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Fall 2017 Edition | Volume I

Johnny Li, P.h.D., FSA
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Preface

Thank you for choosing ACTEX Learning.

Since Exam MFE was introduced in May 2007, there have been quite a few changes to its syllabus and its learning objectives. To cope with these changes, ACTEX decided to launch a brand new study manual, which adopts a completely different pedagogical approach.

The most significant difference is that this edition is fully self-contained, by which we mean that, with this manual, you do not even have to read the “required” text (Derivatives Markets by Robert L. McDonald). By reading this manual, you should be able to understand the concepts and techniques you need for the exam. Sufficient practice problems are also provided in this manual. As such, there is no need to go through the textbook’s end-of-chapter problems, which are either too trivial (simple substitutions) or too computationally intensive (Excel may be required). Note also that the textbook’s end-of-chapter problems are not at all similar (in difficulty and in format) to the questions released by the Society of Actuaries (SoA).

We do not want to overwhelm students with verbose explanations. Whenever possible, concepts and techniques are demonstrated with examples and integrated into the practice problems.

Another distinguishing feature of this manual is that it covers the exam materials in a different order than it occurs in Derivatives Markets. There are a few reasons for using an alternative ordering:

1. Some topics are repeated quite a few times in the textbook, making students difficult to fully understand them. For example, “estimation of volatility” is discussed three times in Derivatives Markets (Sections 10.2, 12.5, 18.5)! In sharp contrast, our study manual presents this topic fully in one single section (Module 3, Lesson 5.1).

2. The focus of the textbook is somewhat different from what the SoA expects from the candidates. According to the SoA, the purpose of the exam is to develop candidates’ knowledge of the theoretical basis. Nevertheless, the first half of the textbook is almost entirely devoted to applications. Therefore, we believe that reading the textbook or following the textbook’s ordering is not the best use of your precious time.

To help you better prepare for the exam, we intentionally write the practice problems and the mock exams in a similar format as the released exam and sample questions. This will enable you to, for example, retrieve information more quickly in the real exam. Further, we have integrated the sample and previous exam questions provided by the SoA into the study manual into our examples, our practice problems, and our mock exams. This seems to be a better way to learn how to solve those questions, and of course, you will need no extra time to review those questions.
Our recommended procedure for use of this study manual is as follows:

1. Read the lessons in order.
2. Immediately after reading a lesson, complete the practice problems we provide for that lesson. Make sure that you understand every single practice problem.
3. After studying all 20 lessons, work on the mock exams.

If you find a possible error in this manual, please let us know at the “Customer Feedback” link on the ACTEX homepage (www.actexmadriver.com). Any confirmed errata will be posted on the ACTEX website under the “Errata & Updates” link.
**A Note on Rounding and the Normal Distribution Calculator**

To achieve the desired accuracy, we recommend that you store values in intermediate steps in your calculator. If you prefer not to, please keep at least six decimal places.

In this study guide, normal probability values and $z$-values are based on a normal distribution calculator instead of a normal table in other exams. In the actual examination you would also use the same normal distribution calculator.

The calculator is very easy to use. Simply go to


Recall that $N(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \, dz$ is the cumulative distribution function of a standard normal random variable. To find $N(x)$, you may use the first panel of the calculator. Type in the value of $x$ and press “Normal CDF”. Then you would get $N(x)$. For example, when $x = -1.282$, the calculator would report 0.09992.

To find the $100p$th percentile of the standard normal random variable (i.e. to find the value of $x$ such that $N(x) = p$), enter $p$ into the cell adjacent to $N(x)$, and press “Inverse CDF”. Then you would get $x$. For example, when $N(x) = 0.25$, the calculator would report –0.67449.

If you do not want to go online every time when you follow the examples and work on the practice problems, you can set up your own normal distribution calculator using Excel. Open a blank workbook, and set up the following

```
Cell A1:  x
Cell A2:  N(x)
Cell B1: -1.282
Cell B2: = round(normsdist(B1), 5)

Cell A5:  N(x)
Cell A6:  x
Cell B5: 0.25
Cell B6: = round(normsinv(B5), 5)
```

Cell B2 would report 0.09992 and Cell B6 would report –0.67449 if you are using Excel 2010 or more recent versions of Excel. You can alter the values in B1 and B5 to calculate any probability and percentile. Save your workbook for later use.
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Lesson 1: Stock as an Underlying Asset

Lesson 1  Stock as an Underlying Asset

OBJECTIVES

1. To understand the long and short position in a stock and a stock index paying dividends continuously

2. To understanding the following terms: bid-ask spread, lease payment, credit risk, exchange, clearinghouse

3. To understand the payoff of European calls and puts

There are thousands of financial instruments in today’s financial world. In Exam MFE, you would be introduced an important class of financial instruments known as derivatives. As its name suggests, derivatives are “derived” from some more fundamental financial instruments known as underlying assets. Before we start our journey of derivatives, we need to understand the underlying assets. In this first lesson we focus on stocks.

1.1.1 Financial Markets

This section provides with you some factual information. The chance that you would be tested on these materials is slim.

You must have heard of the term “financial market”. In economics,

- a market refers to a variety of systems, institutions, procedures and also the possible buyers and sellers of a certain good or service;

- a financial market is a market in which people trade different kinds of financial securities, including bonds that you have learnt in Exam FM.

Shopping in a financial market is quite different from shopping in a mall. When you enter a grocery store, you become a potential buyer. The store, which is the seller, lists the prices of the goods. You pay the price, and then you get the goods. Sometimes you may ask for the goods to be delivered to you within (say) 10 days after purchase if the goods is too bulky or is stored in a
warehouse. You can also be a seller, too, if you open your own grocery store, in which case you may be setting prices. Notice that in any transaction, there would be two parties: the buyer and the seller. Later on we would use “long” to refer to buyers and “short” to refer to sellers.

Now let us consider what would happen if you want to trade (which can mean “buy” or “sell”) stocks, bonds, or derivatives. Such financial assets are certainly not traded in a grocery. As a matter of fact, many financial assets do not physically exist; they only exist on electronic records and represent an ownership or rights to do something. Nowadays you would not get a large pile of bond certificates and coupons when you buy a coupon bond! The trading would typically involve at least 4 steps.

**Step 1:** The buyer and seller locate each other and agree on a price.

**Step 2:** The trade is then cleared. It means that the obligations of the buyer and the seller are specified. For example, the buyer agrees to pay the seller by a specified date and the seller agrees to deliver the asset upon receiving payment.

**Step 3:** The trade is then settled. It means that the buyer and the seller fulfill the obligations.

**Step 4:** A change of ownership of the financial asset is recorded. The trade is completed.

In real life, it is hard for buyers and sellers to find each other. **Step 1** is facilitated by brokers, dealers and sometimes exchanges. Stocks are usually traded in an organized exchange, where rigorous rules that govern trading and information flows exist. In the US, we have, for example, the New York Stock Exchange (NYSE) for stocks and the Chicago Board Options Exchange (CBOE) for many derivatives. For other assets you can go to an over-the-counter (OTC) market. For OTC markets there is not a physical location where trading takes place. Trading is also less formal. In both cases the buyer or seller would contact a broker, who then contacts a market maker to create a trade.

Market makers are traders who will buy assets from customers who wish to sell, and sell financial assets from customers who wish to buy.

Just like a supermarket which buys commodities from producers at a low wholesale price and then sells at a high retail price, market makers buy financial assets from customers at a low bid price and sell financial assets to customers (who ask the market maker for the financial asset) at a high ask price (also called offer price). The difference between the two prices is called the bid-ask spread. Moreover, for every trade you have to pay brokerage fee to the broker. When an investor buys stocks from an exchange (at ask price), he or she is actually buying from the market maker. When an investor sells stock to the exchange, he or she is actually selling to the market maker (at bid price). The bid-ask spread is small for liquid assets.

Brokers may keep the financial asset for the investors. One may ask for a physical delivery of the asset (e.g. the certificate of ownership for a stock, if that ever exists) but usually investors would not do that (or else he has to delivery it to the broker if he wishes to sell it later).

There is also a clearinghouse that would keep track or all obligations and payments for **Step 2** and **Step 3**.
Example 1.1.1

Stock XYZ is bid at $49.75 and offered at $50, and the brokerage fee is 0.3% of the bid or ask price. Suppose you buy 100 shares, and then sell the 100 shares after half an hour. What is the round-trip transaction cost if the bid price does not change?

Solution

Time 0: Buy 100 shares at $50: pay $50 \times (1 + 0.3\%) \times 100 = 5015$

Time 0.5 hrs: Sell 100 shares at $49.75: receive $49.75 \times (1 - 0.3\%) \times 100 = 4960.075$

Total transaction cost $= 5015 - 4960.075 = 54.925.$

We can break down the transaction cost into two components:

- Transaction cost as brokerage fee: $50 \times 0.3\% \times 100 + 49.75 \times 0.3\% \times 100 = 29.925$

- Transaction as bid-ask spread: $0.25 \times 100 = 25$

[END]

1.1.2. Stocks and Stock Indexes

You buy stocks to make profit if you expect the stock price to go up. To put it simply, we assume that a stock is very liquid and that there is no bid-ask spread, so that there is a single price:

1. You buy one share at the current price $S_0$.
2. You own the stock, and hence you are entitled to cash dividends (if any) of the stock.
3. You sell the stock at time $T$ when the price is $S_T$. So you earn $S_T - S_0$, ignoring interest lost on initial investment of $S_0$.

Of course you can elaborate the whole story by incorporating some of the fine details in the previous section. Here is the more complete story:

Buying Stocks

- You purchase stocks from an exchange. To buy one share of a stock, you pay the current (ask) price of one share. The broker then purchases the stock from a stock exchange. Later when you sell the stock, you contact the broker again and the broker would do the necessary work for you. You will receive the stock (bid) price at the time when you sell the stock.

- Unlike purchasing commodities, you will not hold the share physically. What you have is a long position in the stock in a stock account.

- Some stocks provide dividends. For example, if you have 100 shares of a stock that is currently priced at 25 and is going to pay a dividend of 0.1 per share tomorrow, then tomorrow you will receive $100 \times 0.1 = 10$ dollar amount of dividends.
After paying dividends, the per share price drops by the amount of dividend per share. In the illustration above, the cum-dividend (bid) price is 25, and the ex-dividend price is $25 - 0.1 = 24.9$.

Selling Stocks

If you think the price of the stock is going to decline, how can you make a profit? You can borrow some stock from a broker and do a short-selling:

1. **You borrow one share from a lender.**
2. **You sell the stock at (bid) price $S_0$ to a market maker and thus receive a cash of $S_0$; the amount $S_0$ is usually called the **proceeds from short selling**.
3. **Later you buy back the stock from the market at a lower (ask) price $S_T$ and return it to the lender, so you capture a profit of $(S_0 - S_T)$, ignoring interest received by investing $S_0$.**
4. **If the stock pays any dividends (or interest or coupons if you were short selling a bond) before you cover the short position, you have to pay dividends to the lender. If the asset is a commodity, such payments are called **lease payments**. Lease payment is the payment that one has to make when he borrows an asset.**

In the above the short seller must always be prepared to cover the short position (since the lender has the right to sell his assets any time), so he has a liability of $S_t$ at time $t$ before closing the position. However, in practice a short seller typically borrows through a broker (e.g. investment bank), who usually holds a large amount of assets for other investors who go long (e.g. pension funds, mutual funds). If the lender of the asset, who in most cases does not even know that his assets are borrowed, would like to sell the asset, the broker can transfer the asset from another investor to the lender.
To avoid the short seller from going away after receiving $S_0$ without covering the short (such is credit risk), the broker may seize $S_0$ at the beginning as collateral. When the short seller covers the position, the broker returns $S_0$, plus interest. The rate paid on $S_0$ is called the repo rate in bond markets and short rebate in stock markets.

**Example 1.1.2**

You short-sell 400 shares of XYZ, which has a bid price of $25.12 and an ask price of $25.34. You cover the short position 6 months later when the bid price is $22.91 and the ask price is $23.06. There is a 0.3% brokerage commission in the short-sale.

(a) What profit do you earn in the short sales? Ignore interest on the proceeds from short sells.

(b) Suppose that the 6-month (non-continuously compounded) interest rate is 3.5% and that you are paid 2.5% on the short-sale proceeds. How much interest do you lose?

**Solution**

(a) At the beginning you would receive $400 \times 25.12 \times (1 - 0.3\%) = 10017.856$

At the end you have to close the position using $400 \times 23.06 \times (1 + 0.3\%) = 9251.672$. Thus the total profit is 766.184.

(b) $10017.856 \times 1\% = 100.18$

**Stock Indexes**

You would probably have heard of stock indexes. S&P 500 is a prominent example in the US stock market. It tracks the movement of the US stock market. There are also indexes that track the performance of stocks of a particular sector (e.g. NASDAQ).

A stock index is an average of a collection of stock prices.

The weighting in the average is usually not uniform. Stocks with a greater market capitalization would be more heavily weighted. The collection of stocks used can also change with time.

One can try to replicate a stock index by purchasing the component stocks at the correct weighting. (It is more easily said than done. S&P500 contains 500 stocks, as its name suggests!). Different stocks in the collection pay dividends at different time. When we look at a large portfolio of stocks, dividends would be paid very frequently. As an approximation,

We assume that the dividends from a stock index are paid continuously at a rate that is **proportional to the level of the index**. The **dividend yield** of an index is defined as the annualized dividend payment divided by the stock index.
The Mathematics of Continuous Dividends

From this subsection onwards we assume no any transaction costs for simplicity.

- Let $S_t$ be the (ex-dividend) stock index at time $t$ and $\delta$ be the dividend yield (which is assumed to be constant). In an infinitesimally short time interval $(t, t + dt)$, the non-annualized dividend yield is $\delta dt$, and the dollar amount of dividends per “share” is $S_t (\delta dt)$.

- Suppose that we own 1 “share” of the stock index at time 0 and we use the dividends to buy extra shares. The number of shares we own would gradually increase.

Let $n_t$ be the number of shares at time $t$. Since the additional number of shares purchased in $(t, t + dt)$ is $dn_t$, we have

$$\text{cost of purchasing extra shares } = \text{dividend income at } (t, t + dt)$$

$$S_t dn_t = n_t (\delta S_t dt)$$

or

$$\frac{dn_t}{dt} = \delta n_t.$$

Since $n_0 = 1$, the solution is $n_t = \exp(\delta t)$.

So if we have 1 share at the beginning, then we will have $e^{\delta T}$ shares at time $T$.

![Diagram: 1 share at time 0, $e^{\delta T}$ shares at time $T$]

The result above should be compared with the situation when we deposit one dollar in a bank account which credits interest continuously at a rate of $r$.

![Diagram: 1 dollar at time 0, $e^{r T}$ dollars at time $T$]

What about discounting? If we want to have 1 dollar at time $T$, we only need to put $e^{-r T}$ in a bank account at time 0 and then reinvest all interests in the bank account. Similarly, if we want to have 1 “share” of the index at time $T$, we only need to buy $e^{-\delta T}$ “shares” of the index at time 0 and reinvest all dividends in the index:

![Diagram: $e^{-\delta T}$ shares at time 0, 1 share at time $T$]
Sometimes we would like to calculate the profit or loss when we close out all positions in an investment. For stock and stock indexes, cash flows would involve the initial cost of buying the stock (or proceeds from short-selling), dividends received (or paid) before closing out all positions, and the final value of the stock.

The profit (or net payoff) is computed as follows:

Profit over investment horizon \([0, T]\) = value of the final position at \(T\) 
+ accumulated value of any income received in \((0, T)\) 
− the accumulated cost to set up the position at time 0.

Example 1.1.3

(a) Suppose that stock \(X\) is currently priced at 30 per share and the company has announced that it is going to pay a dividend of 0.3 per share after 2 months. You purchase 100 shares of stock \(X\) and invest all dividends received at a continuously compounded risk-free interest rate of 5%. After 3 months, you sell the stock when the stock price is 35.4. Calculate the 3-month profit.

(b) Suppose that stock index \(Y\) is currently valued priced at 25 and the index pays dividends continuously at a rate proportional to its price at a constant rate of 3%. You purchase 200 units of the index and invest all dividends into the index. The continuously compounded risk-free interest rate is 5%. After 4 months, you close out all positions when the index value is 25.9. Calculate the 4-month profit.

Solution

(a) At \(t = 2 / 12\), the stock pays a dividend. The dollar amount of dividends you will receive is 

\[0.3 \times 100 = 30.\]

The accumulated value at \(T = 3 / 12\) is 

\[30e^{0.05 \times 3/12} = 30.1253.\]

The 3-month profit is 

\[35.4 \times 100 – 30 \times 100 \times e^{0.05 \times 3/12} + 30.1253 = 532.390.\]

(b) After 4 months, you have \(200 \times e^{0.03 \times 4/12} = 202.0100\) shares. The 4-month profit is 

\[25.9 \times 202.0100 – 25 \times 200 \times e^{0.05 \times 4/12} = 148.0273.\]

Shorting a Stock Index

Recall that the simplified story for short selling a stock is as follows:

(1) Borrow one share from a broker.
(2) Sell the stock at a price of $S_0$ and thus receive $S_0$ dollars of cash.

(3) Later you buy back the stock from the market at a (lower) price $S_t$ and return it to the broker. So you capture a profit of $(S_0 - S_t)$.

(4) If the stock pays dividends before you cover the short position, then you will need to pay cash dividends to the broker.

If the underlying asset is a stock index that pays dividends at a rate proportional to its price at a constant rate $\delta$, then you can pay dividends by borrowing extra shares. It is like you borrow one dollar and the lender credits interest continuously at rate $r$. If you decide not to pay interest you repaying the principal, the lender would assume the interest to accumulate. This would generate even more interest payments and at the end you need to return $e^{\delta T}$. For a stock index, you will end up with a short position of $e^{\delta T}$ shares at time $T$, and you have to pay $e^{\delta T} S_T$ to cover the short.

**Example 1.1.4**

Assume a continuously compounded risk-free interest rate of 5% for this question.

(a) Stock $X$ is currently priced at 30 per share and the company has announced that it is going to pay a dividend of 0.3 per share after 1 month. You short-sell 50 shares of stock $X$. After 3 months, you cover the short position when the stock price is 33.3.

Calculate the 3-month profit.

(b) Stock $Y$ is currently priced at 25 per share and it pays dividends continuously at a rate proportional to its price at a constant rate of 3%. You short-sell 500 shares of stock $Y$ and repay dividends by borrowing extra shares of stock $Y$. After 2 months, you cover the short position when the stock price is 24.

Calculate the 2-month profit.

**Solution**

(a) At $t = 0$, you receive a cash of $30 \times 50 = 1500$ from short-selling. At $t = 1/12$, you need to pay $0.3 \times 50 = 15$.

The accumulated value at $T = 3 / 12$ is $15 \times \exp(0.05 \times 2/12) = 15.1256$.

The 3-month profit is

$$1500 \times \exp(0.05 \times 3/12) - 33.3 \times 50 - 15.1256 = -161.258.$$ 

(b) After 2 months, you have borrowed $500 \times \exp(0.03 \times 2/12) = 502.5063$ shares. The 2-month profit is

$$25 \times 500 \times \exp(0.05 \times 2/12) - 24 \times 502.5063 = 544.45.$$ 

[END]
Module 1: Introductory Derivatives
Lesson 1: Stock as an Underlying Asset

1.1.3 Derivative Securities

A derivative security is a financial instrument or contract that has a value determined by the price of something else. Recall that the “something else” here is called an underlying asset. The interest rate swap that you have learnt in Exam FM is a derivative on future interest rates.

You have also seen something similar in Exam P. Consider a reinsurance contract. Suppose that an insurance company has a risk exposure of $100 million to hurricane and wants to limit losses. It can enter into annual reinsurance contracts that cover on a pro rata basis 70% of its losses, subject to a deductible of $10 million. If in a year the total hurricane claims total $60 million, then the reinsurance company would pay the insurance company $35 million and the losses of the insurance company would only be $25 million.

In what follows we use the notation \( x_+ = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \) in payoffs. The payoff on the reinsurance contract is

\[ Y = 0.7 \max(X - 10, 0) = 0.7(X - 10)_+, \]

where \( X \) is the actual claim size. As we shall see immediate below, this contract is actually a European call on the claim.

A derivative’s payoff can be dependent on a variety of assets. In most cases, the underlying asset is usually a commodity or a financial asset:

(a) Commodity includes metals (e.g. gold, copper), agricultural products (e.g. hog, corn) and energy (e.g. crude oil, electricity, natural gas). Even temperature can be an underlying “asset”.

(b) Financial asset includes stocks, stock indexes, bonds and also futures and foreign currencies that we are going to introduce in Lesson 4 of this module.

Now let us look at a class of derivatives known as European options. The term “European” refers to the property that the option only gives payoffs at maturity. Unless otherwise stated, options are understood to be European. Suppose we are standing at time 0. The time-\( T \) stock price, \( S_T \), is random. Let \( K \) be a positive constant.

There are two basic types of options:

- A call option gives the holder the right (but not the obligation) to buy the underlying asset on a certain date \( T \) for a certain price \( K \).
- A put option gives the holder the right (but not the obligation) to sell the underlying asset on a certain date \( T \) for a certain price \( K \).

\( T \) is called the expiration date or maturity date.

\( K \) is called the strike price or exercise price of the option.
We denote the price / premium of a $T$-year $K$-strike European call option at time 0 by $c(S_0, K, T)$. The time-$T$ payoff is

$$(S_T - K)_+ = \max(S_T - K, 0)$$

because the option holder would choose not to exercise the call and walk away if the stock is not worth $K$ after $T$ years.

Similarly, we denote the price premium of a $T$-year $K$-strike European put option at time 0 by $p(S_0, K, T)$. The time-$T$ payoff is

$$(K - S_T)_+ = \max(K - S_T, 0)$$

because the option holder may choose not to exercise the option and walk away if the stock is worth more than $K$ after $T$ years.

The payoff diagrams for a long call and a long put position are respectively

![Payoff Diagrams](image)

The payoff diagrams for a short call and a short put position (i.e. the position of the seller / writer of the option) are

A short position in a call option:  

![Payoff Diagrams](image) 

A short position in a put option:  

![Payoff Diagrams](image)
The payoffs, profits and costs for long and short positions are always opposite in signs. Later on if you find the payoff or cost for a particular short position hard to compute or visualize, you can simply calculate the associated quantity assuming that the position is long and then flip the sign, as follow:

**Long stock:**

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Cost at time 0</th>
<th>Payoff at time $T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy one share and invest cash dividends at rate $r$</td>
<td>$S_0$</td>
<td>$S_T + \text{accumulated value of dividends}$</td>
</tr>
</tbody>
</table>

Time-$T$ profit = $S_T + \text{accumulated value of dividends} - S_0e^{rT}$

**Short stock:**

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Cost at time 0</th>
<th>Payoff at time $T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short sell one share and repay cash dividends at rate $r$</td>
<td>$-S_0$</td>
<td>$-S_T - \text{accumulated value of dividends}$</td>
</tr>
</tbody>
</table>

Time-$T$ profit = $S_0e^{rT} - S_T - \text{accumulated value of dividends}$

**Long call:**

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Cost at time 0</th>
<th>Payoff at time $T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy one $T$-year strike-$K$ European call</td>
<td>$c(S_0, K, T)$</td>
<td>$(S_T - K)_+$</td>
</tr>
</tbody>
</table>

Time-$T$ profit = $(S_T - K)_+ - c(S_0, K, T)e^{rT}$

**Short call:**

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Cost at time 0</th>
<th>Payoff at time $T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell one $T$-year strike-$K$ European call</td>
<td>$-c(S_0, K, T)$</td>
<td>$-(S_T - K)_+$</td>
</tr>
</tbody>
</table>

Time-$T$ profit = $c(S_0, K, T)e^{rT} - (S_T - K)_+$

---

**Example 1.1.5 [SoA FM Sample #11]**

Stock XYZ has the following characteristics:

- The current price is 40.
- The price of a 35-strike 1-year European call option is 9.12.
- The price of a 40-strike 1-year European call option is 6.22.
- The price of a 45-strike 1-year European call option is 4.08.

The annual effective risk-free interest rate is 8%.

Let $S$ be the price of the stock price one year from now.

All call positions being compared are long.
Determine the range for $S$ such that the 45-strike call produces a higher profit than the 40-strike call, but a lower profit than the 35-strike call.

(A) $S < 38.13$
(B) $38.13 < S < 40.44$
(C) $40.44 < S < 42.31$
(D) $S > 42.31$
(E) The range is empty.

--- Solution ---

We first analyze the 45-strike call and the 35-strike call. The profit for the 45-strike call is

$$(S - 45)_+ - 4.08 \times 1.08 = \begin{cases} S - 49.4064 & S > 45 \\ -4.4064 & S \leq 45 \end{cases}$$

The profit for the 35-strike call is

$$(S - 35)_+ - 9.12 \times 1.08 = \begin{cases} S - 44.8496 & S > 35 \\ -9.8496 & S \leq 35 \end{cases}$$

For better comparison, we rewrite the above as follows and make a rough sketch:

<table>
<thead>
<tr>
<th>Profit</th>
<th>$S &lt; 35$</th>
<th>$35 \leq S \leq 45$</th>
<th>$S &gt; 45$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35-strike</td>
<td>$-9.8496$</td>
<td>$S - 44.8496$</td>
<td>$S - 44.4064$</td>
</tr>
<tr>
<td>45-strike</td>
<td>$-4.4064$</td>
<td>$-4.4064$</td>
<td>$S - 49.4064$</td>
</tr>
</tbody>
</table>

$S > S^*$ for the 45-strike call to have a higher profit. To determine $S^*$, we set

$$-4.4064 = S^* - 44.8496.$$  

Then $S^* = 40.4432$. If we also incorporate the 40-strike call, we have
So the range is $40.4432 < S < 42.3112$. The correct choice is (C).

[ END ]
Exercise 1.1

Unless otherwise stated, you may assume that there is no transaction cost.

Stocks and Stock Indexes

1. For Stock XYZ, you are given:
   (i) The following bid and ask prices
   |
   
<table>
<thead>
<tr>
<th>Time</th>
<th>Bid price</th>
<th>Ask price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time 0</td>
<td>49.7</td>
<td>50.2</td>
</tr>
<tr>
<td>After half an hour</td>
<td>50.1</td>
<td>50.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
   (ii) The brokerage fee is 0.3% of the bid or ask price.

   You buy 50 shares at time 0, and then sell all shares after half an hour.
   (a) How much is received from the broker after half an hour?
   (b) What is the round-trip brokerage fee?

2. For a stock, the bid-ask spread is $0.4. The current bid price is $100. You buy 10 shares and sell the stock after 1 day, when the ask price becomes $100. Ignoring brokerage fee, interest and commissions, calculate the 1-day profit.

3. You buy 200 shares of XYZ, which has a bid price of $25.12 and an ask price of $25.34. You sell all shares after 6 months later when the bid price is $24.91 and the ask price is $25.06. There is a 0.3% brokerage commission, and the borrowing and lending interest rates are 6%, compounded 4 times a year. XYZ pays a single dividend of 0.4 at the end of 3 months.

   Calculate the 6-month profit.

4. Stock Y is currently priced at 25 per share and it pays dividends continuously at a rate proportional to its price at a constant rate of 3.5%. You purchase 100 shares of stock Y and invest all dividends by purchasing extra shares of stock Y. After 4 months, you close out all positions when the stock price is $S_4$.

   If the 4-month profit is zero when the continuously compounded risk-free interest rate is 2.5%, find $S_4$.

5. A stock index pays dividends continuously at a rate proportional to its price. The rate is 3% during the next 4 months. Afterwards the rate is 1%.

   Howard invests in the index at time 0 and reinvests all dividends by purchasing extra units of the index. After 8 months, he owns 120 units of the index. If the current index value is 25.4, what is the initial cost for Howard’s position?
6. Explain the rationale behind the following.
   (a) Sometimes the broker might require the short seller to deposit extra cash (called haircut) apart from the proceeds from short sales as collateral.
   (b) Sometimes a trustworthy third party (e.g. a bank), instead of the broker, would hold the proceedings from short sales.

**Derivative Securities**

7. [SoA old FM Sample #42] An investor purchases a non-dividend-paying stock and writes a $t$-year, European call option for this stock, with call premium $C$. The stock price at time of purchase and strike price are both $K$. Assume that there are no transaction costs. The risk-free annual force of interest is a constant $r$. Let $S(K)$ represent the stock price at time $t$.

Determine an algebraic expression for the investor’s profit at expiration.

(A) $Ce^{rt}$
(B) $C(1 + rt) - S + K$
(C) $Ce^{rt} - S + K$
(D) $Ce^{rt} + K(1 - e^{rt})$
(E) $C(1 + r)^t + K[1 - (1 + r)^t]$

8. [SoA old FM Sample #35] A customer buys a 50-strike put on an index when the market price of the index is also 50. The premium for the put is 5. Assume that the option contract is for an underlying 100 units of the index.

Calculate the customer’s profit if the index declines to 45 at expiration.

(A) $-$1000
(B) $-$500
(C) 0
(D) 500
(E) 1000

9. [SoA old FM Sample #49] The market price of Stock A is 50. A customer buys a 50-strike put contract on Stock A for 500. The put contract is for 100 shares of A.

Calculate the customer’s maximum possible loss.

(A) 0
(B) 5
(C) 50
(D) 500
(E) 5000
10. [SoA old FM Sample #48] For a certain stock, Investor A purchases a 45-strike call option while Investor B purchases a 135-strike put option. Both options are European with the same expiration date. Assume that there are no transaction costs.

If the final stock price at expiration is $S$, Investor A’s payoff will be 12.

Calculate Investor B’s payoff at expiration, if the final stock price is $S$.

(A) 0  
(B) 12  
(C) 36  
(D) 57  
(E) 78

11. [SoA old FM Sample #12] Consider a European put option on a stock index without dividends, with 6 months to expiration and a strike price of 1,000. Suppose that the annual nominal risk-free rate is 4% convertible semi-annually, and that the put costs 74.20 today.

Calculate the price that the index must be in 6 months so that being long in the put would produce the same profit as being short in the put.

(A) 922.83  
(B) 924.32  
(C) 1,000.00  
(D) 1,075.68  
(E) 1,077.17

12. [SoA old FM Sample #62] The price of an asset will either rise by 25% or fall by 40% in 1 year, with equal probability. A European put option on this asset matures after 1 year.

Assume the following:
- Price of the asset today: 100
- Strike price of the put option: 130
- Put option premium: 7
- Annual effective risk free rate: 3%

Calculate the expected profit of the put option.

(A) 12.79  
(B) 15.89  
(C) 22.69  
(D) 27.79  
(E) 30.29
13. Repeat the previous question, assuming that the strike price of the put option is 110.

14. [SoA old FM Sample #15] The current price of a non-dividend-paying stock is 40 and the continuously compounded annual risk-free rate of return is 8%. You enter into a short position on 3 call options, each with 3 months to maturity, a strike price of 35, and an option premium of 6.13. Simultaneously, you enter into a long position on 5 call options, each with 3 months to maturity, a strike price of 40, and an option premium of 2.78.

All options are held until maturity.

Calculate the maximum possible profit and the maximum possible loss for the entire option portfolio.

<table>
<thead>
<tr>
<th>Maximum Profit</th>
<th>Maximum Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) 3.42</td>
<td>4.58</td>
</tr>
<tr>
<td>(B) 4.58</td>
<td>10.42</td>
</tr>
<tr>
<td>(C) Unlimited</td>
<td>10.42</td>
</tr>
<tr>
<td>(D) 4.58</td>
<td>Unlimited</td>
</tr>
<tr>
<td>(E) Unlimited</td>
<td>Unlimited</td>
</tr>
</tbody>
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