These practice exams should be used during the last month prior to your exam.

This exam has 30 questions, corresponding to an exam of about 3 hours.

Each problem is similar to a problem in my study guide, sold separately. Solutions to problems are at the end of each practice exam.

prepared by
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1. On January 1, 2012, a common stock is priced at $78.
   You are given the following:
   (i) Dividends of equal amounts will be paid on January 31, April 30, July 31, and October 31.
   (ii) A 9-month 70-strike European call option on this stock has a premium of $12.57.
   (iii) A 9-month 70-strike European put option on this stock has a premium of $3.86.
   (iv) The continuously compounded risk-free interest rate is 4.8%.
   Calculate the amount of each dividend.
   (A) $0.57  (B) $0.58  (C) $0.59  (D) $0.60  (E) $0.61

2. Use the following Binomial tree:
   \[
   \begin{align*}
   S_{uu} &= 72 \\
   S_u &= 60 \\
   S_0 &= 50 \\
   S_{ud} &= 48 \\
   S_d &= 40 \\
   S_{dd} &= 32
   \end{align*}
   \]
   \( h = \text{six months}. \quad r = 4\%. \quad \delta = 0\%. \)
   What is the premium for a 1 year European Lookback call?
   A. 8.10  B. 8.30  C. 8.50  D. 8.70  E. 8.90

3. The current price of a non-dividend paying stock is 82.
   You buy 100 shares of stock.
   You also buy 100 80-strike 6-month puts for a premium of 7 per put.
   You sell 100 90-strike 6-month calls for a premium of 12 per call.
   You will sell your shares in 6 months.
   The continuously compounded risk-free interest rate is 10%.
   What is the range of your possible profit in 6 months?
   A. -95 to 905  B. 0 to 1000  C. 100 to 1100  D. 200 to 1200  E. 300 to 1300
4. Currently one can buy 1 U.S. Dollar for 122 Yen. 
Dollar and Yen interest rates are 4.5% and 2.0%, respectively. 
The price of a 120 Yen strike 2-year put option to sell one U.S. dollar is 16 Yen. 
What is the price in Yen of a similar call? 
A. Less than 13 
B. At least 13, but less than 14 
C. At least 14, but less than 15 
D. At least 15, but less than 16 
E. At least 16

5. \( S_0 = 200. \ \alpha = 12\% \text{ per year.} \ \delta = 2\% \text{ per year.} \ \sigma = 30\% \text{ per year.} \) 
Simulate the path of this stock’s price over each of the next 3 months. 
Use the following three random numbers from (0, 1): 
0.7324, 0.4681, 0.1515. 
Determine the average simulated stock price: \( (S_{1/12} + S_{2/12} + S_{3/12}) / 3. \) 
A. 200 \hspace{1cm} B. 202 \hspace{1cm} C. 204 \hspace{1cm} D. 206 \hspace{1cm} E. 208

6. Consider the following information about an at-the-money European put option on stock ABC: 
• The current stock price is $100. 
• The time to expiration is 2 years. 
• The continuously compounded risk-free rate is 9% annually. 
• The stock pays dividends at a continuous rate of 2%. 
• The price is calculated using a 2-step Binomial Model where each step is one year in length. 
The stock price tree is shown below: 

```
  121
 /   \
110   
/     \
100   
/     \
  99
 /   \
90   
/     \
  81
```
Calculate the price of the put on stock ABC. 
A. Less than 0.30 
B. At least 0.30, but less than 0.35 
C. At least 0.35, but less than 0.40 
D. At least 0.40, but less than 0.45 
E. At least 0.45
7. Assume the Black-Scholes framework.
Suppose the spot exchange rate is $1.40 per Euro.
Assume \( r = 0.05 \), \( r_{\text{Euros}} = 0.035 \), volatility = 0.15.
What is the premium in dollars for an at-the-money put option that expires in 10 months?
A. Less than 0.05
B. At least 0.05, but less than 0.06
C. At least 0.06, but less than 0.07
D. At least 0.07, but less than 0.08
E. At least 0.08

8. You are given the following information on European Calls:

<table>
<thead>
<tr>
<th>( S )</th>
<th>( K )</th>
<th>( \sigma )</th>
<th>( r )</th>
<th>( T )</th>
<th>( \delta )</th>
<th>Option Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>120</td>
<td>20%</td>
<td>6%</td>
<td>3 years</td>
<td>2%</td>
<td>10.55</td>
</tr>
<tr>
<td>110</td>
<td>120</td>
<td>20%</td>
<td>6%</td>
<td>3 years</td>
<td>2%</td>
<td>15.76</td>
</tr>
<tr>
<td>100</td>
<td>130</td>
<td>20%</td>
<td>6%</td>
<td>3 years</td>
<td>2%</td>
<td>7.88</td>
</tr>
<tr>
<td>100</td>
<td>120</td>
<td>20%</td>
<td>6.5%</td>
<td>3 years</td>
<td>2%</td>
<td>11.10</td>
</tr>
<tr>
<td>100</td>
<td>120</td>
<td>20%</td>
<td>6%</td>
<td>3.5 years</td>
<td>2%</td>
<td>12.21</td>
</tr>
<tr>
<td>100</td>
<td>120</td>
<td>20%</td>
<td>6%</td>
<td>3 years</td>
<td>1%</td>
<td>12.03</td>
</tr>
</tbody>
</table>

Estimate rho (per percent), the option Greek, for a 120 strike 3 year call with
\( S_0 = 100 \), \( \sigma = 20\% \), \( \delta = 2\% \), and \( r = 6\% \).
A. 0.3   B. 0.5   C. 0.7   D. 0.9   E. 1.1

9. The future price of a stock follows a LogNormal Distribution with parameters \( m = 5 \) and \( \nu = 0.8 \).
Which of the following is a 95% confidence interval for the price of this stock?
A. [40, 553]
B. [31, 712]
C. [23, 954]
D. [19, 1165]
E. None of the above
10. A stock has a three-year forward price of 108.23.
The stock pays dividends at a continuously compounded rate of 1.4%.
r = 6.1% compounded continuously.
What is the current price of this stock?
A. 93  B. 94  C. 95  D. 96  E. 97

The premium per call is 11.
The risk free continuously compounded annual interest rate is 6%.
Let X be your profit if the spot price of the stock at expiration is 72.
Let Y be your profit if the spot price of the stock at expiration is 96.
Determine X + Y.
A. -8  B. -7  C. -6  D. -5  E. -4

12. Barrier call option prices are shown in the table below.
Each option has the same underlying asset.

<table>
<thead>
<tr>
<th>Option</th>
<th>Strike</th>
<th>Barrier</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>down-and-out Call</td>
<td>80</td>
<td>85</td>
<td>6.11</td>
</tr>
<tr>
<td>up-and-out Call</td>
<td>80</td>
<td>200</td>
<td>12.18</td>
</tr>
<tr>
<td>down-and-in Call</td>
<td>80</td>
<td>85</td>
<td>X</td>
</tr>
<tr>
<td>up-and-in Call</td>
<td>80</td>
<td>200</td>
<td>14.67</td>
</tr>
<tr>
<td>down-and-out Put</td>
<td>70</td>
<td>35</td>
<td>3.43</td>
</tr>
<tr>
<td>up-and-out Put</td>
<td>70</td>
<td>110</td>
<td>Y</td>
</tr>
<tr>
<td>down-and-in Put</td>
<td>70</td>
<td>35</td>
<td>3.71</td>
</tr>
<tr>
<td>up-and-in Put</td>
<td>70</td>
<td>110</td>
<td>2.09</td>
</tr>
</tbody>
</table>

Determine |X - Y|.
A. Less than 9
B. At least 9, but less than 11
C. At least 11, but less than 13
D. At least 13, but less than 15
E. At least 15
13. Let $S(t)$ be the price of a stock at time $t$.
$S(0) = 90$. $\sigma = 35\%$. $r = 5\%$. $\delta = 1\%$.
At time 1, a contract pays $\text{Max} [\text{Min} [S(1), 110], 85]$.
Determine the time-zero premium of this contract.

(A) 90       (B) 91       (C) 92       (D) 93       (E) 94

14. The underlying stock for a European exchange option has $S = 110$,
continuous rate of dividends = 1.2%, volatility = 0.24.
The strike stock has $S = 140$, continuous rate of dividends = 1.8%, volatility = 0.32.
The two stocks have a correlation coefficient of 0.60.
If the exchange option expires in 2 years, what is the price of the call using the Black-Scholes
approach?
A. Less than 4
B. At least 4, but less than 5
C. At least 5, but less than 6
D. At least 6, but less than 7
E. At least 7

15. The current price of a non-dividend paying stock is 67.
Mr. White buys a one-year 60-80 collar on this stock.
Elmer buys a one-year 60-80 call bull spread on the same stock.
The effective annual risk free rate is 7%.
Determine the difference of Mr. White’s premium minus Elmer’s premium.
A. Less than -12
B. At least -12 but less than -4
C. At least -4, but less than 4
D. At least 4, but less than 12
E. 12 or more

16. An investor bought a 1200-strike European put option on an index with one year to expiration.
The premium for this option was 134.
The investor also wrote a 1300-strike European put option on the same index with one year
to expiration.
The premium for this option was 171.
The continuously compounded annual interest rate is 7%.
Calculate the index price at expiration that will allow the investor to break even.
(A) 1220       (B) 1240       (C) 1260       (D) 1280       (E) 1300
17. Use the following information:
• The risk-free rate, compounded continuously, is 10%.
• European options on one share of a stock expiring in eight months have the following prices:

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>Call option price</th>
<th>Put option price</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>32.09</td>
<td>3.21</td>
</tr>
<tr>
<td>100</td>
<td>15.61</td>
<td>14.79</td>
</tr>
<tr>
<td>120</td>
<td>9.23</td>
<td>27.12</td>
</tr>
</tbody>
</table>

An investor purchases asymmetric butterfly spreads on this stock, using the above options such that the investor sells 100 of the middle strike options.
Let V be the investor’s profit if the spot price of the stock at expiration is 60.
Let X be the investor’s profit if the spot price of the stock at expiration is 80.
Let Y be the investor’s profit if the spot price of the stock at expiration is 110.
Let Z be the investor’s profit if the spot price of the stock at expiration is 130.
Determine $V + X + Y + Z$.
A. Less than -150
B. At least -150, but less than -50
C. At least -50, but less than 50
D. At least 50, but less than 150
E. 150 or more

18. Which of the following statements is false?
A. The control variate method uses the insight that for every simulated realization, there is an opposite and equally likely realization.
B. The continuation value is the value of leaving an option unexercised.
C. Latin hypercube sampling is a generalization of stratified sampling when the payoff depends on more than one random variable.
D. When valuing options using true probabilities, the discount rates are not the same along the different paths of future stock prices.
E. None of A. B. C. or D is false.
19. The following tree was produced by the Black-Derman-Toy Model:

Year 0  Year 1  Year 2

17.2%  12.6%  9.0%  9.3%  10.6%

Determine the yield to maturity of a 3 year zero-coupon bond.
A. 11.0%  B. 11.2%  C. 11.4%  D. 11.6%  E. 11.8%

20. For a lognormally distributed stock, \( S_0 = 150, \alpha = 10\% , \delta = 3\% , \) and \( \sigma = 30\% . \) What is the probability that the stock price at time 5 is between 200 and 250?
(A) 12%  (B) 14%  (C) 16%  (D) 18%  (E) 20%
21. The following five charts are profit diagrams for option strategies. Each strategy is constructed with the purchase or sale of 1-year European options. In each chart, the profit is shown with one month to expiration.

Which of the following correctly matches charts with option strategies?

A. Chart 1 = Bull Spread, Chart 3 = Written Straddle.
B. Chart 2 = Straddle, Chart 3 = Butterfly Spread.
C. Chart 4 = Butterfly Spread, Chart 5 = Bull Spread.
D. Chart 1 = Straddle, Chart 4 = Bear Spread.
E. Chart 2 = Written Straddle, Chart 5 = Bear Spread.
22. You own a one-year 80 strike American call option on a dividend paying stock. The stock currently sells at 83, and is expected to sell at either 81 or 90 one year from now. The stock pays dividends at a continuously compounded rate of 2.5% per year. What annual continuously compounded risk-free interest rate would equate the value of exercising the call immediately with the present value of waiting until the expiration to exercise?
A. Less than 3.0%
B. At least 3.0%, but less than 3.5%
C. At least 3.5%, but less than 4.0%
D. At least 4.0%, but less than 4.5%
E. At least 4.5%

23. Warren owns one thousand shares of a stock. Because he worries that its price may drop over the next three years, he decides to employ a rolling insurance strategy, which entails obtaining one year European put options on the stock every year, with the first one being bought immediately. Use the following information:
(i) The continuously compounded risk-free interest rate is 10%.
(ii) The continuously compounded dividend rate is 3%.
(iii) The stock’s volatility is 20%.
(iv) The current stock price is 80.
(v) The strike price for each option is equal to the then-current stock price.
His broker will sell him the options but will charge him for their total cost now. Under the Black-Scholes framework, how much does Warren now pay his broker?
(A) 10,700  (B) 10,800  (C) 10,900  (D) 11,000  (E) 11,100

24. Use the following information:
• The current price of a stock is 91.
• 33% = standard deviation per year of the (continuously compounded) rate of return on the stock
• 7% = continuously compounded annual risk free force of interest
• 2% = (continuously compounded) annual rate at which dividends are paid on this stock
A 100-strike European call has 9 months until expiration. Determine Theta per year.
A. Less than -11
B. At least -11, but less than -10
C. At least -10, but less than -9
D. At least -9, but less than -8
E. -8 or more
25. Assume the Black-Scholes framework.
A stock has $S_0 = 81$, $\alpha = 12\%$, $\sigma = 33\%$, $\delta = 2.5\%$. $r = 5.7\%$.
A market maker, sells ten thousand 2-year 70-strike European puts on this stock.
The market maker will delta hedge his position by using 2-year 70-strike European calls on this stock.
How many calls should he sell?
A. less than 4000
B. at least 4000 but less than 5000
C. at least 5000 but less than 6000
D. at least 6000 but less than 7000
E. at least 7000

26. Stock XYZ has a current price of 70.
The forward price for delivery of this stock in 1 year is 72.14.
The effective annual interest rate is r.
Stock XYZ will pay a single dividend of size d in 6 months.
In which of the following situations is there no comparative advantage to investing in the stock versus investing in the forward contract or vice-versa?
(A) $r = 5\%$ $d = 1$
(B) $r = 5\%$ $d = 2$
(C) $r = 6\%$ $d = 1$
(D) $r = 6\%$ $d = 2$
(E) None of A, B, C, or D

27. You are given the following regarding the foreign currency Bongo Bucks:
(i) One Bongo Buck currently sells for $0.06.
(ii) Six months from now a Bongo Buck will sell for either $0.05 or $0.07.
(iii) The continuously compounded risk-free interest rate on Bongo Bucks is 7\%.
(iv) The continuously compounded risk-free interest rate on U.S. dollars is 4\%.
While reading the Bongo Congo Times, Leonardo notices that a six-month $0.065 strike European call written on Bongo Bucks is selling for $0.0025.
Use the binomial option pricing model to determine if an arbitrage opportunity exists.
What transactions should Leonardo enter into to exploit the arbitrage opportunity (if one exists)?
(A) No arbitrage opportunity exists.
(B) Short Bongo Bucks, lend at the $\$ $ risk-free rate, and buy the call.
(C) Buy Bongo Bucks, borrow at the $\$ $ risk-free rate, and buy the call.
(D) Buy Bongo Bucks, borrow at the $\$ $ risk-free rate, and short the call.
(E) Short Bongo Bucks, lend at the $\$ $ risk-free rate, and short the call.
28. The price of a stock is $120. The stock pays dividends at the continuous rate of 1.5%. The price of a European call with a strike price of $125 is $9.72 and the price of a European put with a strike price of $125 is $12.83. Both options expire in six months. Calculate the annual continuously compounded risk-free rate on a synthetic T-Bill created using these options.

A. Less than 1%
B. At least 1%, but less than 2%
C. At least 2%, but less than 3%
D. At least 3%, but less than 4%
E. At least 4%

29. For a non-dividend stock you are given the following information:
• The current stock price is 70.
• The volatility of the return on the stock is 30%.
• The continuously compounded risk-free interest rate is 5%.
Using the Black-Scholes formula, calculate the cost of an at-the-money one year straddle.
A. Less than 16
B. At least 16, but less than 17
C. At least 17, but less than 18
D. At least 18, but less than 19
E. At least 19

30. The current price of a stock is 90. \( r = 5\% \). continuous rate of dividends = 1.5\%. volatility = 0.3.
Use a Binomial Tree with 4 month time periods, in order to determine the premium of a 100-strike one-year European call option.
A. Less than 8
B. At least 8, but less than 9
C. At least 9, but less than 10
D. At least 10, but less than 1
E. At least 11

END OF PRACTICE EXAM
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### Solutions:

1. **D.** By put-call parity, \( C = P + S - PV[Div] - Ke^{-rT} \). Let \( x \) be the size of each dividend.

\[
12.57 = 3.86 + 78 - x\{e^{-(0.048)(1/12)} + e^{-(0.048)(4/12)} + e^{-(0.048)(7/12)}\} - 70 e^{-(0.048)(9/12)}.
\]

\[
\Rightarrow x = 1.7652 / 2.9525 = 0.598.
\]

Comment: Similar to Q. 9.70 (MFE, 5/07, Q.1).

In put-call parity we only include those dividends paid from the time when we buy the option to when it expires; in this case we only include the dividends paid in January, April, and July.

2. **B.** \( p^* = (e^{0.04/2} - 0.8) / (1.2 - 0.8) = 55.05\% \).

For each of four possible paths, we calculate the minimum, and the payoff on the lookback call which is: \( S_T - \text{minimum} \).

<table>
<thead>
<tr>
<th>Path</th>
<th>Minimum</th>
<th>Payoff on Lookback Call</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>50, 60, 72</td>
<td>50</td>
<td>22</td>
<td>0.5505^2 = 0.3031</td>
</tr>
<tr>
<td>50, 60, 48</td>
<td>48</td>
<td>0</td>
<td>(0.5505)(1 - 0.5505) = 0.2474</td>
</tr>
<tr>
<td>50, 40, 48</td>
<td>40</td>
<td>8</td>
<td>(0.5505)(1 - 0.5505) = 0.2474</td>
</tr>
<tr>
<td>50, 40, 32</td>
<td>32</td>
<td>0</td>
<td>(1 - 0.5505)^2 = 0.2021</td>
</tr>
</tbody>
</table>

Premium for lookback call is: \((22)(0.3031) + (8)(0.2474)) / e^{0.04} = 8.31\).

Comment: Similar to Q. 54.3.

Payoff of a lookback call is: \( S_T - \text{Min}[S_t | 0 \leq t \leq T] \).

Payoff of a lookback put is: \( \text{Max}[S_t | 0 \leq t \leq T] - S_T \).

3. **A.** The accumulated cost of the hedge is: \( (100)(12 - 7) \exp(0.1/2) = 525.64 \).

The profit is: \( 100S - (100)(82) \exp(0.1/2) + 525.64 + 100(80-S)_+ - 100(90)_+ \).

For \( S \leq 80 \), the profit is: \(-8620.42 + 525.64 + 8000 = -94.78 \).

For \( 80 \leq S \leq 90 \), the profit is: \( 100S - 8620.42 + 525.64 = 100S - 8094.78 \).

For \( S \geq 90 \), the profit is: \(-8620.42 + 525.64 + 9000 = 905.22\).

The range of profit is \textbf{from -95 to 905}.

Comment: Q. 5.56 (MFE Sample Introductory Q.3).

4. **A.** \( C_P(x_0, K, T) = P_P(x_0, K, T) + x_0 \exp[-rS_T] - Ke^{-rT} \).

\[
= 16 + (122)e^{-(0.045)} - (120)e^{-(0.02)} = 12.2 \text{ Yen}.
\]

Comment: Similar to Q. 11.8.
5. D. $\Phi^{-1}(0.7324) = 0.62009, \Phi^{-1}(0.4681) = -0.08005, \Phi^{-1}(0.1515) = -1.03002.$

The return over one month is Normal with parameters:

$(0.12 - 0.02 - 0.3^2/2)/12 = 0.004583,$ and $0.3 \sqrt{1/12} = 0.0866.$

$S_{1/12} = (200) \exp[0.004583 + (0.62009)(0.0866)] = 212.00.$

$S_{2/12} = (212.00) \exp[0.004583 + (-0.08005)(0.0866)] = 211.50.$

$S_{3/12} = (211.50) \exp[0.004583 + (-1.03002)(0.0866)] = 194.34.$

$(212.00 + 211.50 + 194.34)/3 = 205.95.$

Comment: Similar to Q. 59.3.

The stock price in month two is dependent on the stock price in month one.

If $R_1$ is the simulated return in month 1, and $R_2$ is the simulated return in month 2, then

$S_{1/12} = S_0 \exp[R_1], \text{ and } S_{2/12} = S_0 \exp[R_1 + R_2] = S_{1/12} \exp[R_2].$

6. E. $p^* = \frac{\exp[h(r - \delta)] - d}{u - d} = \frac{\exp[(1)(0.09 - 0.02)] - 0.9}{1.1 - 0.9} = 86.25\%.$

The premium for the put is:

$\frac{(0.8625)^2(0) + (2)(0.8625)(1 - 0.8625)(100 - 99) + (1 - 0.8625)^2(100 - 81)}{\exp[(2)(0.09)]} = \$0.498.$

Comment: Similar to Q. 21.20 (CAS3, 5/07, Q.14).

7. C. At the money means that $K = S = 1.40.$

$d_1 = \frac{\ln(1.4/1.4) + (0.05 - 0.035 + 0.15^2/2)(10/12)}{(0.15) \sqrt{10/12}} = 0.15975.$

$d_2 = 0.15975 - (0.15) \sqrt{10/12} = 0.02282.$

$N[0.15975] = 0.56346. \quad N[0.02282] = 0.50910.$

Value of this put option is:

$= (1.4) \exp[-(0.05)(10/12)](1 - 0.50910) - \exp[-(0.035)(10/12)](1.4)(1 - 0.56346) = \$0.0656.$

Comment: Similar to Q. 29.1.

Euros act as the asset; replace $\delta$ everywhere by $r_{Euros}.$

8. E. $\rho = \frac{\text{change in option premium}}{\text{change in } r} = \frac{11.10 - 10.55}{6.5 - 6} = 1.10.$

Comment: Similar to Q. 34.15.
9. B. A 95% confidence interval for lnY is:
\[5 - (1.960)(0.8) , 5 + (1.960)(0.8)\] = [3.432, 6.568].
A 95% confidence interval for Y is: \([e^{3.432}, e^{6.568}] = [31, 712]\).
Comment: Similar to Q. 25.15.

10. B. Forward price = \(S_0 \exp[(r - \delta)T]\). \(\Rightarrow 108.23 = S_0 \exp[(6.1\% - 1.4\%)(3)]\). \(\Rightarrow S_0 = 94.00\).
Alternately, the prepaid forward price is the forward price discounted for interest:
\[\frac{108.23}{e^{0.183}} = 90.13.\]
\((\text{current price}) e^{-\delta T} = (\text{current price}) e^{-0.042} = \text{prepaid forward price} = 90.13.\)
\(\Rightarrow \text{current price} = 90.13 e^{0.042} = 94.00.\)
Comment: Similar to Q. 6.12.

11. B. \(X = (72 - 80) - 11 \exp[(6\%)(9/12)] = -11.51.\)
\(Y = (96 - 80) + 11 \exp[(6\%)(9/12)] = 4.49.\)
\(X + Y = -7.02.\)
Comment: Similar to Q. 3.6.

12. E. Working with calls with a barrier of 200, the value of the regular Call with \(K = 80\) is:
Working with calls with a barrier of 85:
26.85 = down-and-out 80 strike Call + down-and-in 80 strike Call = 6.11 + X. \(\Rightarrow X = 20.74.\)
Working with puts with a barrier of 35, the value of the regular Put with \(K = 70\) is:
Working with puts with a barrier of 110:
7.14 = up-and-out 70 strike Put + up-and-in 70 strike Put = Y + 2.09. \(\Rightarrow Y = 5.05.\)
\(|X - Y| = |20.74 - 5.05| = 15.69.\)
Comment: Similar to Q. 47.27 (CAS3, 5/07, Q.34).
Based on the Black-Scholes framework, with \(S_0 = 90, \sigma = 0.4, r = 5\%, T = 2, \delta = 1\%.\)
13. A. \[ \text{Max}[\text{Min}[S(1), 110], 85] = \begin{cases} 
110 & \text{if } S(1) > 110 \\
S(1) & \text{if } 110 \geq S(1) \geq 85 \\
85 & \text{if } S(1) < 85 
\end{cases} \]

By checking all three cases, one can verify that,
\[ \text{Max}[\text{Min}[S(1), 110], 85] = 85 + (S(1) - 85)_+ - (S(1) - 110)_+. \]
Thus, taking the actuarial present value in the risk-neutral environment, the premium is:
\[ 85e^{-0.05} + (\text{premium of a 1-year 85-strike call}) - (\text{premium of a 1-year 110-strike call}) \]
Working at 85:
\[ d_1 = \frac{\ln(90/85) + (1)(0.05 - 0.01 + 0.35^2/2)}{0.35 \sqrt{t}} = 0.45260. \]
\[ d_2 = d_1 - 0.35 \sqrt{t} = 0.10260. \]

Premium of a 1-year 85-strike call is: \( 90e^{-0.01} \Phi[-0.10260] - 90e^{-0.05} \Phi[-0.45260] = (89.10)(0.38818) - (89.10)(0.26302) = 7.06. \)

Thus the premium of this contract is: \( 85e^{-0.05} + 7.06 = 90.17. \)

Alternately, \( \text{Max}[\text{Min}[S(1), 110], 85] = 110 + (85 - S(1))_+ - (110 - S(1))_+. \)
Thus, taking the actuarial present value in the risk-neutral environment, the premium is:
\[ 110e^{-0.05} + (\text{premium of a 1-year 85-strike put}) - (\text{premium of a 1-year 110-strike put}) \]
Premium of a 1-year 85-strike put is: \( 85e^{-0.05} \Phi[-0.28406] - 90e^{-0.01} \Phi[-0.45260] = (80.85)(0.45914) - (89.10)(0.32542) = 8.13. \)

Premium of a 1-year 110-strike put is: \( 110e^{-0.05} \Phi[0.63406] - 90e^{-0.01} \Phi[0.28406] = (104.64)(0.73698) - (89.10)(0.61182) = 22.60. \)

Thus the premium of this contract is: \( 110e^{-0.05} + 8.13 - 22.60 = 90.17. \)

Comment: Similar to Q. 64.9.
If \( S(1) > 110 \), then \( \text{max} \{ \text{min} [S(1), 110], 85 \} = 110. \)
\[ 85 + (S(1) - 85)_+ - (S(1) - 110)_+ = 85 + \{S(1) - 85\} - \{S(1) - 110\} = 110. \] Try \( S(1) = 115. \)

If \( 110 > S(1) > 85 \), then \( \text{max} \{ \text{min} [S(1), 110], 85 \} = S(1). \)
\[ 85 + (S(1) - 85)_+ - (S(1) - 110)_+ = 85 + \{S(1) - 85\} + 0 = S(1). \] Try \( S(1) = 100. \)

If \( 85 > S(1) \), then \( \text{max} \{ \text{min} [S(1), 110], 85 \} = 85. \)
\[ 85 + (S(1) - 85)_+ - (S(1) - 110)_+ = 85 + 0 + 0 = 85. \] Try \( S(1) = 80. \)

Since the two expressions are equal for all three cases, they are indeed equal.
14. E. volatility = $\sqrt{0.24^2 + 0.32^2} - (2)(0.6)(0.24)(0.32) = 0.26046$.

$$d_1 = \frac{\ln(110/140) + (0.018 - 0.012 + 0.26046^2 / 2)(2)}{0.26046 \sqrt{2}} = -0.43796.$$ 

$$d_2 = -0.43796 - (0.26046) \sqrt{2} = -0.80631.$$ 

$N[-0.43796] = 0.33071$. $N[-0.80631] = 0.21003$.

Premium for this call option is: 

$$\exp[-(0.012)(2)] (110) (0.33071) - (140) \exp[-(0.018)(2)] (0.21003) = \$7.15.$$ 

Comment: Similar to Q. 50.3.

15. B. Collar: Buy a 60-strike put and sell a 80-strike call.

Call Bull Spread: Buy a 60-strike call and sell a 80-strike call.

Using put-call parity: $P(60) - C(60) = 60/1.07 - 67 = -10.93$

Thus, the difference in their premiums is: $P(60) - C(60) = -10.93$.

Comment: Similar to Q. 9.40.

16. C. Initially the investor receives: $171 - 134 = 37$.

Thus, the profit is: $37 e^{0.07} + (1200 - S)_+ - (1300 - S)_+$.

For $S \geq 1300$, the profit is 39.68.

For $1200 \leq S \leq 1300$, the profit is: $39.68 - (1300-S) = S - 1260.32$.

For $S \leq 1200$, the profit is: $39.68 + (1200-S) - (1300-S) = -60.32$.

In order for the profit to be zero, $S = 1260.32$.

Comment: Similar to Q. 4.41 (MFE Sample Introductory Q.50).
17. **A.** \( \lambda = \frac{(120 - 100)}{(120 - 70)} = 0.4. \)

Thus the investor buys 40 70-strike, sells 100 100-strike, and buys 60 120-strike options.

If the investor uses calls, his payoff is: \( 40 \ (S-70)_+ + 60 \ (S-120)_+ - 100 \ (S-100)_+. \)

His premium is: \( (40)(32.09) + (60)(9.23) - (100)(15.61) = 276.40. \)

The future value is: \( 276.40 \ e^{(2/3)(0.10)} = 295.45. \)

Thus his profit is: \( 40 \ (S-70)_+ + 60 \ (S-120)_+ - 100 \ (S-100)_+ - 295.45. \)

\( V = -295.45. \)

\( X = (40)(80-70) - 295.45 = 104.55. \)

\( Y = (40)(110-70) - (100)(110-100) - 295.45 = 304.55. \)

\( Z = (40)(130-70) + (60)(130-120) - (100)(130-100) - 295.45 = -295.45. \)

\( V+X+Y+Z = -181.8. \)

If instead the investor uses puts, his payoff is: \( 40 \ (70 - S)_+ + 60 \ (120 - S)_+ - 100 \ (100 - S)_+. \)

His premium is: \( (40)(3.21) + (60)(27.12) - (100)(14.79) = 276.60. \)

The future value is: \( 276.60 \ e^{(2/3)(0.10)} = 295.67. \)

Thus his profit is: \( 40 \ (70 - S)_+ + 60 \ (120 - S)_+ - 100 \ (100 - S)_+ - 295.67. \)

\( V = (40)(70-60) + (60)(120-60) - (100)(100-60) - 295.67 = -295.67. \)

\( X = (60)(120-80) - (100)(100-80) - 295.67 = 104.33. \)

\( Y = (60)(120-110) - 295.67 = 304.33. \)

\( Z = -295.67. \)

\( V+X+Y+Z = -182.7. \)

**Comment:** Similar to Q. 5.45.

As with butterfly spreads, for asymmetric butterfly spreads, the profit graphs are the same (subject to rounding) whether one uses puts or calls.
18. A. A is false, see page 632 of McDonald. A would be true if it said antithetic variate method. B is true. C is true, see page 633 of McDonald. D is true.


19. B. Using the geometric progression of the values in a column of the BDT tree, the missing value is: $\sqrt{(0.172)(0.106)} = 0.135$.

Year 0    Year 1    Year 2

17.2% 12.6% 13.5%
9.0% 9.3% 10.6%

Assume for simplicity that the bond pays $1 upon maturity.

In order to get the price of the 3 year bond, we discount along of each of four possible paths:

$$\frac{1}{(1.09)(1.126)(1.172)} + \frac{1}{(1.09)(1.126)(1.135)} + \frac{1}{(1.09)(1.093)(1.135)} + \frac{1}{(1.09)(1.093)(1.106)} / 4$$

$= 0.7279.$

Yield to maturity is: $(1/0.7279)^{1/3} - 1 = 11.17\%$.

Comment: Similar to Q. 66.6.

20. A. $S_5$ is a LogNormal with parameters:

$\ln 150 + (0.10 - 0.03 - 0.3^2/2)(5) = 5.1356$, and $0.3 \sqrt{5} = 0.67082.$

$F(250) - F(200) = \Phi[\ln(250) - 5.1356 / 0.67082] - \Phi[\ln(200) - 5.1356 / 0.67082]$

$= \Phi[0.57521] - \Phi[0.24256] = 0.71743 - 0.59583 = 12.160\%$.

Comment: Similar to Q. 27.4.
21. E. The profit at one month until expiration should approach the profit at expiration, which is the payoff at expiration minus the original cost to set up the position accumulated for interest. Thus, the profit at expiration should look like the payoff graph, translated either up or down.

Bull Spread: The purchase of an option together with the sale of an otherwise identical option with a higher strike price.

If we use calls with strikes of 90 and 110, then the payoff is: \((S - 90)_+ - (S - 110)_+\).
This is: 0 if \(S < 90\), \(S - 90\) if \(90 \leq S \leq 110\), and 20 if \(S > 110\).

![Chart 1](image)

Chart 1 appears to be approaching, other than a constant, the payoff for a **Bull Spread**.

Written Straddle: Sell a put and a call, with the same strike and time until expiration.

If the strike is 100, then the payoff is: \(-(100 - S)_+ - (S - 100)_+ = -|S - 100|\).

![Chart 2](image)

Chart 2 appears to be approaching, other than a constant, the payoff for a **Written Straddle**.
Butterfly Spread: Buying a K strike option, selling two K + ΔK strike options, and buying a K + 2ΔK strike option.

If we use calls with strikes of 90, 100, and 110, then the payoff is:

\[(S - 90)^+ + (S - 110)^+ - 2(S - 100)^+.\]

This is: 0 if \(S < 90\), \(S - 90\) if \(90 \leq S \leq 100\), \(110 - S\) if \(90 \leq S \leq 110\), and 0 if \(S > 110\).

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**Chart 3** appears to be approaching, other than a constant, the payoff for a Butterfly Spread.

Straddle: Buy a put and a call, with the same strike and time until expiration.

If the strike is 100, then the payoff is: \((100 - S)^+ + (S - 100)^+ = |S - 100|\).

---

**Chart 4** appears to be approaching, other than a constant, the payoff for a Straddle.
Bear Spread: The purchase of an option together with the sale of an otherwise identical option with a lower strike price.
If we use calls with strikes of 90 and 110, then the payoff is: \((S - 110)_+ - (S - 90)_+\).
This is: 0 if \(S < 90\), \(90 - S\) if \(90 \leq S \leq 110\), and -20 if \(S > 110\).

Chart 5 appears to be approaching, other than a constant, the payoff for a **Bear Spread**.
Comment: Similar to Q. 36.16 (MFE Advanced Sample Exam Q.40).
If we buy a Bull Spread, we are hoping that the stock price goes up, which is chart 1. If we buy a Bear Spread, we are hoping that the stock price goes down, which is chart 5.
Graphs were constructed for \(S_0 = 100\), \(\sigma = 30\%\), \(r = 6\%\), and \(\delta = 1\%\).

22. A. If we exercise right away, we get 83 - 80 = 3.
If the stock price goes up the payoff is 10, while if it goes down the payoff is 1.
The continuation value is: \(\frac{10p^* + 1(1 - p^*)}{e^r} = \frac{9p^* + 1}{e^r}\).
\[
p^* = \frac{83 \exp[r - 0.025] - 81}{90 - 81} = 9.2222e^{r-0.025} - 9.
\]
We want: \(3 = \frac{9p^* + 1}{e^r} \Rightarrow 3e^r = 9p^* + 1 = 83e^{r-0.025} - 80.\)
\[\Rightarrow e^r = \frac{80}{(83e^{0.025} - 3)} = 1.0263 \Rightarrow r = 2.6\%.
\]
Alternately, the payoff on the similar 80-strike put is zero at both the up and down node; \(P = 0\).
We want \(C = 3\), so that the continuation value is equal to the exercise value.
Thus \(C = P + S_0 e^{-\delta T} - K e^{rT}. \Rightarrow 3 = 0 + 83 \exp[-(2.5\%)(1)] - 80 \exp[-r]. \Rightarrow r = 2.6\%.
\]
Comment: Similar to Q. 19.42 (5B, 5/99, Q.34).
\[
p^* = \frac{83 \exp[0.026 - 0.025] - 81}{90 - 81} = 23.14\%.
\]
The continuation value is: \(\frac{(10)(0.2314) + (1)(1 - 0.2314)}{e^{0.026}} = 3.00.\)
23. B. For a forward start put, with forward start date $t_1$, expiration time $T$, with strike price equal to the current stock price, the premium is:

$$S_0 \exp[-\delta t_1] \{\exp[-r(T - t_1)] \Phi[-d_2] - \exp[-\delta(T - t_1)] \Phi[-d_1]\},$$

where $d_1 = \frac{-\ln[1] + (r - \delta + \sigma^2/2)(T - t_1)}{\sigma \sqrt{T - t_1}}$, and $d_2 = d_1 - \sigma \sqrt{T - t_1}$.

$$d_1 = \frac{-\ln[1] + (10\% - 3\% + 0.2^2/2)(1)}{0.2 \sqrt{1}} = 0.45. \quad d_2 = 0.45 - 0.2 \sqrt{1} = 0.25$$

$$\Phi[0.45] = 0.67364. \quad \Phi[0.25] = 0.59871.$$  

Premium for the first forward start put is:

$$80 \{\exp[-10\%](1 - 0.59871) - \exp[-3\%](1 - 0.67364)\} = 3.7110.$$  

For the second forward start put, $T - t_1 = 2 - 1 = 1$. Thus $d_1$ and $d_2$ are the same as before.

The premium for the second option is:

$$S_0 \exp[-\delta] \{\exp[-r] \Phi[-d_2] - \exp[-\delta] \Phi[-d_1]\}.$$  

This is the premium for the first put times $\exp[-\delta]$: $3.7110 \times e^{-0.03} = 3.6013$.

Similarly, the third put has premium:

$$S_0 \exp[-2\delta] \{\exp[-r] \Phi[-d_2] - \exp[-\delta] \Phi[-d_1]\}.$$  

This is the premium for the first put times $\exp[-2\delta]$: $3.7110 \times e^{-0.06} = 3.4949$.

Therefore, the premium for the rolling insurance strategy is:

$$(1,000)(3.7110 + 3.6013 + 3.4949) = 10,807.$$  

Comment: Similar to Q. 51.7 (MFE Advanced Sample Exam, Q.33).

The first “forward start put” is just an ordinary put.

24. D. $d_1 = \frac{\ln(S/K) + (r - \delta + \sigma^2/2)T}{\sigma \sqrt{T}} = \frac{\ln(91/100) + (0.07 - 0.02 + 0.33^2/2)(3/4)}{(0.33) \sqrt{3/4}} = -0.05589. \quad d_2 = d_1 - \sigma \sqrt{T} = -0.05589 - 0.33 \sqrt{3/4} = -0.34168.$$

$$\Phi[d_2] = \Phi[-0.34168] = \exp[-(-0.34168)^2/2] / \sqrt{2\pi} = 0.37632.$$  

$$\Phi[d_1] = 0.47771. \quad \Phi[d_2] = 0.36630.$$  

Theta = $\delta e^{-\delta T} S \Phi[d_1] - r K e^{-r T} \Phi[d_2] - K e^{-r T} \phi[d_2] \frac{\sigma}{2 \sqrt{T}} =$

$$(0.02) e^{-0.02(3/4)} (91) (0.47771) - (0.07)(100) e^{-0.07(3/4)} (0.36630) - (100) e^{-0.07(3/4)} (0.37632) \frac{0.33}{2 \sqrt{3/4}} = -8.38.$$  

Comment: Similar to Q. 34.38.
25. A. \[ d_1 = \frac{\ln(S/K) + (r - \delta + \sigma^2/2)T}{\sigma \sqrt{T}} = \]
\[ \ln(81/70) + (0.057 - 0.025 + 0.33^2/2)(2) \]
\[ \frac{0.33 \sqrt{2}}{0.68322} = 0.68322. \quad \Phi[0.68322] = 0.75277. \]
\[ \Delta_{\text{call}} = e^{-\delta T} \Phi[d_1] = \exp[-(0.025)(2)] (0.75277) = 0.71606. \]
\[ \Delta_{\text{put}} = -e^{-\delta T} \Phi[-d_1] = -\exp[-(0.025)(2)] (1 - 0.75277) = -0.23517. \]
Having sold 10,000 puts, his delta is: \[ (-10,000)(-0.23517) = 2351.7. \]
In order to get his delta to be zero, he needs to sell calls: \[ 2351.7/0.71606 = 3284.2. \]
Comment: Similar to Q. 42.21.

26. D. The relationship between the current price and forward price should be:
\[ (70)(1+r) - d(1+r)^{1/2} = 75. \]
This holds for \( r = 6\% \) and \( d = 2. \)
Comment: Similar to Q. 6.61 (MFE Sample Introductory Q.10).
27. D. This is a $ denominated call, and therefore dollars act as money and Bongo Bucks act as the asset. \( r = 4\% \). \( \delta = 7\% \).

\[
\Delta = e^{-\delta h} \frac{C_u - C_d}{S_0 (u - d)} = e^{-0.07/2} \frac{0.005 - 0}{0.07 - 0.05} = 0.241.
\]

\[
B = e^{-rh} \frac{u C_d - d C_u}{u - d} = e^{-0.04/2} \frac{(7/6)(0) - (5/6)(0.005)}{7/6 - 5/6} = -$0.01225.
\]

Therefore, the call premium should be: \((0.06)(0.241) - 0.01225 = 0.00221\).

Therefore, the call is overpriced at $0.0025.

We can create a synthetic call by replicating its cashflows; buy \( \Delta \) Bongo Bucks and sell bonds.

We want to buy a synthetic call and sell one of the real overpriced call.

Therefore, Leonardo can exploit the arbitrage opportunity by selling (shorting) a call, buying Bongo Bucks, and selling risk-free bonds (borrowing money.)

Alternately, \( p^* = \frac{\exp[(0.04 - 0.07)(1/2)] - 5/6}{7/6 - 5/6} = 0.4553 \).

Therefore, the call premium should be: \((0.005) e^{-0.04/2} (0.4553) = 0.00223\).

Therefore, the call is overpriced at $0.0025. Proceed as before.

Comment: Similar to Q.16.33 (MFE/3F, 5/09, Q.3).

If \( u \) is too large or \( d \) is too small, then there is arbitrage inherent in the binomial tree itself.

However, here \( d < \exp[(r-\delta)h] < u \). Even if the values of \( u \) and \( d \) are fine, as they are here, one can not pick the price of an option out of thin air as was done in this question. We were given enough information to price the option. If the option is selling for other than this fair market price, then there is an opportunity for arbitrage. (We are implicitly assuming that the model given in the question is correct.)

28. E. Using put call parity, \( C = P - Ke^{rT} + Se^{-\delta T} \).

\[
9.72 = 12.83 - 125 e^{r/2} + 120 e^{-0.015/2} \Rightarrow e^{r/2} = 122.21/125. \Rightarrow r = 4.5\%.
\]

Comment: Similar to Q. 14.7 (CAS3, 5/07, Q.13).
29. B. Straddle: Purchase a call and the otherwise identical put. In this case both have \( K = S_0 = 70 \).

\[
d_1 = \frac{\ln(S/K) + (r - \delta + \sigma^2/2)T}{\sigma \sqrt{T}} = \frac{\ln(1) + (0.05 - 0 + 0.3^2/2)(1)}{0.3 \sqrt{1}} = 0.31667.
\]

\[
d_2 = d_1 - \sigma \sqrt{T} = 0.31667 - (0.3) \sqrt{1} = 0.01667.
\]

\[
\Phi[0.31667] = 0.62425. \quad \Phi[0.01667] = 0.50665.
\]

Call premium is: \( e^{-\delta T} S \Phi[d_1] - K e^{-rT} \Phi[d_2] = \)

\[
\exp[-(0)(1)] (70) (0.62425) - (70) \exp[-(0.05)(1)] (0.50665) = 9.9617.
\]

Put premium is: \( K e^{-rT} \Phi[-d_2] - e^{-\delta T} S \Phi[-d_1] = \)

\[
= (70) \exp[-(0.05)(1)] (1 - 0.50665) - \exp[-(0)(1)] (70) (1 - 0.62425) = 6.5477.
\]


30. B. \( u = \exp[(5\% - 1.5\%)/3 + 0.3 \sqrt{1/3}] = 1.2031. \)

\[
d = \exp[(5\% - 1.5\%)/3 - 0.3 \sqrt{1/3}] = 0.8508.
\]

\[
p^* = \frac{\exp[(5\% - 1.5\%)/3] - 0.8508}{1.2031 - 0.8508} = 0.4568.
\]

\[
S_{uuu} = (90)(1.2031^3) = 156.7. \quad S_{duu} = (90)(0.8508)(1.2031^2) = 110.8.
\]

\[
S_{ddu} = (90)(0.8508^2)(1.2031) = 78.4. \quad S_{ddd} = (90)(0.8508^3) = 55.3.
\]

The Binomial probabilities are: 0.4568^3 = 0.0953, (3)(0.4568^2)(1 - 0.4568) = 0.3400, \( (3)(0.4568)(1 - 0.4568)^2 = 0.4044, (1 - 0.4568)^3 = 0.1603. \)

Discounting back in the risk-neutral environment, the premium of the call is:

\[
\frac{(156.7 - 100)(0.0953) + (110.8 - 100)(0.3400)}{\exp[0.05]} = 8.6.
\]

Comment: Similar to Q. 19.5.

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