for an ordinary deductible of $3d$.

(A) 0.3	(B) 0.4	(C) 0.5
(D) 0.6	(E) 0.7	

3. A reinsurer is pricing an excess of loss reinsurance treaty covering the layer 1,500,000 excess of 500,000.

You are given:

(i) Historical data for four claims with losses greater than the attachment point, evaluated as of 12/31/2023:

Loss ID	**Accident Year**	**Trended Loss and ALAE**
1	2021	650,000
2	2022	2,300,000
3	2022	1,250,000
4	2023	1,950,000

(ii) On-level subject premium by accident year:

Accident Year	**On-level Subject Premium**
2021	6,500,000
2022	7,000,000
2023	7,500,000

(iii) Accident year development factors applicable to losses in the layer 1,500,000 excess of 500,000:

Period	**Loss Development Factors**
12 month-Ultimate	1.40
24 month-Ultimate	1.15
36 month-Ultimate	1.05
48 month-Ultimate	1.00

Calculate the trended and developed loss as a percentage of the on-level subject premium.

(A) 21% (B) 22% (C) 23%
(D) 24% (E) 25%

4. Losses (in thousands) on an insurance company's policies follow a distribution with probability density function

$$f(x) = \frac{3\theta^3}{(x+\theta)^4}, \qquad 0 < x < \infty.$$

You are given:

(i) 70% of the policies are standard policies, for which $\theta = 3$.

(ii) 30% of the policies are preferred policies, for which $\theta = 1$.

Calculate the variance of the loss of a randomly selected policy.

(A) 4.8 (B) 4.9 (C) 5.0
(D) 5.1 (E) 5.2

5. An actuary is fitting a member of the $(a, b, 0)$ class of distributions to a set of count data of insurance policies. He produces a plot of $k(n_k/n_{k-1})$ against k, where n_k is the number of policies with k claims, for $k = 1, 2, 3, \ldots$, and finds that the plotted points lie closely on a straight line. The slope and intercept of the line are 0.5 and 1, respectively.

Calculate the estimated variance of the number of claims of a policy.

(A) 3 (B) 4 (C) 5
(D) 6 (E) 7

6. You are given that the annual number of claims follows a zero-modified negative binomial distribution with parameters $p_0^M = 0.6$, $r = 2$, and $\beta = 0.5$.

Calculate the probability that three claims occur during a year.

(A) 0.03 (B) 0.04 (C) 0.05
(D) 0.06 (E) 0.07

7. For a collective risk model, you are given:

(i) The number of losses has a geometric distribution with $\beta = 4$.

(ii) The common distribution of the individual losses is $f_X(x) = 0.25$ for $x = 1, 2, 3, 4$.

(iii) The number of losses and the loss amounts are independent.

Calculate the probability that the aggregate loss does not exceed 4.

(A) 0.21 (B) 0.26 (C) 0.31
(D) 0.36 (E) 0.41

8. For an aggregate loss model, you are given:

(i) The coefficient of variation of the number of claims is 1.4142.

(ii) The coefficient of variation of the severity is 0.4472.

(iii) The coefficient of variation of the aggregate loss is 1.4832.

Calculate the expected number of claims.

(A) 1 (B) 2 (C) 4
(D) 6 (E) 8

9. You are asked to consider whether the standard deviation principle

$$\rho(X) = \mathrm{E}[X] + k\sqrt{\mathrm{Var}(X)},$$

where k is a constant, is a coherent risk measure.

Determine the number of properties of a coherent risk measure satisfied by $\rho(X)$.

(A) 0 (B) 1 (C) 2
(D) 3 (E) 4

10. For a group of policies, you are given:

(i) Losses follow a Weibull distribution with parameters $\tau = 0.5$ and θ.

(ii) A random sample of 21 losses resulted in the following:

Interval	Number of Losses
$[0, 100]$	6
$(100, 400]$	7
$(400, \infty)$	8

Calculate the maximum likelihood estimate of θ.

(A) 20 (B) 50 (C) 100
(D) 200 (E) 500

11. You are given:

(i) Ground-up losses follow a uniform distribution on the interval $(0, \theta)$.

(ii) For each loss, there is an ordinary deductible of 5 and a maximum covered loss of 30.

(iii) A sample of ground-up losses is:

$$6, \quad 9, \quad 12, \quad 15, \quad 16, \quad 18, \quad 21, \quad 24, \quad 25, \quad 28, \quad 30^+, \quad 30^+,$$

where + indicates that the original loss exceeds the maximum covered loss.

Calculate the maximum likelihood estimate of θ.

(A) 30 (B) 33 (C) 35
(D) 36 (E) 40

12. You are given:

(i) The number of claims follows a Poisson distribution.

(ii) Claim severity is independent of the number of claims and has an exponential distribution.

A full credibility standard is determined so that the total number of claims is within 5% of the expected number with 99% probability.

If the same expected number of claims for full credibility is applied to the total cost of claims, then the actual total cost would be within $100r\%$ of the expected cost with 95% probability.

Calculate r.

(A) 0.050 (B) 0.052 (C) 0.054
(D) 0.056 (E) Cannot be determined from the information given.

13. For a certain accident year, you are given:

(i) Losses paid-to-date $= 300,000$

(ii) Loss reserve using the chain-ladder method $= 150,000$

(iii) Loss reserve using the Bornhuetter–Ferguson method $= 180,000$

Calculate the loss reserve using the expected loss ratio method.

(A) 200,000 (B) 210,000 (C) 220,000
(D) 230,000 (E) 240,000

14. Company DEF sells homeowners insurance policies. You are given:

(i) The loss costs by policy year are:

Policy Year	Loss Cost	Weight
PY3	200	40%
PY4	220	60%

(ii) The straight line fitted to the natural log of the loss costs indicates a calculated trend factor of 10% per year compounded continuously.

(iii) New rates take effect on November 1, CY6 for one-year policies and will be in effect for one year.

Calculate the projected expected loss cost for these new rates.

(A) 280 (B) 290 (C) 300
(D) 310 (E) 320

15. You are given the following earned premiums for three calendar years:

Calendar Year	**Earned Premium**
CY5	3,000
CY6	3,300
CY7	3,500

All policies have a one-year term and policy issues are uniformly distributed through each year.

The following rate changes have occurred:

Date	**Rate Change**
July 1, CY3	+10%
July 1, CY5	+8%
April 1, CY7	+5%

Rates are currently at the level set on April 1, CY7.

Calculate the sum of the earned premiums for CY5, CY6, and CY7 at the current rate level.

(A) 10,100 (B) 10,200 (C) 10,300
(D) 10,400 (E) 10,500

16. The price of a non-dividend paying stock over a one-year period follows a one-period binomial model with $S_0 = 10$, $u = 1.2214$, and $d = 0.9048$.

A 1-year at-the-money European call option on the stock has a price of 0.9645.

Calculate the continuously compounded risk-free interest rate.

(A) 3% (B) 4% (C) 5%
(D) 6% (E) 7%

17. Assume the Black–Scholes framework. You are given:

(i) The price of a non-dividend paying stock at time $t = 0$ is 80.
(ii) The volatility of the stock is 30%.
(iii) The continuously compounded risk-free interest rate is 8%.

Consider a 3-month 90-strike European put option issued at time $t = 0$ on the stock.

Calculate the number of units of stock in the replicating portfolio for the put option at time $t = 0$.

(A) -0.72 (B) -0.42 (C) -0.28
(D) 0.28 (E) 0.72

18. Which of the following statements about disability income insurance is not true?

(A) The benefits are often capped at 50–70% of the salary that is being replaced.
(B) The elimination period is the time between the beginning of a period of disability and the beginning of the benefit payments.
(C) If the policyholder can do some work, but not at the full earning capacity established before the period of disability, then they may be eligible for a benefit based on partial disability.
(D) The off period is typically selected by the policyholder from a list offered by the insurer.
(E) Return to work assistance offsets costs associated with returning to work after a period of disability.

19. You are given:

$$\mu_{30+t} = \begin{cases} 0.01, & \text{if } 0 \le t < 10, \\ 0.02, & \text{if } 10 \le t < 20, \\ 0.03, & \text{if } 20 \le t. \end{cases}$$

Calculate the 80th percentile of the future lifetime of (30).

(A) 16 (B) 32 (C) 48
(D) 64 (E) 80

20. Fatman, aged 50, is a really fat man. Fully aware of his poor health conditions, he is determined to adopt a healthy lifestyle and lead a normal life one year from now.

You are given:

(i) The force of mortality of Fatman is

$$\mu^*_{50+t} = \begin{cases} 10\mu_{50+t}, & \text{if } 0 \le t \le 1, \\ \mu_{50+t}, & \text{if } t > 1, \end{cases}$$

where μ_{50+t}, $t \ge 0$, is the force of mortality of a life aged 50 whose mortality follows the Standard Ultimate Life Table.

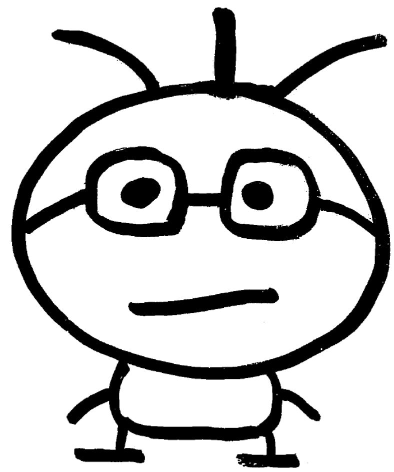
(ii) In the Standard Ultimate Life Table, $e_{50} = 36.09$.

Calculate e^*_{50}, the curtate expectation of life for Fatman.

(**Fatman:** Fat Lives Matter!!!!)

(A) 35.5 (B) 35.6 (C) 35.7
(D) 35.8 (E) 35.9

21. Professor L (shown on the right) only has 3 hairs left on his head and, sadly, he won't be growing any more.



You are given:

(i) The future mortality of each hair follows

$$_{k|}q_x = 0.1(k+1), \quad k = 0, 1, 2, 3$$

and x is Professor L's current age, which is an integer.

(ii) Hair loss is uniformly distributed over each year of age.

(iii) The future lifetimes of the 3 hairs are independent.

Calculate the probability that Professor L is bald (has no hair left) at age $x + 2.5$.

(A) 0.09 (B) 0.11 (C) 0.13
(D) 0.15 (E) 0.17

22. For a 2-year select and ultimate mortality table, you are given:

(i) Ultimate mortality follows the Standard Ultimate Life Table.

(ii) $q_{[x]} = 0.5q_x$ for all x.

(iii) $q_{[x]+1} = 0.7q_{x+1}$ for all x.

Calculate $l_{[90]}$.

(A) 38,000 (B) 39,000 (C) 40,000
(D) 41,000 (E) 42,000

23. The Actuarial Science Society (ASS)[2] consists of n members all age x today with independent future lifetimes. Having passed Exam FAM all with grade 10, the members apply their expertise in life contingencies to establish a pooled fund to pay a death benefit of \$100 at the end of the year of death for each member. In return, each member contributes a one-time amount of \$50 to this fund at inception.

You are given:

(i) $A_x = 0.455$

(ii) ${}^2A_x = 0.235$

Using the normal approximation without the continuity correction, determine the smallest integer n so that there is at least a 95% probability that the pooled fund will be sufficient to cover the present value of all promised death benefits.

(A) 38 (B) 42 (C) 46

(D) 50 (E) 54

24. For a 2-year term insurance of 1,000 on a smoker aged 65, you are given:

(i) The death benefit is paid at the end of the year of death.

(ii) Mortality of non-smokers follows the Standard Ultimate Life Table.

(iii) The force of mortality of smokers is twice the force of mortality of non-smokers at all ages.

(iv) $i = 0.03$

Calculate the expected present value of this insurance.

(A) 12 (B) 15 (C) 18

(D) 21 (E) 24

25. You are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) Deaths are uniformly distributed between integer ages.

(iii) $i = 0.05$

Calculate $1,000\,\mathrm{Var}(\bar{a}_{\overline{T_{55}}|})$.

(A) 8586 (B) 8596 (C) 8606

(D) 8616 (E) 8626

[2] I think a student society with a similar name really exists. See `https://www.melbourneactuary.com/`. ☺

26. In his life contingencies final exam, Guy was asked to calculate

$$\ddot{a}_{x:\overline{n}|}, \qquad a_{x:\overline{n}|}, \qquad \ddot{a}^{(m)}_{x:\overline{n}|}, \qquad a^{(m)}_{x:\overline{n}|},$$

where $m > 1$ is a positive integer, based on the same survival model and positive interest rate. Unprepared for the exam, he cheated and peeked at the answers of his classmate, but only managed to jot the four values in a random order *(bad eyesight!)*:

$$6.373, \qquad 6.539, \qquad 6.128, \qquad 6.789$$

Assuming that the answers of Guy's classmate are correct, calculate m.

(A) 2 (B) 4 (C) 6
(D) 8 (E) 12

27. You are given:

(i) Mortality follows the Standard Ultimate Life Table, which in turn follows the Makeham's law with parameters $A = 0.00022$, $B = 2.7 \times 10^{-6}$, and $c = 1.124$.

(ii) $i = 0.05$

Calculate $1,000\,{}_{20|}\ddot{a}^{(12)}_{45}$ using the three-term Woolhouse formula.

(A) 4710 (B) 4711 (C) 4712
(D) 4713 (E) 4714

28. For a fully discrete whole life insurance of 1,000, you are given:

(i) $q_x = 0.10$

(ii) $i = 0.05$

(iii) The annual net premium for this insurance at issue age x is 145.30.

Calculate the annual net premium for this insurance at issue age $x + 1$.

(A) 156 (B) 157 (C) 158
(D) 159 (E) 160

29. For a fully discrete whole life insurance with a face amount of 100,000 issued to (45), you are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) Initial expenses are 100 plus 50% of the first year's premium.

(iii) Renewal expenses are 5% of the renewal premiums.

(iv) Claim expenses of 500 are incurred when the death benefit is paid.

(v) $i = 0.05$

(vi) L_0 is the gross loss at issue random variable for this policy.

(vii) The standard deviation of L_0 is 13,095.

Calculate $\mathrm{E}[L_0]$.

(A) $-1,800$ (B) $-1,900$ (C) $-2,000$
(D) $-2,100$ (E) $-2,200$

30. For a fully continuous whole life insurance of 1,000 on (x), you are given:

(i) $\mu_{x+t} = 0.04$ for all $t \geq 0$.

(ii) $\delta = 0.06$

(iii) The premium rate per year is 45.

Calculate the probability that the insurer makes a profit on this policy.

(A) 0.43 (B) 0.47 (C) 0.50
(D) 0.53 (E) 0.57

31. For a 20-year deferred whole life annuity-due of 1,000 per year issued to (45), you are given:

(i) Level annual net premiums of 363 are payable for 20 years.

(ii) $\ddot{a}_{45} = 17.4106$, $\ddot{a}_{55} = 15.7098$, and $\ddot{a}_{65} = 13.4130$.

(iii) ${}_{10}E_{45} = 0.5972$ and ${}_{20}E_{45} = 0.3457$.

Calculate the net premium policy value at the end of year 10.

(A) 4600 (B) 4700 (C) 4800
(D) 4900 (E) 5000

32. For a fully discrete 3-year term insurance of 1,000 on (50), you are given:

(i) $i = 0.06$

(ii) $q_{50} = 0.025$

(iii) The annual net premium is 56.05.

(iv) The net premium policy value at the end of the second year is 29.14.

Calculate q_{51}.

(A) 0.05 (B) 0.06 (C) 0.07

(D) 0.08 (E) 0.09

33. For a fully discrete whole life insurance, you are given:

(i) Level annual premiums, calculated based on the equivalence principle, are paid at the beginning of each year.

(ii) Initial expenses are 50% of the annual gross premium and renewal expenses are 10% of the annual gross premium.

(iii) There are no other expenses.

Determine which of the following statements is/are true.

I. The gross premium is larger than the net premium.

II. The gross premium policy value is larger than the net premium policy value in renewal years.

III. The expense policy value is positive in renewal years.

(A) I only (B) II only (C) III only

(D) I, II, and III (E) The correct answer is not given by (A), (B), (C), or (D).

34. For a fully discrete 20-year endowment insurance of 1,000 on (70), you are given:

(i) Mortality follows the Standard Ultimate Life Table.

(ii) $i = 0.05$

Calculate the full preliminary term reserve for this insurance at the end of year 10.

(A) 350 (B) 360 (C) 370

(D) 380 (E) 390

****END OF PRACTICE EXAM 1****

Solutions to Practice Exam 1

Answer Key

Question #	Answer	FAM Topic
1	A	1
2	C	1
3	C	1
4	E	2
5	D	2
6	C	2
7	E	2
8	A	2
9	D	2
10	E	3
11	C	3
12	C	4
13	E	5
14	B	5
15	E	5
16	C	6
17	A	6

Question #	Answer	FAM Topic
18	D	7
19	D	8
20	C	8
21	A	8
22	A	8
23	A	9
24	E	9
25	C	9
26	B	9
27	B	9
28	B	10
29	B	10
30	E	10
31	D	10
32	C	10
33	A	10
34	C	10

Illustrative Solutions

1. [Section 1.1] **(Coverage modifications: Given the expected cost per loss, solve for the policy limit; uniform loss)**

Similar example(s)/problem(s): Problems 1.1.22 (for Pareto) and 1.1.24

Solution. You can directly consider

$$\mathrm{E}[X \wedge u] \overset{(1.1.2)}{=} \int_0^u S_X(x)\,\mathrm{d}x = \int_0^u \left(1 - \frac{x}{1000}\right)\mathrm{d}x = u - \frac{u^2}{2000} = 455,$$

which requires solving a quadratic equation in u.

An easier solution results from considering $\mathrm{E}[(X-u)_+] \overset{(1.1.7)}{=} \mathrm{E}[X] - \mathrm{E}[X \wedge u] = \frac{1000}{2} - 455 = 45$. Because the excess loss variable $X - u \mid X > u$ follows a $\mathrm{U}(0, 1000-u)$ distribution,

$$\mathrm{E}[(X-u)_+] \overset{(1.1.19)}{=} e_X(u)S_X(u) \overset{(1.1.17)}{=} \left(\frac{1000-u}{2}\right)\left(\frac{1000-u}{1000}\right) = \frac{(1000-u)^2}{2000}.$$

Equating this with 45 gives

$$(1000-u)^2 = 45(2000) = 90,000 \quad \Rightarrow \quad u = 1000 - \sqrt{90,000} = \boxed{700}. \quad \textbf{(Answer: (A))} \quad \square$$

Remark. (i) The solution above based on $\mathrm{E}[(X-u)_+]$, which involves the perfect square $(1000-u)^2$, dispenses with the need to solve a quadratic equation.

(ii) If you work with $\mathrm{E}[X \wedge u] = 455$, then instead of solving the quadratic equation

$$u - \frac{u^2}{2000} = 455$$

by an "honest" means, you may see which of the five values of u satisfies the given equation. After all, FAM is a multiple-choice exam. (This trial and error approach does not work for written-answer exams like Exam ASTAM.)

2. **[Section 1.1] (Coverage modifications: LER for an exponential distribution)**

Similar example(s)/problem(s): Example 1.1.5 and Problem 1.1.37

Solution. For an exponential distribution with parameter θ and an ordinary deductible of d, the LER is

$$\mathrm{LER}_d \overset{(1.1.8)}{=} \frac{\mathrm{E}[X \wedge d]}{\mathrm{E}[X]} \overset{\substack{\text{(Section A.3.3.1}\\ \text{of FAM-S tables)}}}{=} \frac{\theta(1 - \mathrm{e}^{-d/\theta})}{\theta} = 1 - \mathrm{e}^{-d/\theta}.$$

From (iv),

$$\frac{\mathrm{LER}_{2d}}{\mathrm{LER}_d} = \frac{1 - \mathrm{e}^{-2d/\theta}}{1 - \mathrm{e}^{-d/\theta}} \overset{\text{(see remark)}}{=} \frac{(1 + \mathrm{e}^{-d/\theta})(1 - \mathrm{e}^{-d/\theta})}{1 - \mathrm{e}^{-d/\theta}} \overset{\text{(cancel)}}{=} 1 + \mathrm{e}^{-d/\theta} = 1.8,$$

so $\mathrm{e}^{-d/\theta} = 0.8$.

For an ordinary deductible of $3d$, $\mathrm{LER}_{3d} = 1 - \mathrm{e}^{-3d/\theta} = 1 - 0.8^3 = \boxed{0.488}$. **(Answer: (C))** □

Remark. (i) The solution above used the identity $1 - x^2 \equiv (1 + x)(1 - x)$ with $x = \mathrm{e}^{-d/\theta}$. There is no need to solve a quadratic equation (you can, if you love algebra!).

(ii) As in Problem 1.1.37, there is no need (and, no way, given the available information) to calculate the individual values of d and θ.

3. **[Section 1.2] (Insurance/reinsurance coverages: Experience rating for excess of loss reinsurance)**

Similar example(s)/problem(s): Example 1.2.2 and Problem 1.2.9

Solution. The following table shows the amount of trended loss that falls within the treaty layer for each claim:

Loss ID	**(1) Accident Year**	**(2) Trended Loss**	$\mathbf{((2) \wedge 2,000,000 - 500,000)_+}$ **Trended Loss in Treaty Layer**
1	2021	650,000	150,000
2	2022	2,300,000	1,500,000
3	2022	1,250,000	750,000
4	2023	1,950,000	1,450,000

Then we group the losses by AY, develop the losses to ultimate values, and calculate the loss ratio: (Note that losses in AYs 2021, 2022, and 2023 are in development months 36, 24, and 12, respectively, as of 12/31/2023.)

AY	(4) Trended Losses in Treaty Layer	(5) Loss Development Factor	(4) × (5) Trended and Developed Losses in Treaty Layer
2021	150,000	1.05	157,500
2022	2,250,000	1.15	2,587,500
2023	1,450,000	1.40	2,030,000

The total trended and developed losses equal $157,500 + 2,587,500 + 2,030,000 = 4,775,000$. The total on-level subject premium is $6,500,000 + 7,000,000 + 7,500,000 = 21,000,000$, so the estimated loss ratio is $4,775,000/21,000,000 = \boxed{22.74\%}$. **(Answer: (C))** □

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(D) Overlooked the 2,000,000 limit and mistreated the trended loss for Claim 2 as 1,800,000.

4. **[Section 2.1] (Severity models: Variance of a mixture)**

Similar example(s)/problem(s): Problems 2.1.43 and 2.1.53

Solution. Observe that the losses are a 70-30 mixture of two Pareto distributions, one with parameters $\alpha = 3$ and $\theta = 3$, and the other with parameters $\alpha = 3$ and $\theta = 1$. From Section A.2.3.1 of the FAM-S tables, the mean and second moment of a Pareto distribution are $\frac{\theta}{\alpha-1}$ and $\frac{2\theta^2}{(\alpha-1)(\alpha-2)}$, respectively. Using the fact that the raw moments of a mixture are a mixture of the constituent raw moments, we have

$$\begin{aligned}
\mathrm{E}[X] &\overset{(2.1.7)}{=} 0.7\left(\frac{3}{3-1}\right) + 0.3\left(\frac{1}{3-1}\right) = 0.7(1.5) + 0.3(0.5) = 1.2, \\
\mathrm{E}[X^2] &\overset{(2.1.7)}{=} 0.7\left[\frac{2(3)^2}{(3-1)(3-2)}\right] + 0.3\left[\frac{2(1)^2}{(3-1)(3-2)}\right] = 0.7(9) + 0.3(1) = 6.6,
\end{aligned}$$

so $\mathrm{Var}(X) = \mathrm{E}[X^2] - \mathrm{E}[X]^2 = 6.6 - 1.2^2 = \boxed{5.16}$. **(Answer: (E))** □

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(C) This is the weighted average of the two constituent variances: $0.7(9-1.5^2)+0.3(1-0.5^2) = 4.95$.

5. **[Section 2.2] (Frequency models: What can you deduce from the graphical method?)**

Similar example(s)/problem(s): Problem 2.2.46

Solution. Because

$$k\left(\frac{n_k}{n_{k-1}}\right) \approx k\left(\frac{p_k}{p_{k-1}}\right) = \underset{\text{(slope)}}{a} \times k + \underset{\text{(intercept)}}{b},$$

this problem uses an indirect way to tell you that $a = 0.5$ and $b = 1$. Using the variance formula at the beginning of Section B.2 of the FAM-S tables,

$$\mathrm{Var}(N) = \frac{a+b}{(1-a)^2} = \frac{0.5+1}{(1-0.5)^2} = \boxed{6}. \quad \textbf{(Answer: (D))}$$

□

Remark. As a post-check, you can deduce from $a = 0.5$ and $b = 1$ that N is negative binomial with $r = 3$ and $\beta = 1$, thus $\text{Var}(N) = r\beta(1+\beta) = 3(1)(2) = 6$.

6. **[Section 2.2] (Frequency models: Probabilities of an $(a, b, 1)$ distribution)**

Solution 1 ($p_3 \longrightarrow p_3^M$). The proportionality constant relating p_k^M's and p_k's for $k = 1, 2, \ldots$ is

$$c \overset{(2.2.5)}{=} \frac{1-p_0^M}{1-p_0} = \frac{1-0.6}{1-(1+0.5)^{-2}} = 0.72.$$

Then the probability that three claims occur during a year is

$$p_3^M \overset{(2.2.4)}{=} cp_3 = 0.72\left[\frac{2(3)(4)}{3!}\right]\left[\frac{0.5^3}{1.5^5}\right] = \boxed{0.0474}. \quad \textbf{(Answer: (C))}$$

□

Solution 2 ($p_3^T \longrightarrow p_3^M$). From Section B.3.1.5 of the FAM-S tables,

$$p_3^T = \frac{r(r+1)(r+2)}{k![(1+\beta)^r - 1]}\left(\frac{\beta}{1+\beta}\right)^k = \frac{2(2+1)(2+2)}{3![(1+0.5)^2-1]}\left(\frac{0.5}{1.5}\right)^3 = 0.118519.$$

Then the probability that three claims occur during a year is

$$p_3^M \overset{(2.2.7)}{=} (1-p_0^M)p_3^T = (1-0.6)(0.118519) = \boxed{0.0474}. \quad \textbf{(Answer: (C))}$$

□

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(E) This is $p_3 = 0.0658$.

7. **[Section 2.3] (Aggregate loss models: $F_S(x)$ with a discrete severity)**

Similar example(s)/problem(s): Example 2.3.1 and Problem 2.3.12

Solution. For the given geometric distribution, $a = \frac{\beta}{1+\beta} = \frac{4}{1+4} = 0.8$ and $b = 0$, and the $(a, b, 0)$ recursive formula gives $p_k = 0.8p_{k-1}$ for $k = 1, 2, 3, 4$, so

$$p_0 = \frac{1}{1+4} = 0.2, \quad p_1 = 0.16, \quad p_2 = 0.128, \quad p_3 = 0.1024, \quad p_4 = 0.08192.$$

The following table shows the n-fold convolution $F_X^{*n}(4)$ for $n = 0, 1, 2, 3, 4$:

x	$f_X(x)$	$F_X^{*0}(x)$	$F_X^{*1}(x)$	$F_X^{*2}(x)$	$F_X^{*3}(x)$	$F_X^{*4}(x)$
0	0	1	0	0	0	0
1	0.25	1	0.25	0	0	0
2	0.25	1	0.5	0.0625	0	0
3	0.25	1	0.75	0.1875	0.015625	0
4	0.25	**1**	**1**	**0.375**	**0.0625**	**0.00390625**
p_n		0.2	0.16	0.128	0.1024	0.08192

By (2.3.7) with $x = 4$,

$$\begin{aligned} F_S(4) &= p_0F_X^{*0}(4) + p_1F_X^{*1}(4) + p_2F_X^{*2}(4) + p_3F_X^{*3}(4) + p_4F_X^{*4}(4) \\ &= 0.2(1) + 0.16(1) + 0.128(0.375) + 0.1024(0.0625) + 0.08192(0.00390625) \\ &= \boxed{0.41472}. \quad \textbf{(Answer: (E))} \end{aligned}$$

□

Remark. (i) ⚠ Possible reason(s) for getting some of the incorrect answers:

(A) Missed the term $p_0F_X^{*0}(4) = 0.2$.

(ii) If you prefer to calculate $f_S(x)$ for $x = 0, 1, 2, 3, 4$ by direct enumeration, note that for $x = 4$,

$$f_S(4) = p_1f_X(4) + p_2[f_X(2)^2 + 2f_X(1)f_X(3)] + p_3[3f_X(1)^2f_X(2)] + p_4f_X(1)^4$$

If you find it difficult to exhaust all possibilities, then consider using the method of convolution!

8. **[Section 2.3] (Aggregate loss models: Relating the coefficients of variation of S, N, X)**

Similar example(s)/problem(s): Problems 2.3.28 and 2.3.29

Solution. From (2.3.9), $\mathrm{Var}(S) = \mathrm{E}[N]\mathrm{Var}(X) + \mathrm{Var}(N)\mathrm{E}[X]^2$. Dividing both sides by $\mathrm{E}[S]^2 = \mathrm{E}[N]^2\mathrm{E}[X]^2$ yields the following relationships among the three coefficients of variation:

$$\mathrm{CV}_S^2 = \frac{\mathrm{CV}_X^2}{\mathrm{E}[N]} + \mathrm{CV}_N^2,$$

where the subscripts identify the random variables. Plugging in the values given in (i) to (iii), we have

$$1.4832^2 = \frac{0.4472^2}{\mathrm{E}[N]} + 1.4142^2 \quad \Rightarrow \quad \mathrm{E}[N] = \boxed{1}. \quad \textbf{(Answer: (A))}$$

□

Remark. (i) From $\mathrm{CV}_N = \sqrt{\mathrm{Var}(N)}/\mathrm{E}[N]$, you can also get $\mathrm{Var}(N) = 2$.

(ii) ⚠ Possible reason(s) for getting some of the incorrect answers:

(D) Missed the squares and used $\mathrm{CV}_S = \frac{\mathrm{CV}_X}{\mathrm{E}[N]} + \mathrm{CV}_N$.

9. **[Section 2.4] (Risk measures: Coherence/incoherence of the standard deviation principle)**

Similar example(s)/problem(s): Example 2.4.1, and Problems 2.4.1 and 2.4.3

Solution. As shown in Problem 2.4.3, the standard deviation principle is positively homogeneous, translation invariant, subadditive, but not monotonic. In other words, it satisfies $\boxed{\text{three}}$ of the four properties of a coherent risk measure. **(Answer: (D))** □

Remark. It is not too hard to investigate each of the four properties of a coherent risk measure for the standard deviation principle from first principles, but that will take you quite some time on the real exam (especially subadditivity).

10. **[Section 3.1] (MLE: Weibull, grouped data)**

Similar example(s)/problem(s): Example 3.1.2 and Problem 3.1.21

Ambrose's comments: This is a version of Example 3.1.2, where exponential is changed to Weibull.

Solution. From Section A.3.2.3 of the FAM-S tables, the distribution function of a Weibull distribution with $\tau = 0.5$ is $F(x) = 1 - \mathrm{e}^{-(x/\theta)^{0.5}}$. The likelihood function is

$$\begin{aligned}
L(\theta) &= [F(100)]^6[F(400) - F(100)]^7[S(400)]^8 \\
&= (1-\mathrm{e}^{-(100/\theta)^{0.5}})^6(\mathrm{e}^{-(100/\theta)^{0.5}} - \mathrm{e}^{-(400/\theta)^{0.5}})^7(\mathrm{e}^{-(400/\theta)^{0.5}})^8 \\
&= (1-\mathrm{e}^{-10/\theta^{0.5}})^6(\mathrm{e}^{-10/\theta^{0.5}} - \mathrm{e}^{-20/\theta^{0.5}})^7(\mathrm{e}^{-20/\theta^{0.5}})^8 \\
&\overset{(\text{factorized in terms of } \mathrm{e}^{-10/\theta^{0.5}})}{=} (1-\mathrm{e}^{-10/\theta^{0.5}})^6[\mathrm{e}^{-10/\theta^{0.5}}(1-\mathrm{e}^{-10/\theta^{0.5}})]^7(\mathrm{e}^{-10/\theta^{0.5}})^{16} \\
&\overset{(\text{simplify})}{=} (\mathrm{e}^{-10/\theta^{0.5}})^{23}(1-\mathrm{e}^{-10/\theta^{0.5}})^{13},
\end{aligned}$$

which is of the Bernoulli form maximized at

$$\mathrm{e}^{-10/\hat{\theta}^{0.5}} = \frac{23}{23+13} \quad \Rightarrow \quad \hat{\theta} = \boxed{498.19}. \quad \textbf{(Answer: (E))}$$

□

Remark. I set the group boundaries $0, 100, 400, \infty$ so that the likelihood function can be rearranged into a Bernoulli form. Otherwise, it would be difficult, if not impossible, to find the mle of θ in closed form.

11. **[Section 3.2] (MLE: Uniform, right-censored and left-truncated data)**

Similar example(s)/problem(s): Example 3.2.14 and Problem 3.2.69

Solution. Upon subtraction by $d = 5$, the corresponding values of the excess loss random variable $X - 5 \mid X > 5$, which has a $\mathrm{U}(0, \theta - 5)$ distribution, are

$$1, \quad 4, \quad 7, \quad 10, \quad 11, \quad 13, \quad 16, \quad 19, \quad 20, \quad 23, \quad 25^+, \quad 25^+,$$

By (3.2.8) with $u = 25$, $n_u = 10$, and $n = 12$, the mle of θ is

$$\hat{\theta} - 5 = \frac{12}{10}(25) \quad \Rightarrow \quad \hat{\theta} = \boxed{35}. \quad \textbf{(Answer: (C))}$$

□

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(A) This is the maximum ground-up loss and would be the mle for complete, individual data, or $\frac{12}{10}(25) = 30$, which would arise if you forgot to adjust the parameter of the excess loss random variable from θ to $\theta - 5$.

(D) This is $\hat{\theta} = \frac{12}{10}(30) = 36$, which would be the mle if there were no left truncation.

12. **[Section 4.1] (Full credibility: Standards for the number of claims vs. aggregate loss)**

Similar example(s)/problem(s): Example 4.1.4 and Problem 4.1.22

Solution.

- *(Number of claims)* By (4.1.5) with $r = 0.05$ and $p = 0.99$, the expected number of claims needed for full credibility is $\lambda_0 = (2.57583/0.05)^2 = 2653.9601$.
- *(Aggregate loss)* For exponential claim severity, the square of its CV is

$$\mathrm{CV}_Y^2 = \frac{\sigma_Y^2}{\mu_Y^2} = \frac{\theta^2}{\theta^2} = 1.$$

By (4.1.6) with $p = 0.95$, the expected number of claims needed for full credibility is

$$\lambda_0(1 + \mathrm{CV}_Y^2) = \left(\frac{1.95996}{r}\right)^2 (1+1) = 2653.9601,$$

so $r = \boxed{0.0538}$. **(Answer: (C))** □

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(A) Incorrectly treated $y_p = \Phi^{-1}(p)$ instead of $\Phi^{-1}\left(\frac{1+p}{2}\right)$.

(E) Thought that the value of the exponential mean θ was needed for getting the answer.

13. **[Section 5.1] (Loss reserving: Relating the three loss reserves)**

Similar example(s)/problem(s): Example 5.1.7 and Problem 5.1.22

Solution 1 (Using the weighted average formula). From (i) and (ii), we get

$$300,000(f_{\text{ult}} - 1) = 150,000 \quad \Rightarrow \quad f_{\text{ult}} = 1.5.$$

Then by the weighted average formula,

$$180,000 = R_{\text{BF}} \overset{(5.1.9)}{=} \left(1 - \frac{1}{f_{\text{ult}}}\right) R_{\text{LR}} + \frac{1}{f_{\text{ult}}} R_{\text{CL}} = \left(1 - \frac{1}{1.5}\right) R_{\text{LR}} + \frac{1}{1.5}(150,000),$$

which gives $R_{\text{LR}} = \boxed{240,000}$. **(Answer: (E))** □

Solution 2 (Without using the weighted average formula). As in Solution 1, we get $f_{\text{ult}} = 1.5$. Then from the Bornhuetter–Ferguson loss reserve,

$$\begin{array}{c}\text{Estimated}\\ \text{ultimate losses}\end{array} \times \left(1 - \frac{1}{1.5}\right) \overset{(5.1.7)}{=} 180,000 \quad \Rightarrow \quad \begin{array}{c}\text{Estimated}\\ \text{ultimate losses}\end{array} = 540,000$$

based on the expected loss ratio method. Finally, loss reserve $\overset{(5.1.1)}{=} 540,000 - 300,000 = \boxed{240,000}$. **(Answer: (E))** □

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(A) Mixed up the two weights in (5.1.9) and got

$$180,000 = \frac{1}{1.5}R_{\text{LR}} + \left(1 - \frac{1}{1.5}\right)(150,000),$$

resulting in $R_{\text{LR}} = 195,000$.

14. **[Section 5.2] (Ratemaking: Loss trending for policy year data)**

Similar example(s)/problem(s): Example 5.2.6, and Problems 5.2.13 and 5.2.16

Solution. Note that:

- The mid-point of PY3 is 01/01/CY4 (not 07/01/CY3).
- The mid-point of PY4 is 01/01/CY5.
- The mid-point of the future policy period is 11/01/CY7.

Therefore, the PY3 loss cost will be projected 3 years, 10 months, or 23/6 years, and the PY4 loss cost will be projected 2 years, 10 months, or 17/6 years. Based on a 40/60 weighted average, the projected loss cost is

$$0.4(200\mathrm{e}^{0.1(23/6)}) + 0.6(220\mathrm{e}^{0.1(17/6)}) = \boxed{292.61}. \quad \textbf{(Answer: (B))}$$

□

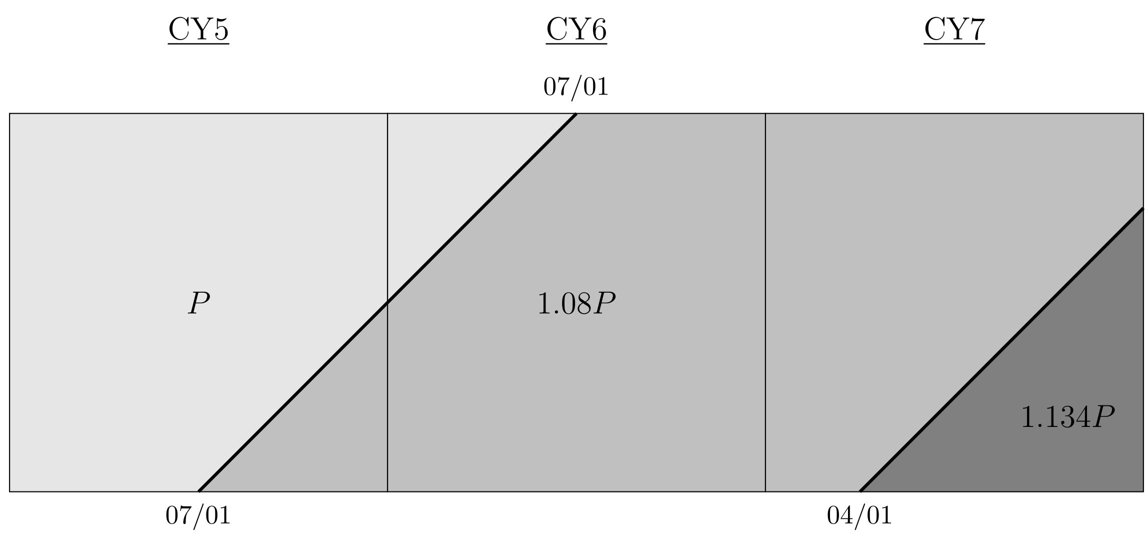
Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(D) Mistreated the PY data as AY data and got $0.4(200\mathrm{e}^{0.1(13/3)}) + 0.6(220\mathrm{e}^{0.1(10/3)}) = 307.61$.

15. **[Section 5.2] (Ratemaking: Parallelogram method)**

Similar example(s)/problem(s): Example 5.2.10, and Problems 5.2.35 and 5.2.36

Solution. Let P be the rate effective from 07/01/CY3 (i.e., after the 07/01/CY3 rate change). The following diagram depicts the distribution of the premium rates in CY5, CY6, and CY7:



The following table shows the rates in effect in each CY, their respective areas, and the average rate:

CY5		CY6		CY7	
Rate	Area	Rate	Area	Rate	Area
$1.08P$	$\frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{8}$	P	$\frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{8}$	$1.134P$	$\frac{1}{2}\left(\frac{9}{12}\right)^2 = \frac{9}{32}$
P	$1 - \frac{1}{8} = \frac{7}{8}$	$1.08P$	$1 - \frac{1}{8} = \frac{7}{8}$	$1.08P$	$1 - \frac{9}{32} = \frac{23}{32}$
Average rate $= 1.01P$		Average rate $= 1.07P$		Average rate $= 1.0951875P$	

The current rate is $1.134P$, so the sum of the three on-level earned premiums is

$$1.134\left(\frac{3,000}{1.01} + \frac{3,300}{1.07} + \frac{3,500}{1.0951875}\right) = \boxed{10,489.74}. \quad \textbf{(Answer: (E))}$$

□

Remark. The rate change on July 1, CY3 (= 10%) has no role to play in this problem.

16. **[Section 6.2] (The binomial model: Given an option price, deduce r)**

Similar example(s)/problem(s): Example 6.2.3, and Problems 6.2.8 and 6.2.9

Solution. The two possible payoffs of the call are $C_u = (1.2214(10) - 10)_+ = 2.214$ at the u node and $C_d = (0.9048(10) - 10)_+ = 10$ at the d node. Using the risk-neutral pricing formula and noting that $1 - q = (e^{rh} - d)/(u - d)$, we get

$$C_0 \overset{(6.2.6)}{=} e^{-rh}[(1-q)C_u + qC_d] = e^{-r}\left(\frac{e^r - 0.9048}{1.2214 - 0.9048}\right)(2.214) \overset{\text{(given)}}{=} 0.9645,$$

giving $r = \boxed{0.0484}$. **(Answer: (C))** □

17. **[Section 6.3] (The Black–Scholes model: Calculating the delta of an option)**

Similar example(s)/problem(s): Problem 6.3.20

Solution. The number of units of stock in the replicating portfolio for the put option at time $t = 0$ is the put delta at that time. With

$$d_1 = \frac{\ln(80/90) + (0.08 + 0.3^2/2)(0.25)}{0.3\sqrt{0.25}} = -0.57689,$$

the delta of the put at time $t = 0$ is $-\Phi(-d_1) = \boxed{-0.71799}$. **(Answer: (A))** □

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(E) Missed the negative sign of the put delta.

(D) This is $\Phi(d_1)$, which is the delta of the call with the same strike price and expiry time as the given put.

18. **[Section 13.4] (Facts about DII)**

Similar example(s)/problem(s): Problems 13.6.15 and 13.6.17

Ambrose's comments: This question tests the specific details of disability income insurance. To complete this question, you need to know the nuts and bolts of DII quite well.

Solution. All statements are true except (D). Unlike the waiting (elimination) period, the off period is typically set by the insurer, not selected by the policyholder from a list offered by the insurer. **(Answer: (D))** □

19. **[Section 7.2] (Percentile of a future lifetime with a piecewise constant μ)**

Similar example(s)/problem(s): Problems 7.1.3 and 7.2.11

Solution. By (7.2.6), the survival function of T_{30} is given by

$$
{}_tp_{30} = \exp\left(-\int_0^t \mu_{30+s}\,\mathrm{d}s\right) = \begin{cases} \mathrm{e}^{-0.01t}, & \text{if } 0 \le t < 10, \\ \mathrm{e}^{-0.01(10)-0.02(t-10)}, & \text{if } 10 \le t < 20, \\ \mathrm{e}^{-0.01(10)-0.02(10)-0.03(t-20)}, & \text{if } 20 \le t. \end{cases}
$$

The 80th percentile of T_{30} is the value of t such that ${}_tp_{30} = 0.2$. Since ${}_{20}p_{30} = 0.7408 > 0.2$, the 80th percentile must be larger than 20 and the third expression above applies. Solving

$$
{}_tp_{30} = \mathrm{e}^{-0.01(10)-0.02(10)-0.03(t-20)} = 0.2
$$

yields $t = \boxed{63.65}$ as the required 80th percentile. **(Answer: (D))** □

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(A) This is the 20th percentile of T_{30}, obtained by solving ${}_tp_{30} = 0.8$.

20. **[Section 7.5] (e_x with an abnormal first-year mortality)**

Similar example(s)/problem(s): Example 7.5.3, and Problems 7.5.10 and 7.5.11

Solution. We can regard the first year ($t = 0$ to $t = 1$) as the period of abnormal mortality, where the SULT force of mortality is multiplied by 10.

- *Standard mortality:* By the one-step recursive formula for e_x,

  $$
  e_{50} \overset{(7.5.3)}{=} p_{50}(1 + e_{51}) = (1 - 0.001209)(1 + e_{51}) = 36.09,
  $$

  so $e_{51} = 35.1337$.

- *Abnormal mortality:* Since $\mu^*_{50+t} = 10\mu_{50+t}$ for all $t \in [0, 1]$, the one-year survival probability for Fatman is

  $$
  p^*_{50} \overset{(7.2.9)}{=} p^{10}_{50} = (1 - 0.001209)^{10} = 0.987976,
  $$

  Another application of the one-step recursive formula with $e^*_{51} = e_{51} = 35.1337$ yields

  $$
  e^*_{50} = p^*_{50}(1 + e^*_{51}) = 0.987976(1 + 35.1337) = \boxed{35.70}. \quad \textbf{(Answer: (C))}
  $$
  □

21. [**Section 8.2**] **(Simple probability calculations with UDD)**

Similar example(s)/problem(s): Example 8.2.2

Ambrose's comments: This question about the "mortality" of hair is adapted from an old SOA question (Course 3 Spring 2001 Question 13) testing a fractional age assumption no longer in the exam syllabus.

Solution 1 (Using probability symbols). Using (i) with $k = 0, 1, 2$ and the "$p \times q$" formula for ${}_{u|t}q_x$, we get the mortality rates at ages x, $x + 1$, and $x + 2$:

$$\begin{cases} {}_{0|}q_x = q_x = 0.1 \\ {}_{1|}q_x \overset{(7.1.6)}{=} p_x q_{x+1} = 0.2 \\ {}_{2|}q_x \overset{(7.1.6)}{=} p_x p_{x+1} q_{x+2} = 0.3 \end{cases} \quad \Rightarrow \quad \begin{cases} q_x = 0.1 \\ q_{x+1} = 2/9 \\ q_{x+2} = 3/7 \end{cases}.$$

The probability that each hair will "die" before age $x + 2.5$ is

$${}_{2.5}q_x = 1 - p_x(p_{x+1})({}_{0.5}p_{x+2}) \overset{(8.2.1)}{=} 1 - (1 - 0.1)\left(1 - \frac{2}{9}\right)\left[1 - 0.5\left(\frac{3}{7}\right)\right] = 0.45.$$

As the three hairs have independent future lifetimes, the probability that Professor L is bald at age $x + 2.5$ is $({}_{2.5}q_x)^3 = \boxed{0.0911}$. **(Answer: (A))** □

Solution 2 (Using life table symbols). Assume $l_x = 1$ (any number will do). For $k = 0, 1, 2$,

$${}_{k|}q_x \overset{(8.1.4)}{=} \frac{l_{x+k} - l_{x+k+1}}{l_x} = l_{x+k} - l_{x+k+1} \quad \Rightarrow \quad \begin{cases} l_{x+1} = 1 - 0.1 = 0.9 \\ l_{x+2} = 0.9 - 0.2 = 0.7 \\ l_{x+3} = 0.7 - 0.3 = 0.4 \end{cases}.$$

By the UDD assumption, $l_{x+2.5} \overset{(8.2.5)}{=} 0.5(l_{x+2} + l_{x+3}) = 0.55$. Then ${}_{2.5}q_x = 1 - l_{x+2.5}/l_x = 0.45$. As the three hairs have independent future lifetimes, the probability that Professor L is bald at age $x + 2.5$ is $({}_{2.5}q_x)^3 = \boxed{0.0911}$. **(Answer: (A))** □

Remark. The CF assumption would lead to ${}_{2.5}q_x = 0.470850$.

22. [**Section 8.3**] **(Going from l_x to $l_{[x]}$)**

Similar example(s)/problem(s): Example 8.3.4 and Problem 8.3.11

Ambrose's comments: This question tests the first type of common exam question about select tables discussed in Subsection 8.3.3 (calculating $l_{[x]}$ in the select table based on the values of l_x in the ultimate table and the relationship between select and ultimate mortality rates).

Solution. With a two-year select period, we can relate $l_{[90]}$ (select, to be determined) and $l_{[90]+2} = l_{92}$ (ultimate, available from SULT) via $l_{[90]}({}_2p_{[90]}) = l_{92}$. From (ii) and (iii), the two-year survival probability is

$$\begin{aligned} {}_2p_{[90]} &= p_{[90]}p_{[90]+1} \\ &= (1 - 0.5q_{90})(1 - 0.7q_{91}) \\ &= [1 - 0.5(0.100917)][1 - 0.7(0.112675)] \\ &= 0.874649. \end{aligned}$$

Then $l_{[90]} = \dfrac{l_{92}}{{}_2p_{[90]}} = \dfrac{33,379.9}{0.874649} = \boxed{38,163.78}$. **(Answer: (A))** □

Remark. For comparison, $l_{90} = 41,841.1$, corresponding to (E), is larger.

23. [Section 9.1] **(Normal approximation for insurances: How many members should ASS recruit?)**

Similar example(s)/problem(s): Example 9.1.5 and Problem 11.2.51

Ambrose's comments: Instead of testing probabilities or percentiles of a total present value random variable for a given n (size of the group) based on normal approximation, which is regularly featured in past exams, this question turns things around and requires you to calculate the smallest n for a given probability of loss.

Solution. Let $S = \sum_{i=1}^{n} Z_i$, where Z_i is the PVRV of the whole life insurance of \$100 on the ith life, be the total present value of all death benefits. We are interested in the smallest integral value of n such that $\mathrm{P}(S \le 50n) \ge 0.95$, which means that $50n$ should be greater than the 95th percentile of S. From (i) and (ii), we get

$$\mathrm{E}[S] = 100(0.455)n = 45.5n \quad \text{and} \quad \mathrm{Var}(S) = 100^2(0.235 - 0.455^2)n = 279.75n.$$

Using the normal approximation, we solve

$$50n \ge \pi_{0.95}(S) \overset{\text{(normal approx.)}}{=} \mathrm{E}[S] + 1.645\sqrt{\mathrm{Var}(S)} = 45.5n + 1.645\sqrt{279.75n},$$

which gives $\sqrt{n} \ge 6.1142$ or $n \ge 37.3832$. In other words, at least $\boxed{38}$ members are needed. **(Answer: (A))** □

Remark. Possible reason(s) for getting some of the incorrect answers:

(E) Used 1.96 (97.5th standard normal percentile) in place of 1.645 (95th percentile).

24. [Section 9.1] **(EPV of a 2-year discrete term insurance for a smoker)**

Similar example(s)/problem(s): Example 9.1.11 and Problem 9.1.42

Solution. As $\mu_x = 2\mu_x^{\text{SULT}}$ for all $x \ge 0$, we have

$$\begin{aligned} p_{65} &\overset{(7.2.9)}{=} (p_{65}^{\text{SULT}})^2 = 0.994085^2 = 0.988205, \\ p_{66} &= (p_{66}^{\text{SULT}})^2 = 0.993381^2 = 0.986806. \end{aligned}$$

The EPV of the 2-year term insurance on a smoker is

$$\begin{aligned} 1,000A^{1}_{65:\overline{2}|} &\overset{(9.1.2)}{=} 1,000(vq_{65} + v^2 p_{65} q_{66}) \\ &= 1,000\left[\frac{1 - 0.988205}{1.03} + \frac{0.988205(1 - 0.986806)}{1.03^2}\right] \\ &= \boxed{23.74}. \quad \textbf{(Answer: (E))} \end{aligned}$$

□

Remark. Possible reason(s) for getting some of the incorrect answers:

(A) This is the value of $1,000A^{1}_{65:\overline{2}|}$ for non-smokers.

(C) This is the average of $1,000A^1_{65:\overline{2}|}$ for non-smokers and $1,000A^1_{65:\overline{2}|}$ for smokers. Note that you are specifically told to deal with a "2-year term insurance of 1,000 on a *smoker*" and nothing is known about the proportion of smokers and non-smokers in the population.

25. **[Sections 9.3 and 10.1] (Variance of a continuous annuity PVRV under UDD)**

Similar example(s)/problem(s): Problem 10.1.45

Ambrose's comments: Although this question is not hard, it is educational in the sense that it tests the material in two chapters, Chapters 9 (UDD formulas for insurances) and 10 (variance for annuity PVs).

Solution. By (10.1.10), $\mathrm{Var}(\bar{a}_{\overline{T_{55}}|}) = \dfrac{{}^2\bar{A}_{55} - \bar{A}_{55}^2}{\delta^2}$. Under the UDD assumption, we can calculate the two endowment EPVs as

$$\begin{aligned} \bar{A}_{55} &\overset{(9.3.1)}{=} \frac{i}{\delta}A_{55} = \frac{0.05}{\ln 1.05}(0.23524) = 0.241073, \\ {}^2\bar{A}_{55} &\overset{(9.3.4)}{=} \frac{2i+i^2}{2\delta}({}^2A_{55}) = \frac{2(0.05)+0.05^2}{2\ln 1.05}(0.07483) = 0.078603. \end{aligned}$$

Thus $1,000\,\mathrm{Var}(\bar{a}_{\overline{T_{55}}|}) = 1,000\left[\dfrac{0.078603 - 0.241073^2}{(\ln 1.05)^2}\right] = \boxed{8,606.17}$. **(Answer: (C))** □

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(B) This is $1,000\mathrm{Var}(\ddot{a}_{\overline{K_{55}+1}|}) = 1,000\left[\dfrac{0.07483 - 0.23524^2}{(0.05/1.05)^2}\right] = 8,596.03$.

26. **[Section 10.2] (Comparing annuity EPVs, due vs. immediate, annual vs. $1/m$thly)**

Similar example(s)/problem(s): Problem 10.2.7

Ambrose's comments: This ambitious and rather challenging question tests both the chain of inequalities in (10.2.7) and the due-immediate formulas relating $a_{x:\overline{n}|}$, $\ddot{a}_{x:\overline{n}|}$ and $a^{(m)}_{x:\overline{n}|}$, $\ddot{a}^{(m)}_{x:\overline{n}|}$.

Solution. By (the temporary version of) (10.2.7), $a_{x:\overline{n}|} \le a^{(m)}_{x:\overline{n}|} \le \ddot{a}^{(m)}_{x:\overline{n}|} \le \ddot{a}_{x:\overline{n}|}$, so

$$a_{x:\overline{n}|} = 6.128, \qquad a^{(m)}_{x:\overline{n}|} = 6.373, \qquad \ddot{a}^{(m)}_{x:\overline{n}|} = 6.539, \qquad \ddot{a}_{x:\overline{n}|} = 6.789.$$

Considering $a_{x:\overline{n}|}$ and $\ddot{a}_{x:\overline{n}|}$, we have

$$6.128 = a_{x:\overline{n}|} \overset{(10.1.15)}{=} \ddot{a}_{x:\overline{n}|} - 1 + {}_nE_x = 6.789 - 1 + {}_nE_x,$$

so ${}_nE_x = 0.339$. Then it follows from $a^{(m)}_{x:\overline{n}|}$ and $\ddot{a}^{(m)}_{x:\overline{n}|}$ that

$$6.373 = a^{(m)}_{x:\overline{n}|} \overset{(10.2.5)}{=} \ddot{a}^{(m)}_{x:\overline{n}|} - \frac{1}{m} + \frac{1}{m}{}_nE_x = 6.539 - \frac{1}{m} + \frac{1}{m}(0.339),$$

which gives $m = \boxed{4}$. **(Answer: (B))** □

27. **[Section 10.3] (Three-term Woolhouse formula for a deferred annuity)**

Similar example(s)/problem(s): Problems 10.3.17 and 10.3.18

Ambrose's comments: Most questions that test the Woolhouse formula center on the two-term formula. This question tests the three-term formula, using the fact that the SULT is based on Makeham's law.

Solution. From the last page of the FAM-L tables, the force of mortality of the given Makeham's distribution at age 65 is

$$\mu_{65} = A + Bc^{65} = 0.00022 + (2.7 \times 10^{-6})(1.124)^{65} = 0.00560485.$$

By the three-term Woolhouse formula,

$$\begin{aligned} {}_{20|}\ddot{a}_{45}^{(12)} &= {}_{20}E_{45}\ddot{a}_{65}^{(12)} \\ &\overset{(10.3.5)}{\approx} {}_{20}E_{45}\left[\ddot{a}_{65} - \frac{12-1}{2(12)} - \frac{12^2-1}{12(12)^2}(\mu_{65}+\delta)\right] \\ &= 0.35994\left[13.5498 - \frac{11}{24} - \frac{143}{1728}(0.00560485 + \ln 1.05)\right] \\ &= 4.710522 \end{aligned}$$

and $1,000\,{}_{20|}\ddot{a}_{45}^{(12)} = \boxed{4,710.52}$. **(Answer: (B))** □

Remark. (i) The Makeham's law is used to get the force of mortality at age 65, which is required for the 3-term Woolhouse formula. If the fact that the SULT is generated from the Makeham's law is not given in the question, then we can apply the approximation

$$\mu_{65} \approx -\frac{1}{2}\ln\left(\frac{l_{66}}{l_{64}}\right) = -\frac{1}{2}\ln\left(\frac{94,020.3}{95,082.5}\right) = 0.005617,$$

which is quite close to the exact value above.

(ii) ⚠ Possible reason(s) for getting some of the incorrect answers:

(A) This is the value based on the UDD approximation:

$${}_{20|}\ddot{a}_{45}^{(12)} \overset{(10.3.1)}{=} {}_{20}E_{45}[\alpha(12)\ddot{a}_{65} - \beta(12)] = 0.35994[1.0002(13.5498) - 0.46651] = 4.7102.$$

(C) This is the value based on the two-term Woolhouse formula:

$${}_{20|}\ddot{a}_{45}^{(12)} \overset{(10.3.6)}{\approx} {}_{20}E_{45}\left[\ddot{a}_{65} - \frac{12-1}{2(12)}\right] = 0.35994\left(13.5498 - \frac{11}{24}\right) = 4.7121.$$

28. **[Section 11.1] (Going from P_x to P_{x+1})**

Similar example(s)/problem(s): Example 11.1.6 and Problems 11.1.21 and 11.1.25

Ambrose's comments: There is no simple recursive relationship between annual net premiums at consecutive ages, but the net premiums for standard whole life and endowment insurances can be expressed solely in terms of insurance or annuity EPVs, which do enjoy recursive formulas. This observation is the key to solving this problem efficiently.

Solution. By (11.1.1) and (iii),

$$1000P_x = 1000\left(\frac{1}{\ddot{a}_x} - d\right) = 1000\left(\frac{1}{\ddot{a}_x} - \frac{0.05}{1.05}\right) = 145.30 \quad \Rightarrow \quad \ddot{a}_x = 5.183521.$$

Then using the recursive formula for $\ddot{a}_x$, we get

$$5.183521 = \ddot{a}_x \overset{(10.1.6)}{=} 1 + vp_x\ddot{a}_{x+1} = 1 + \frac{1-0.1}{1.05}\ddot{a}_{x+1} \quad \Rightarrow \quad \ddot{a}_{x+1} = 4.880775$$

One more application of (11.1.1), this time at age $x+1$, yields the required net premium at age $x+1$:

$$1000P_{x+1} = 1000\left(\frac{1}{\ddot{a}_{x+1}} - \frac{0.05}{1.05}\right) = 1000\left(\frac{1}{4.880775} - \frac{0.05}{1.05}\right) = \boxed{157.27}. \quad \textbf{(Answer: (B))}$$

□

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(C) Mistakenly used $P_x = \dfrac{1}{\ddot{a}_x} - i$ instead of $P_x = \dfrac{1}{\ddot{a}_x} - d$.

29. **[Section 11.2] (From variance to expected value of L_0)**

Similar example(s)/problem(s): Example 11.2.10, and Problems 11.2.26 and 11.2.31

Solution. We can deduce the gross premium from the standard deviation in (vii). By (11.2.1) with $S = 100,000$, $E = 500$, and $G - R = 0.95G$,

$$\sqrt{\mathrm{Var}(L_0^g)} = \sqrt{\left(100,500 + \frac{0.95G}{0.05/1.05}\right)^2 (0.03463 - 0.15161^2)} = 13,095,$$

so $G = 1,045.2111$. Then the expected value of the gross loss at issue random variable is

$$\begin{aligned}\mathrm{E}[L_0^g] &= 100,500A_{45} + (100 + 0.45G) - 0.95G\ddot{a}_{45} \\ &= 100,500(0.15161) + [100 + 0.45(1,045.2111)] - 0.95(1,045.2111)(17.8162) \\ &= \boxed{-1,883.46}. \quad \textbf{(Answer: (B))}\end{aligned}$$

□

30. **[Section 11.2] (Probability of a profit for a fully continuous whole life insurance)**

Similar example(s)/problem(s): Example 11.2.15, and Problems 11.2.44 and 11.2.45

Ambrose's comments: Remember that besides EPV and variance, other actuarial quantities commonly tested in the exam include probabilities and percentiles of PVRVs (of insurances, annuities, or PV of losses). This question tests a probability for a loss at issue random variable.

Solution. In terms of T_x, the loss at issue random variable is

$$L_0 = 1000v^{T_x} - 45\bar{a}_{\overline{T_x}|} = 1000v^{T_x} - 45\left(\frac{1-v^{T_x}}{0.06}\right) = 1750v^{T_x} - 750.$$

Therefore,

$$L_0 < 0 \quad \Leftrightarrow \quad v^{T_x} < \frac{3}{7} \quad \Leftrightarrow \quad \mathrm{e}^{-0.06T_x} < \frac{3}{7} \quad \Leftrightarrow \quad T_x > -\frac{\ln(3/7)}{0.06} = 14.1216,$$

and the required probability is $\mathrm{P}(T_x > 14.1216) = \mathrm{e}^{-0.04(14.1216)} = \boxed{0.5684}$. **(Answer: (E))** □

Remark. ⚠ Possible reason(s) for getting some of the incorrect answers:

(A) This is the probability of a loss, $1 - 0.5684 = 0.4316$.

31. **[Section 12.1] (Policy value of a deferred annuity by definition)**

Ambrose's comments: Calculating the policy value of a deferred annuity from first principles is rarely tested in past exams. This question serves to fill this void.

Solution. By definition, the net premium policy value at the end of year 10 is ${}_{10}V = 1000({}_{10|}\ddot{a}_{55}) - P\ddot{a}_{55:\overline{10}|}$. The two annuity factors can be calculated as

$$\begin{aligned} {}_{10|}\ddot{a}_{55} &\overset{(10.1.18)}{=} {}_{10}E_{55}\ddot{a}_{65} \overset{(9.1.3)}{=} \frac{0.3457}{0.5972}(13.4130) = 7.764357, \\ \ddot{a}_{55:\overline{10}|} &\overset{(10.1.17)}{=} \ddot{a}_{55} - {}_{10|}\ddot{a}_{55} = 15.7098 - 7.764357 = 7.945443, \end{aligned}$$

we have ${}_{10}V = 1000(7.764357) - 363(7.945443) = \boxed{4880.16}$. **(Answer: (D))** □

Remark. The annual net premium given in (i) can be calculated as

$$P = \frac{1000({}_{20|}\ddot{a}_{45})}{\ddot{a}_{45:\overline{20}|}} = \frac{1000({}_{20}E_{45}\ddot{a}_{65})}{\ddot{a}_{45} - {}_{20}E_{45}\ddot{a}_{65}} = \frac{1000(0.34570)(13.4130)}{17.4106 - 0.34570(13.4130)} = 363.0009.$$

This question would probably be too cumbersome if you had to calculate both the net premium and the net premium policy value.

32. **[Section 12.2] (Two applications of the policy value recursive formula)**

Similar example(s)/problem(s): Problems 12.2.20 and 12.2.21

Solution. Applying the policy value recursive formula at $t = 0$ and noting that ${}_0V^n = 0$, we have

$$(0 + 56.05)(1.06) = 0.025(1,000) + (1 - 0.025)({}_1V^n),$$

so ${}_1V^n = 35.295385$. One more application of the recursive formula, in the NAAR form and this time at $t = 1$, yields

$$(35.295385 + 56.05)(1.06) = 29.14 + (1,000 - 29.14)q_{51},$$

which gives $q_{51} = \boxed{0.0697}$. **(Answer: (C))** □

Remark. (i) The fact that the insurance is "3-year term" plays no role in this question.

(ii) ⚠ Possible reason(s) for getting some of the incorrect answers:

(E) This is the value of q_{52}, which is not what the question requires, but can be obtained by applying the recursive formula at $t = 2$:

$$(29.14 + 56.05)(1.06) = 0 + (1,000 - 0)q_{52} \quad \Rightarrow \quad q_{52} = 0.0903.$$

33. [Section 12.2] (Gross vs. net premiums, and gross vs. net premium policy values)

Similar example(s)/problem(s): Problem 12.2.56

Solution. I. True. The gross premium is always larger than the net premium because the former has to fund the expenses as well. Mathematically, $P^g = P^n + P^e > P^n$ because $P^e > 0$.

II-III. False. With front-loaded expenses (which is the case in this question, according to (ii)), the expense loading is larger than the renewal expenses, leading to a negative expense policy value in renewal years, i.e., ${}_tV^e < 0$ for all $t > 0$.

As a result, the gross premium policy value is smaller than the net premium policy value in renewal years. Mathematically, ${}_tV^g = {}_tV^n + {}_tV^e < {}_tV^n$ for all $t > 0$. **(Answer: (A))** □

34. [Section 12.2] (FPT reserve for an endowment insurance)

Similar example(s)/problem(s): Example 12.2.23 and Problem 12.2.74

Solution. By (12.2.14), ${}_{10}V^{\text{FPT}} = 1,000\left(1 - \dfrac{\ddot{a}_{80:\overline{10|}}}{\ddot{a}_{71:\overline{19|}}}\right)$. From the SULT, $\ddot{a}_{80:\overline{10|}} = 6.7885$ and $\ddot{a}_{70:\overline{20|}} = 11.1109$. By the recursive formula for $\ddot{a}_{x:\overline{n|}}$,

$$\ddot{a}_{70:\overline{20|}} \overset{(10.1.14)}{=} 1 + vp_{70}\ddot{a}_{71:\overline{19|}} \quad \Rightarrow \quad \ddot{a}_{71:\overline{19|}} = \frac{11.1109 - 1}{(1 - 0.010413)/1.05} = 10.728157.$$

Hence ${}_{10}V^{\text{FPT}} = 1,000\left(1 - \dfrac{6.7885}{10.728157}\right) = \boxed{367.23}$. **(Answer: (C))** □

Remark. (i) You can also calculate $\ddot{a}_{71:\overline{19|}}$ as $\ddot{a}_{71} - {}_{19}E_{71}\ddot{a}_{90} = \ddot{a}_{71} - \dfrac{1}{1.05^{19}}\left(\dfrac{l_{90}}{l_{71}}\right)\ddot{a}_{90}$, but the calculations are slightly more involved than using the recursive formula for $\ddot{a}_{x:\overline{n|}}$.

(ii) ⚠ Possible reason(s) for getting some of the incorrect answers:

(E) This is the net premium policy value ${}_{10}V^n = 1,000\left(1 - \dfrac{\ddot{a}_{80:\overline{10|}}}{\ddot{a}_{70:\overline{20|}}}\right) = 389.02$.