## Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance

Solutions Manual Fourth Edition

**Robert L. Brown** Ph.D., FSA, FCIA, ACAS **W. Scott Lennox** FSA, FCIA, FCAS

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Requests for permission should be addressed to: ACTEX Learning PO Box 715 New Hartford CT 06057

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#### SOLUTIONS TO THE TEXTBOOK EXERCISES

### **Chapter 1**

- 1.1 (a) The law of large numbers states that as the number of observations increases, the difference between the observed relative frequency of an event and the true underlying probability tends to zero.
  - (b) The risk to the insurance company is not equal to the sum of the individual risks (variance of total outcome) transferred to it.
- $1.2 \quad u(x) = k \cdot \log x$

$$u'(x) = \frac{k}{x} = kx^{-1}$$
$$u''(x) = -kx^{-2}$$

Since u'(x) > 0 and u''(x) < 0, this decision maker is risk averse.

1.3 To reflect the risk attribute, we use utility value rather than monetary value.

EUV(A) = 1(.6) + .5(.1) + 0(.3) = .65EUV(B) = .9(.5) + .8(.3) + .2(.2) = .73

A risk avoider would choose Proposal B.

1.4 (a) EMV(X) = 50,000(.35) - 20,000(.65) = 4500EMV(Y) = 5,000(.55) - 5,000(.45) = 500

Both choose X based on expected monetary value.

(b) A: EUV(X) = 1.00(.35) + .30(.65) = .545EUV(Y) = .55(.55) + .45(.45) = .505

Businessman A chooses X based on expected utility value.

B: 
$$EUV(X) = 1.00(.35) + .55(.65) = .70750$$
  
 $EUV(Y) = .77(.55) + .709(.45) = .74255$ 

Businessman B chooses Y based on expected utility value.

1.5 (a) 
$$u(P) = \sqrt{P - 1000} = (P - 1000)^{1/2}$$
  
 $u'(P) = \frac{1}{2}(P - 1000)^{-1/2}$   
 $u''(P) = -\frac{1}{4}(P - 1000)^{-3/2}$ 

Management is risk averse since u'(P) > 0 but u''(P) < 0.

 (b) (i) EMV(A) = 3000(.10) + 3500(.20) + ... + 5000(.10) = 4000 EMV(B) = 2000(.10) + 3000(.25) + ... + 6000(.10) = 4000 The EMV is the same for both proposals, so management would be indifferent on this basis.

#### (ii) First we find the utility value of each profit amount.

Proposal A			Proposal B			
Utility	Probability	Profit	Utility	Probability		
44.72	.10	2,000	31.62	.10		
50.00	.20	3,000	44.72	.25		
54.77	.40	4,000	54.77	.30		
59.16	.20	5,000	63.24	.25		
63.24	.10	6,000	70.71	.10		
$EUV(A) = 44.72(.10) + 50.00(.20) + \dots + 63.24(.10)$ = 54.536 $EUV(B) = 31.62(.10) + 44.72(.25) + \dots + 70.71(.10)$ = 53.654						
	Proposal Utility 44.72 50.00 54.77 59.16 63.24 (A) = = (B) = =	Proposal A <u>Utility</u> Probability 44.72 .10 50.00 .20 54.77 .40 59.16 .20 63.24 .10 (A) = 44.72(.10) + 5 = 54.536 (B) = 31.62(.10) + 4 = 53.654	Proposal AUtilityProbabilityProfit $44.72$ .102,000 $50.00$ .203,000 $54.77$ .404,000 $59.16$ .205,000 $63.24$ .106,000 $(A)$ = $44.72(.10) + 50.00(.20) + = 54.536$ $(B)$ = $31.62(.10) + 44.72(.25) + = 53.654$	Proposal AProposalUtilityProbability $Profit$ Utility44.72.102,00031.6250.00.203,00044.7254.77.404,00054.7759.16.205,00063.2463.24.106,00070.71 $(A) = 44.72(.10) + 50.00(.20) + \dots + 63.24)$ $= 54.536$ $(B) = 31.62(.10) + 44.72(.25) + \dots + 70.7)$ $= 53.654$		

Management chooses A based on expected utility value. NOTE: We can also see that Proposal B has the larger variance.

1.6 (a) EMV (no insurance) = 10,000(p) + 30,000(1-p)EMV (insurance) = 20,000(p) + 25,000(1-p)Equating and solving for *p* we have

10,000p + 30,000(1-p) = 20,000(p) + 25,000(1-p),which solves for  $p = \frac{5}{15} = \frac{1}{3}.$  (b) The utility values of the various profit amounts are as follows.

Freeze<br/>No InsuranceNo Freeze<br/>71No Freeze<br/>158Insurance123141EUV (no insurance)= 71(p)+158(1-p)EUV (insurance)= 123(p)+141(1-p)Equating and solving for p we have<br/>71p+158(1-p)= 123p+141(1-p),which solves for  $p = \frac{17}{69} = .2464.$ 

1.7 With insurance your utility position is  $\left(\frac{50,000-G}{10,000}\right)^9$ .

Without insurance your expected utility position is

$$\int_{0}^{30,000} \left(\frac{50,000-x}{10,000}\right)^{.9} \left(\frac{1}{30,000}\right) dx$$
  
=  $-\left(\frac{1}{1.9}\right) \left(\frac{1}{10,000}\right)^{.9} \left(\frac{1}{30,000}\right) (50,000-x)^{1.9} \Big|_{0}^{30,000}$   
=  $\left(\frac{1}{1.9}\right) \left(\frac{1}{10,000}\right)^{.9} \left(\frac{1}{30,000}\right) \left[ (50,000)^{1.9} - (20,000)^{1.9} \right]$ 

Equate the two expected utility positions and solve for G.  $(50,000-G)^{.9} = \left(\frac{1}{1.9}\right) \left(\frac{1}{30,000}\right) \left[(50,000)^{1.9} - (20,000)^{1.9}\right].$  = 12,258.46Then we have  $G = 50,000 - (12,258.46)^{10/9} = 15,109.54.$ 

1.8 With no wager your wealth is 20,000 and your expected utility position is  $1 - \exp\left(-\frac{20,000}{100,000}\right) = .1812692$ .

If you wager an amount w, you end up with either 30,000 - w (if you win) or 20,000 - w (if you lose). The expected utility position is

$$\frac{1}{2} \left[ 1 - \exp\left(-\frac{30,000 - w}{100,000}\right) \right] + \frac{1}{2} \left[ 1 - \exp\left(-\frac{20,000 - w}{100,000}\right) \right].$$

Equating and solving for *w* we have

$$.1812692 = 1 - \frac{1}{2} (.7408182) \cdot \exp\left(\frac{w}{100,000}\right)$$
$$-\frac{1}{2} (.8187307) \cdot \exp\left(\frac{w}{100,000}\right)$$
$$.8187308 = (.779774) \cdot \exp\left(\frac{w}{100,000}\right)$$
$$\exp\left(\frac{w}{100,000}\right) = 1.049958$$
$$w = 4,875.05$$

1.9 With no wager the wealth is 3000, with expected utility value  $10,000(3000) - (3000)^2 = 21,000,000$ . With the wager the wealth is either 5000 (if she wins) or 3000 - w (if she loses), with expected utility value

$$.30 [10,000(5000) - (5000)^{2}] + .70 [10,000(3000 - w) - (3000 - w)^{2}].$$

Equating and solving for *w* we have

$$21,000,000 = 7,500,000 + .7[21,000,000 - 4000w - w^{2}],$$

which solves for

$$w = \frac{-2800 \pm \sqrt{(2800)^2 + 4,800,000(.7)}}{2} = 390.46.$$

1.10 
$$\mu = \frac{1}{3}(2000 + 4000 + 6000) = 4000$$
  
 $\sigma^2 = \frac{1}{3}[(2000 - 4000)^2 + (4000 - 4000)^2 + (6000 - 4000)^2]$   
 $= \frac{8,000,000}{3}$   
 $\sigma = \sqrt{\sigma^2} = 2000\sqrt{2/3} = 1632.99$   
 $u = \mu + \frac{\sigma}{6} = 4000 + \frac{1632.99}{6} = 4272.17$ 

The gross premium is 4500. Since this exceeds the expected utility loss, no insurance will be purchased.

1.11 (a) The gross premium is 1.10% of the expected loss, so we have

 $GP = 1.10 \cdot E(L)$ = 1.10[10,000(.15) + 20,000(.04) + 50,000(.01)] = 3080 Utility with insurance = U(525,000 - G) = 13.16527 Utility without insurance = .80u(525,000) + .15u(515,000) + .04u(505,000) + .01u(475,000) = 13.16571441  $\therefore$  Do not buy insurance.

(b) This time the company's gross premium is

$$GP = 1.10 \cdot E(L)$$
  
= 1.10[20,000(.04)(.50) + 50,000(.01)(.50)] = 715

Mr. Smith's expected utility position, with insurance, is (G = 715)

$$\begin{aligned} .80u(525,000-G) + .15u(525,000-10,000-G) \\ &+ .04u(525,000-10,000-G) \\ &+ .01u(525,000-25,000-G) = 13.16564312. \end{aligned}$$

His expected utility position without insurance is 13.165714, from part (a).

: Do not buy insurance

- 1.12 The criteria to review (see Section 1.4) include the following:
  - 1. Economically feasible (OK)
  - 2. Economic value is calculable (common; should be)
  - 3. Loss must be definite (OK)
  - 4. Loss must be accidental (should be if no profit)
  - 5. Exposures in risk class homogeneous (OK)
  - 6. Units spatially and temporally independent (OK)

Yes, the insurance purchase is appropriate.

- 1.13 (a) This is really the same as Exercise 1.12; the risk is insurable.
  - (b) The net single premium is the expected value of the present value of the insurance payment. If *Z* denotes the random present value of payment, then

$$NSP = E[Z]$$
  
= 8000(1.10)<sup>-1</sup>( $\frac{1000}{7000}$ ) + 5000(1.10)<sup>-2</sup>( $\frac{1500}{7000}$ ) + 0 = 1924.44  
(c)  $Var(Z) = E[Z^2] - (E[Z])^2$   
 $E[Z^2] = [8000(1.10)^{-1}]^2(\frac{10}{70}) + [5000(1.10)^{-2}]^2(\frac{15}{70})$   
= 11,215,081  
 $Var(Z) = 11,215,081 - 3,703,469 = 7,511,612$ 

- 1.14 Gambling: Creates risk where none exists or needed to. Takes dollars of high marginal utility. If you win you get dollars of lower marginal utility (if risk averse).
  - Insurance: Transfers risk through pooling techniques. Takes dollars of low marginal utility and protects dollars of high marginal utility. In total, society has higher total utility with insurance than without.
- 1.15 Risk:
   A measure of variation in economic outcomes

   e.g.:
   risk of a monetary loss if house burns down

   Peril:
   A cause of risk

   e.g.:
   fire, wind, theft, illness
  - Hazard: A contributing factor to the peril
  - e.g.: poor wiring, location, moral hazard

### Chapter 2

- 2.1 All-risks or comprehensive covers everything except what is specifically excluded. Specified perils only covers the named perils. All-risks cover will exclude several perils, such as nuclear radio-activity, war, wear and tear, so it is not absolutely all-risks.
- 2.2 (a) Salvage: Once the insurer has paid the policyholder full compensation for damaged property, it assumes ownership of the property and can sell it for its salvage value. This decreases the premium for the coverage.
  - (b) Subrogation: The insurer, having indemnified the policyholder, acquires the legal rights of the policyholder to sue the party at fault and recover costs. This will lower some premiums (e.g., homeowners dwelling coverage or auto collision), and raise the corresponding premium on the liability cover.
- 2.3 A loss is covered by a policy only if a covered peril is the proximate cause of a covered consequence (both are needed). A covered peril is the proximate cause if it is the cause that initiates an unbroken sequence of events leading to a covered consequence.
- 2.4 (a) Objectives of the coinsurance clause:
  - 1. It encourages insurance to value.
  - 2. It creates premium equity between insureds.
  - 3. The overall rate level can be smaller but still adequate.
  - (b) Disadvantages of the coinsurance clause:
    - 1. Not well understood by policyholder.
    - 2. A policyholder who buys less than full coverage is only penalized if there is a claim, since he or she can pay a lower premium and get away with it.

- 3. The 80% coinsurance clause discriminates against those who carry higher levels of insurance.
- 4. Because of the misunderstanding of the coinsurance clause, some costly disputes arise over its use and meaning.
- 5. With high rates of inflation in real estate, a homeowner may unwittingly fall below the coinsurance percentage requirement.
- 6. A coinsurance percentage of less than 100% implies a recommendation to policyholders to buy less than full coverage.
- 2.5 Find X such that  $X\left(\frac{400,000}{80\% \text{ of } 800,000}\right) = 320,000.$ The equation solves for X = 512,000.
- 2.6 Find X such that  $\frac{120,000}{X\% \text{ of } 200,000} \cdot 10,000 = 7,500$ . The equation solves for X = 80%.
- 2.7 The payment would be  $\frac{120,000}{70\% \text{ of } 200,000} \cdot 175,000 = 150,000.$ But the payment will be limited to the policy limit of 120,000.

The deductible is  $X = \frac{2}{3}(0) + \frac{1}{3}(250) = 83.33$ , so the payment is 750 - 83.33 = 666.67.

- 2.9 (a) Contributory negligence: It used to be that if a worker contributed in any way to the injury or sickness, then the worker could not seek compensation.
  - (b) Fellow-servant: If a fellow worker contributed in any way to the worker's injury or sickness, then the employer was not at fault and the worker could not seek compensation.
  - (c) Assumption-of-risk: The ability of the worker to sue was often restricted if the worker had advance knowledge of the inherent dangers of the job.
- 2.10 Objectives of workers compensation:
  - 1. Broad coverage of workers for occupational injury and disease.
  - 2. Substantial protection against loss of income.
  - 3. Sufficient medical care and rehab services.
  - 4. Encouragement of safety (through lower premiums).
  - 5. An efficient and effective administrative system.
- 2.11 Workers compensation benefits:
  - 1. Medical care benefits (normally unlimited).
  - 2. Disability income benefits (after waiting period).
  - 3. Death benefits including a burial allowance plus cash-income payments to eligible survivors.
  - 4. Rehab services and benefits.

- 2.12 Advantages:
  - 1. Gets rid of small claims and their expenses.
  - 2. All losses are reduced by amount of deductible, so premium is lower.
  - 3. Provides an economic incentive for the policyholder to prevent a claim.
  - 4. Policyholders can optimize the use of their limited premium dollars by using deductibles to save money where the utility value of the coverage is not as great.

Disadvantages:

- 1. Insured may be disappointed by being put at risk.
- 2. Can lead to misunderstandings and bad public relations.
- 3. Makes the marketing of the coverage more difficult.
- 4. The insured may just inflate the claim to recover the deductible which, in turn, penalizes the honest policyholder because of the resulting higher premium.
- 2.13 (a) Pay (.80)(12,000) = 9600, which is within the policy limit.
  - (b) Pay (12,000 1,000) = 11,000; but the payment is limited to the policy limit of 10,000.

(c)	Claim	5000	12,000	15,000
	Deductible	5000	X	0

The deductible is  $X = \left(\frac{3}{10}\right)(5000) + \left(\frac{7}{10}\right)(0) = 1,500;$ the payment is 12,000 - 1,500 = 10,500. 2.14 Let *L* denote the loss. If L < d, the claim payment is 0, and if L > d, the claim payment is L - d.

$$E[L] = \frac{1}{10} \left[ \int_0^d 0 \, dL + \int_d^{10} (L-d) \, dL \right]$$
$$= \frac{1}{10} \left[ \frac{1}{2} L^2 - dL \Big|_d^{10} \right] = \frac{1}{10} \left[ 50 - 10d + \frac{1}{2} d^2 \right]$$

Since this must equal 2, we find d from

$$2 = \frac{1}{10} \left[ 50 - 10d + \frac{1}{2}d^2 \right],$$

which solves for d = 3.68.

2.15 250 Deductible:

$$E[L] = \frac{1}{5000} \left[ \int_0^{250} 0 \, dL + \int_{250}^{5000} (L - 250) \, dL \right]$$
$$= \frac{1}{5000} \left[ \frac{1}{2} L^2 - 250L \Big|_{250}^{5000} \right] = 2256.25$$

500 Deductible:

$$E[L] = \frac{1}{5000} \left[ \int_0^{500} 0 \, dL + \int_{500}^{5000} (L - 500) \, dL \right]$$
$$= \frac{1}{5000} \left[ \frac{1}{2} L^2 - 500L \Big|_{500}^{5000} \right] = 2025.00$$

The expected loss payment will be reduced by 2256.25 - 2025.00 = 231.25.

- 2.16 Reasons for policy limits:
  - 1. Clarifies obligation of insurer.
  - 2. Provides an upper bound on risk to insurer, decreasing the probability of insurer insolvency. Also decreases the premium.
  - 3. Makes sure that policyholder cannot profit from a loss.
  - 4. Allows the policyholder to choose the most appropriate coverage at an appropriate price.